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A class of initials-dependent dynamical systems

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ABSTRACT

Nonlinear term is critical for emergence of chaos in autonomous dynamical systems. The sampled time series in chaotic system are dependent on the initial selection of variables, while the attractors are invariant for fixed parameters. In this paper, the dynamical behavior of a class of dynamical system is investigated at fixed parameter region. It is found that the state selection is dependent on the initials and the potential mechanism is discussed. It is confirmed that the system can be switched between stable state, periodical state and even chaotic state by selecting appropriate initials even the parameters are fixed. We think that nonlinear cross terms with higher order could account for the emergence of this behavior. It indicates that initial selection and resetting can be also effective to control some chaotic systems, and these chaotic systems could enhance security for possible secure communication because the chaotic attractor depends on the parameter and initials selection as well. In the case of secure communication, the reconstruction of phase space becomes more difficult because the attractors are changed arbitrarily, thus the safety for secure keys is enhanced. For chaos control, when the initials are reset, the controller can be removed and the system can develop to step into the desired target by itself.

1. Introduction

Chaos is observed in chemical, physical and biological systems, and nonlinear analysis is helpful to understand the dynamical behavior and properties of sampled time series for observable variables from chaotic systems [1–12]. The topics about chaos, hyperchaos and spatiotemporal chaos have been investigated extensively [13,14]. The electrical activities in neuron also show chaotic properties and can be verified in neuron models by setting appropriate parameters and external forcing currents [15–18]. Some researchers prefer to find and design different chaotic, hyperchaotic circuits, dynamical models [19–21]. For example, Azzaz et al. [20] proposed an auto-switched chaotic system and its FPGA implementation was verified. Trejo-Guerra et al. [21] presented a review on the electronic design of chaotic oscillators, the integrated realizations were listed, and the key points for future research on the design of multi-scroll chaotic oscillators were discussed. It is believed that the brain normally works in a chaotic mode, while during attention it shows ordered behavior, as a result, Arama et al. [22] presented a novel model for human memory based on the chaotic dynamics of artificial neural networks. Complex dynamical behaviors are observed in nonlinear dynamical systems for economic models, Tacha et al. [23] presented a scheme adaptive control to regulate the finance system's behavior. Fractional chaotic systems seem to present more complex dynamical behavior, Xu et al. [24] dealed with a synchronization scheme for two fractional chaotic

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systems and its possible application on image encryption was discussed. Mata-Machuca [25] dealed with the synchronization and parameter estimations of an uncertain Rikitake system and its application in secure communications employing chaotic parameter modulation was also discussed. The gains of the receiver system were adjusted continually according to a convenient high order sliding-mode adaptive controller (HOSMAC), until the measurable output errors converged to zero. Ghosh and Chowdhury [26] introduced an adaptive learning rule for estimating all unknown parameters of delay dynamical system using a scalar time series, and Krasovskii–Lyapunov theory was used to derive sufficient condition for synchronization.

More researchers would like to explore more effective schemes to suppress chaos, parameter estimation and realize synchronization between chaotic systems [27–36]. Indeed, many chaotic dynamical systems have been set up for bifurcation analysis and synchronization control. Some researchers thought that fractional-order chaotic systems could be much interesting and more important to be worthy of investigation, for example, Zhou et al. [37,38] investigated the stabilization and synchronization on fractional-order chaotic systems with fractional-order $1 < q < 2$ by using adaptive scheme. It is known that chaotic systems are much dependent on the initials selection and the time series or orbits show much diversity even slight difference occurs in the initial values. However, the attractor and attracted basin could be invariable when the bifurcation parameters for the chaotic system are fixed. These known chaotic and hyperchaotic systems can generate finite attractors while some chaotic systems can produce infinite attractors under appropriate control scheme, for example, jerk circuit [39,40] can be controlled to generate a large number of attractors by applying periodical signal forcing [41]. The dynamical properties can also be discerned by estimating the Hamilton energy [42] on these dimensionless dynamical systems, and it is found that chaotic systems with multi-attractors [43] can hold smaller Hamilton energy and neurons under bursting state also hold smaller Hamilton energy as well. It has been confirmed that nonlinear term is important and necessary for nonlinear dynamical system so that chaotic state can be triggered under appropriate parameters selection. In fact, coupled oscillators and networks can present more complex spatiotemporal dynamics and it is important to explore effective schemes that spatiotemporal chaos can be suppressed and synchronization can be realized in complex network [44–46]. In practical verification, many realistic factors should be considered, for example, the effect of time delay, the control cost (power consumption of controller), as a result, intermittent schemes are used to reach this target. Under the framework of Filippov systems and a linear controller, the exponential synchronization and anti-synchronization criteria for memristor-based neural networks can be guaranteed by the matrix measure and Halanay inequality [47]. Mathiyalagan et al. [48] investigated the impulsive synchronization of memristor based bidirectional associative memory (BAM) neural networks with time varying delays. Then the impulsive time dependent results are derived for the exponential stability of the error system by using linear matrix inequality (LMI) approach.

Nonlinear circuits [49–53] are useful to investigate the chaotic problems, and many researchers thought chaotic circuits could be useful for secure communication and image encryption [54–57]. In fact, nonlinear devices such as negative resistance, negative conductor, negative capacitor are important devices for a setting a chaotic circuit. It is important to mention another important device, memristor [58,59], which the memductance is dependent on the external forcing current and thus it is initial-dependent. As a result, the memristor-coupled oscillators hold more complex dynamical behaviors. The circuit composed of memristor is dependent on the bifurcation parameter and also the initials selection [60]. For most of the well-known chaotic systems, the attractors and basin of attracts keep invariable when the parameters are fixed though the output time series can show some differences by setting different initial values for variables. Therefore, it is interesting to investigate these dynamical systems and its potential mechanism why the developed state also depends on the initial selection. In fact, the initial-independence is associated with memory, it is confirmed that memristor-coupled circuit or oscillator can be switched between different kinds of attractors by resetting the initials. Can we find an effective scheme to develop more chaotic systems which are dependent on initials selection and parameter setting? In this paper, we argue that initial dependence could be associated with nonlinear cubic terms composed of different variables, the potential mechanism on other chaotic systems are discussed. Firstly, the memristor-coupled oscillator is used for preliminary discussion. Secondly, the Rössler model is improved by adding quadratic term into the dynamical system, and the spectrum of Lyapunov exponents, phase portrait and basin of attractor are calculated to discern the dependence of initials selection on attractors.

2. Model, scheme and discussion

It is known that the memductance for memristor is dependent on the initial inputs, as a result, the nonlinear circuits or systems composed of memristor could be dependent on the initial selection for magnetic flux across the memristor. At first, we investigate the dynamics for a class of circuit composed memristor and then explore the potential mechanism for this initial-dependent property on other nonlinear systems so that desired nonlinear circuits can be set up for possible application for secure communication and control. The memristor-coupled circuit [61–63] can be illustrated in Fig. 1. The outputs of circuit can be described by nonlinear equations according to Kirchhoff's law as follows

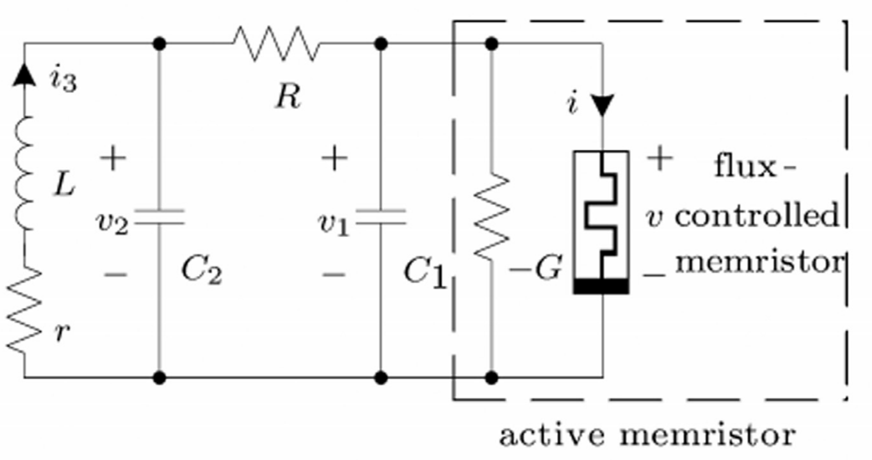

Fig. 1. The schemematic diagram for memristor-coupled circuit, which is improved from the Chua circuit by using a flux-controlled memristor. The memductance is defined by $W(\varphi) = a\varphi + b\varphi^3$.

$$\begin{cases} \dfrac{dv_1}{dt} = \dfrac{1}{RC_1}[v_2 - v_1 + GRv_1 - RW(\varphi)v_1] \\ \dfrac{dv_2}{dt} = \dfrac{1}{RC_2}(v_1 - v_2 + Ri_3) \\ \dfrac{dv_3}{dt} = -\dfrac{1}{L}(v_2 + ri_3) \\ \dfrac{d\phi}{dt} = v_1 \end{cases} \tag{1}$$

where the flux-controlled memristor is defined by $W(\varphi) = a\varphi + b\varphi^3$, and φ is magnetic flux. Furthermore, the circure equations can be mapped into dimensionless dynamical equations after scale transformation by setting $x = v_1$, $y = v_2$, $z = i_3$, $w = \varphi$, $\alpha = 1/C_1$, $\beta = 1/L$, $\gamma = R/L$, $\xi = G$, $C_2 = R = 1$, and a time scale factor k are used reproduce a dimensionless dynamical equations as follows

$$\begin{cases} \dfrac{dx}{dt} = k\alpha(y - x + \xi x - W(w)x) \\ \dfrac{dy}{dt} = k(x - y + z) \\ \dfrac{dz}{dt} = -k(\beta y + \gamma z) \\ \dfrac{dw}{dt} = kx \end{cases} \tag{2}$$

For simplicity, the time scale factor is set $k = 1$, and other parameters are selected as $\alpha = 10$, $\beta = 14$, $\gamma = 0.1$, $\xi = 2.2$, $W(w) = 1 + 3w^2$. The basin and size of attractors depend on the maximal value of variables, for simplicity, the statistical function for the sum of variables are defined by

$$\theta(x, y, z) = (x^2 + y^2 + z^2)_{Max} \tag{3}$$

The developed state depends on the parameter selection but also the initials selection for variable w (or φ), and thus it is very important for the emergence and transition of attractors for the dynamical system. In Fig. 2, the maximal function for Eq. (3) is estimated at fixed initials as $x_0 = 0.0$, $y_0 = 0.001$,$z_0 = 0.0$, w_0 is the initials for variable w mapped from magnetic flux.

To our knowledge, the phase space should be large enough so that complete attractors can be formed, as mentioned in Refs. [64,65], phase compression could be effective to control chaos and spatiotemporal chaos by resetting the boundary of attractors thus appropriate periodical orbits can be selected. As a result, larger values for $\theta(x, y, z)$ in Fig. 2 mean that chaotic attractors can be induced under appropriate initials selection for w, otherwise, periodical attractor with finite basin or size can be found. Extensive numerical studies calculated the Lyapnuov exponents under different initials, and the results are consistent with the chaos occurrence. To present readable illustration for this problem, some phase portraits are plotted by setting different initials on variable w, and the results are plotted in Figs. 3–5.

It is confirmed in Fig. 4 that different chaotic attractors can be generated under appropriate initials selection which also induces different largest Lyapunov exponents. It is also interesting to find appropriate initials so that the dynamical system can be stabilized, and the results are plotted in Fig. 5.

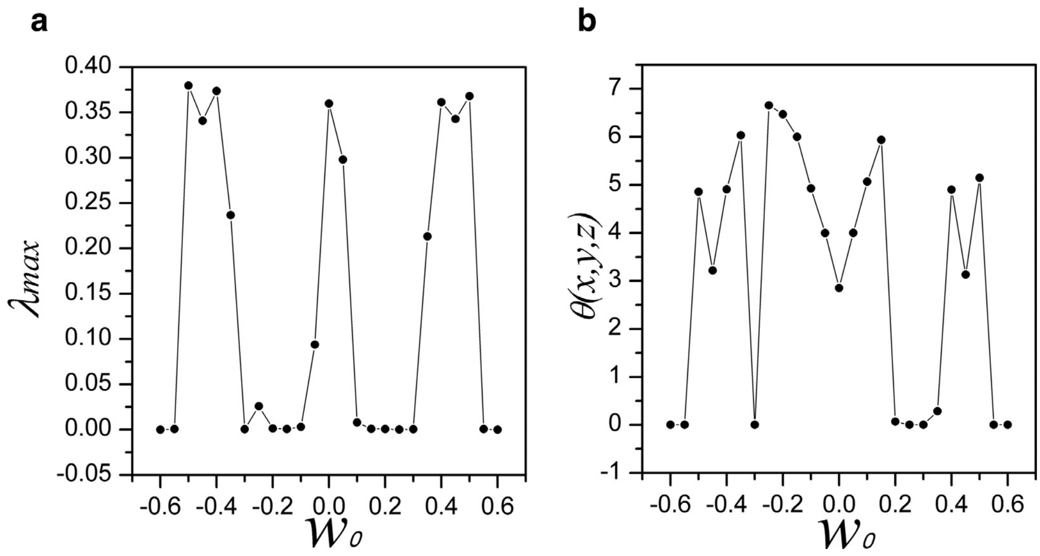

Fig. 2. The largest Lyapunov exponent (a) and maximal function $\theta(x, y, z)$ (b) is respectively calculated under different initials w_0. The other initials are selected as $x_0 = 0.0$, $y_0 = 0.001$, $z_0 = 0.0$.

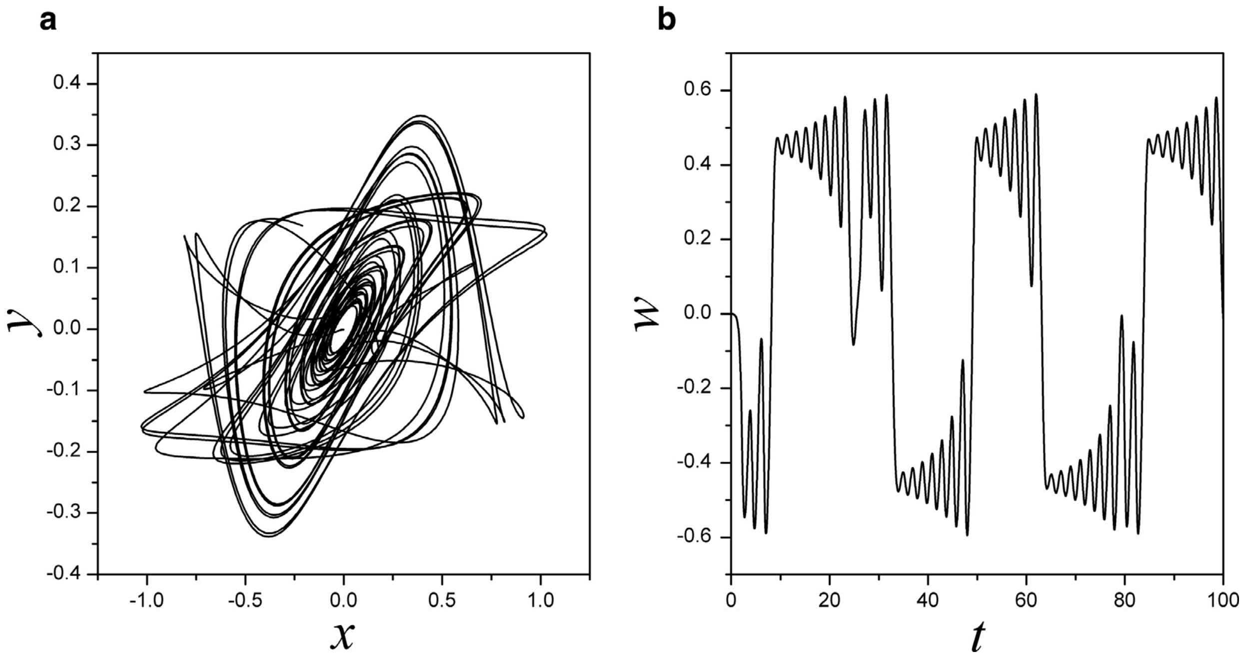

Fig. 3. The phase portrait (a) and time series for variable W(b) are calculated at fixed $w_0 = 0.0$.

It is found in Fig. 5 that limit circle and stable state can be selected under appropriate initials even the parameters were kept the same. It is interesting to check the possible generality in other nonlinear oscillators or systems. In this way, the well-known Rössler model is checked in the next section. The above mentioned results are carried on the oscillator composed of memristor, it is interesting to explore this problem on another chaotic system. For simplicity, further investigation is carried on the Rössler model, and the dynamical equations are described by

$$\begin{cases} \frac{dx}{dt} = -y - z \\ \frac{dy}{dt} = x + ay \\ \frac{dz}{dt} = b + z(x - c) \end{cases} \tag{4}$$

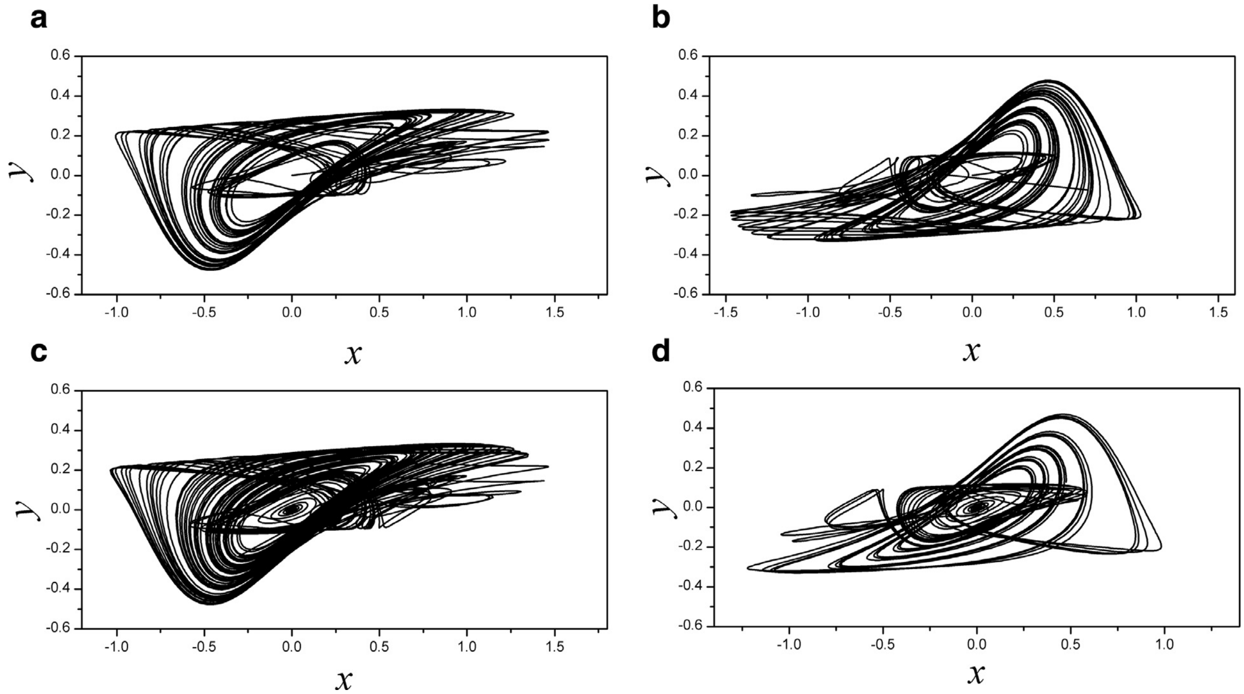

Fig. 4. The phase portrait are calculated at initial values for $w_0 = -0.1$(a), $w_0 = 0.1$(b), $w_0 = -0.4$(c), $w_0 = 0.4$(d).

where a, b, c are parameters, the Eq. (4) can exist chaos at fixed parameters as $a = 0.2$, $b = 0.2$, $c = 0.8$. This chaotic system holds simple form with only one nonlinear term xz being included. To generate a initials-dependent system, the nonlinear term is changed by replacing the variable z in the nonlinear term $z(x - c)$ with quadratic nonlinearities z^2, and it reads as follows

$$\begin{cases} \frac{dx}{dt} = -y - z \\ \frac{dy}{dt} = x + ay \\ \frac{dz}{dt} = b + z^2(x - c) \end{cases} \tag{5}$$

Similar the same discussion for Eq. (2), the largest Lyapunov exponent and statistical function $\theta(x, y, z)$ are calculated under different initials z_0, and the results are plotted in Fig. 6.

It is found that the maximal phase size begins to increase monotonously with increasing the initials for variable z and the decreased from a peak. The maximal Lyapunov exponent is small to zero for most of the initials for variable z, which means that periodic state can be induced under appropriate initials for variable z, extensive numerical studies are carried out and phase portraits are detected in Fig. 7.

The results in Fig. 7 confirmed that different periodic states can be approached under this group of parameters even the initials are changed. It is important to check another group of parameters and find the attractor dependence on initials. For simplicity, the parameters are selected as $a = 0.2$, $b = 0.2$, $c = 2.3$, and the distribution for the maximal Lyapunov exponent and maximal statistical function $\theta(x,y,z)$ are calculated in Fig. 8, furthermore, phase portraits are plotted in Fig. 9.

It is found that periodic state or chaotic state can be selected by applying appropriate initials for variable z. According to the distribution for the largest Lyapunov exponent in Figs. 6(a) and 8(a), it is found that the largest Lyapunov exponent is much small and close to zero with an order 10^{-3}, therefore, most of the initials could be effective to make periodic attractor be dependent on the initials selection. Indeed, the other parameters can be selected appropriately so that the chaotic attractor can be dependent on the initial selection as well. That is to say, under appropriate parameter setting, the improved Rössler system is kept chaos but the chaotic attractor could be different when different initials are used. As a result, initial selection can be used to switch between different chaotic attractors.

It is interesting to discuss the potential mechanism for the state dependence on initials selection though most of us ever believed that chaotic attractors should depend on the parameter selection. In fact, the conductance for memristor $W(\varphi)$ (or $W(w)$) is quadratic nonlinearity could be regarded as a time-varying parameter, the initial selection for φ_0 (or w_0) just triggers the oscillating of the system and then parameter switch occurs for the next time units and bifurcation could

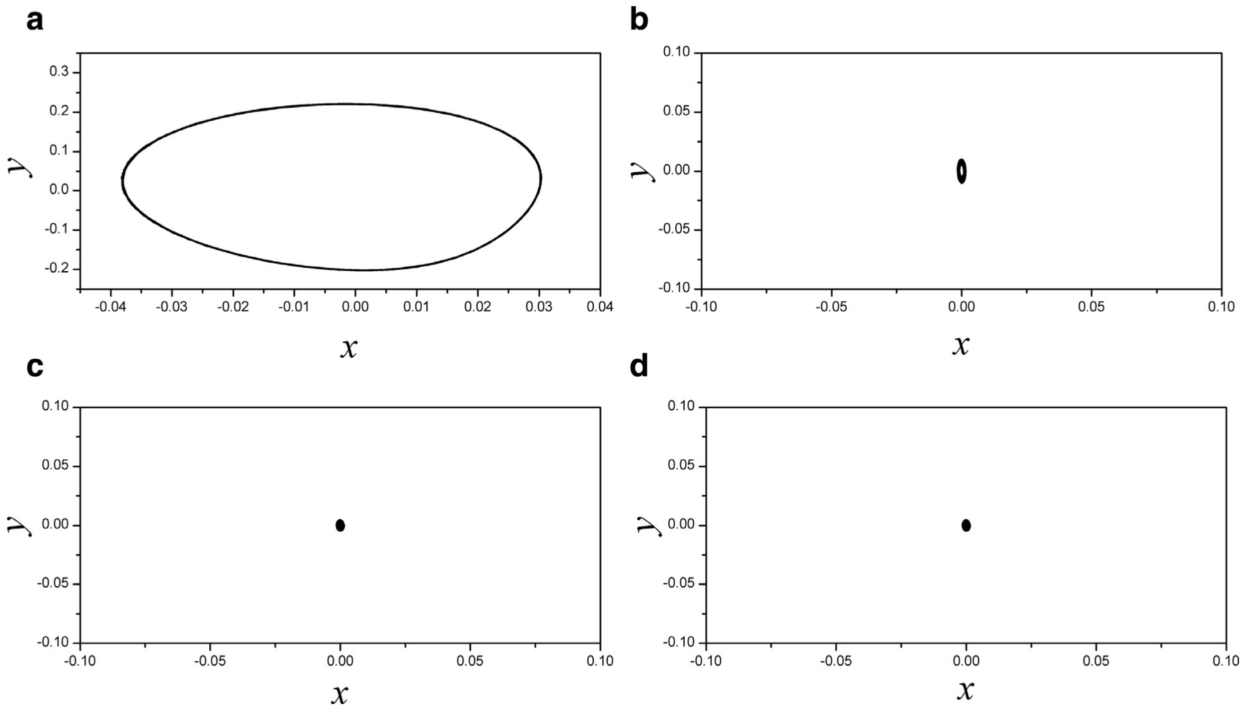

Fig. 5. The phase portrait are calculated at initial values for $w_0 = 2.0$(a), $w_0 = 3.0$(b), $w_0 = -6.0$(c), $w_0 = 6.0$(d).

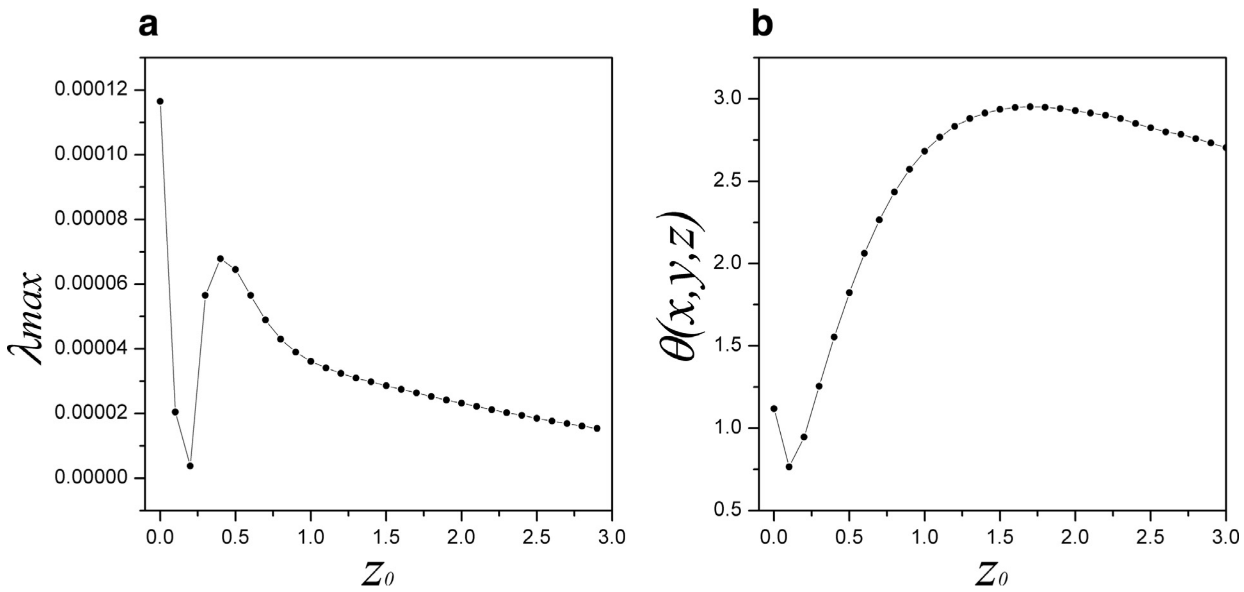

Fig. 6. The largest Lyapunov exponent (a) and maximal function $\theta(x, y, z)$ (b) is respectively calculated under different initials z_0. The other initials are selected as $x_0 = 0.0$, $y_0 = 0.0$, and parameters are given with $a = 0.2$, $b = 0.2$, $c = 0.8$.

be induced under appropriate parameter region. In the case of improved Rössler system, the nonlinear term z^2 plays as sensitive and time-varying parameter, which can switch to another value after the oscillator is cheered up. As a result, we believe that quadratic nonlinearity in nonlinear terms could account for the potential formation mechanism that attractors could be dependent on the initials selection. It is worthy of investigating this scheme on Rössler model by applying other

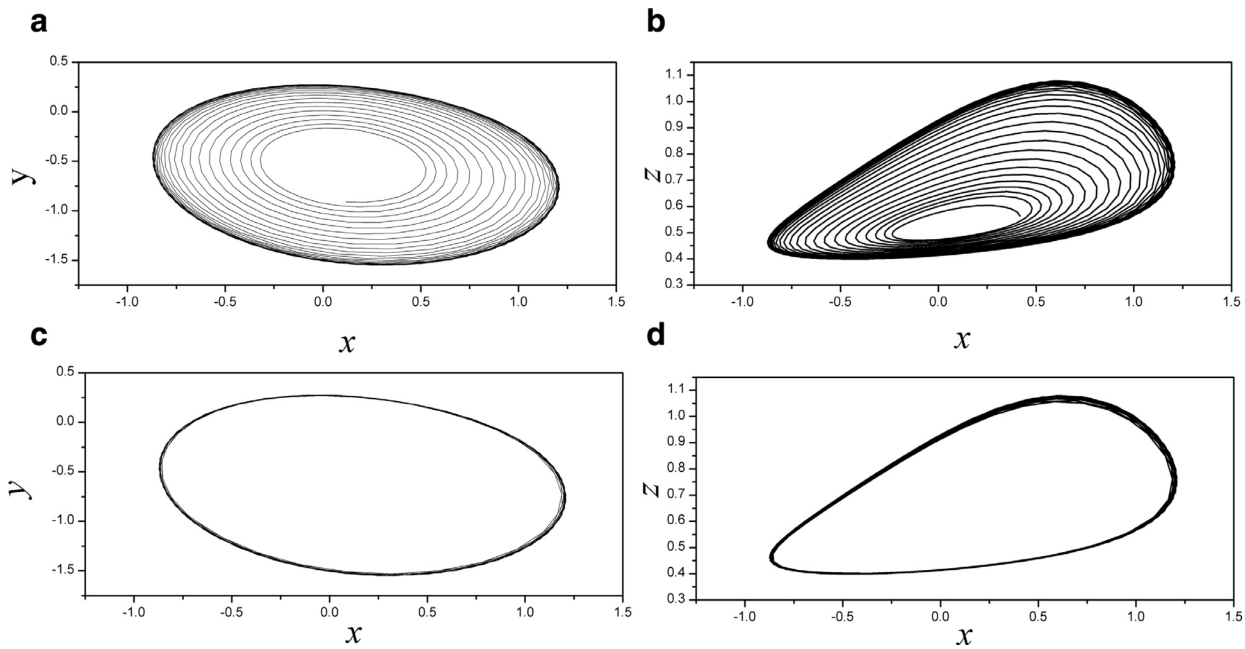

Fig. 7. The phase portrait are calculated at initial values for $z_0 = 0.0$(a), $z_0 = 0.0$(b), $z_0 = 2.0$(c), $z_0 = 2.0$(d).

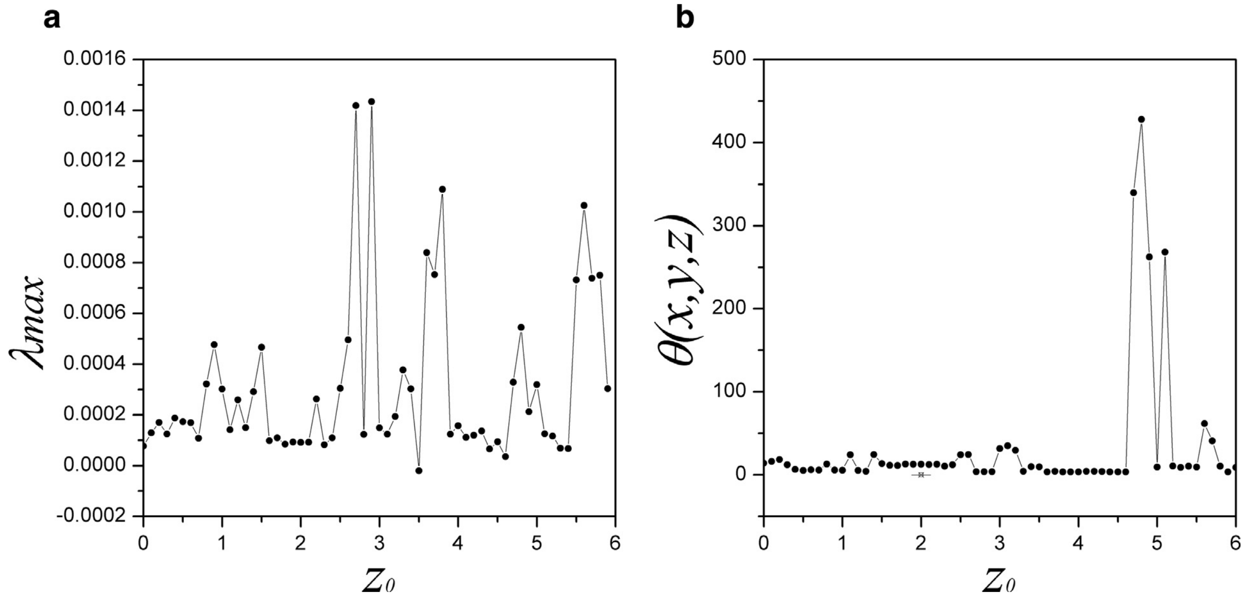

Fig. 8. The maximal Lyapunov exponent (a) and maximal function $\theta(x, y, z)$ (b) is respectively calculated under different initials z_0. The other initials are selected as $x_0 = 0.0$, $y_0 = 0.0$, and parameters are given with $a = 0.2$, $b = 0.2$, $c = 2.3$.

type of quadratic nonlinearity and the improved model is described by

$$\begin{cases} \frac{dx}{dt} = -y - z - kz^2x \\ \frac{dy}{dt} = x + ay \\ \frac{dz}{dt} = b + z(x - c) \end{cases} \tag{6}$$

where k is control parameter, so that the attractors could be selected from periodic, chaotic type if possible. In Fig. 10, the distribution for largest Lyapunov exponent is calculated under different initials at fixed control parameter k.

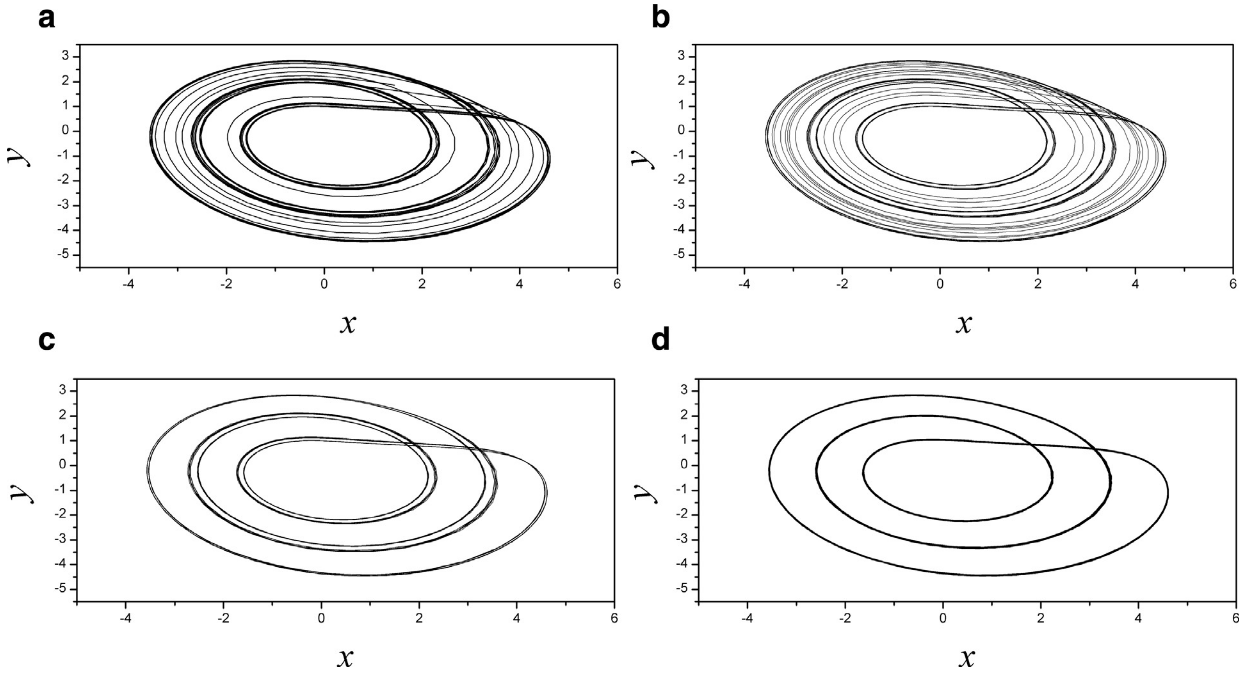

Fig. 9. The phase portrait are calculated at initial values for $z_0 = 1.0$(a), $z_0 = 4.8$(b), $z_0 = 2.0$(c), $z_0 = 4.0$(d).

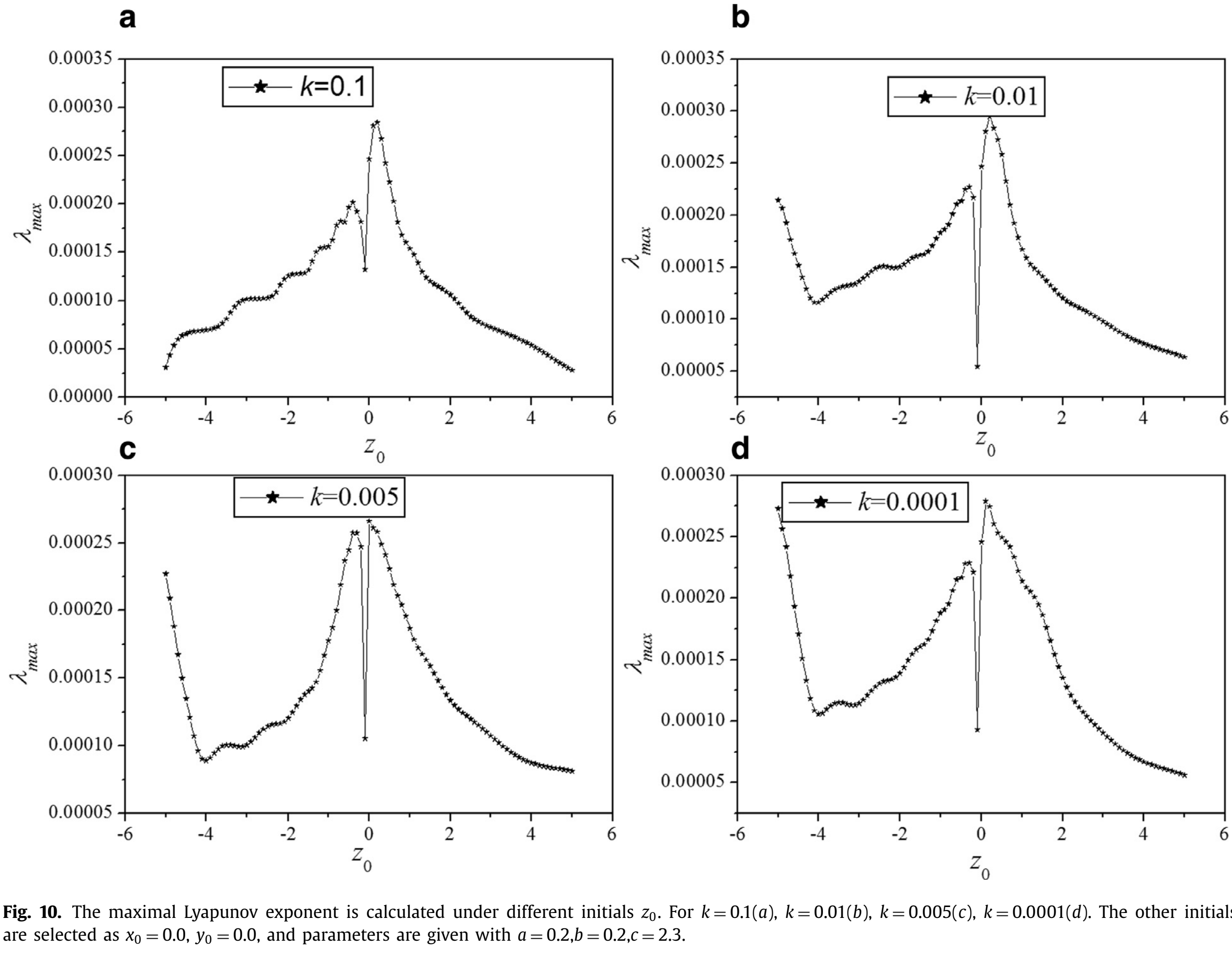

Fig. 10. The maximal Lyapunov exponent is calculated under different initials z_0. For $k = 0.1(a)$, $k = 0.01(b)$, $k = 0.005(c)$, $k = 0.0001(d)$. The other initials are selected as $x_0 = 0.0$, $y_0 = 0.0$, and parameters are given with $a = 0.2$,$b = 0.2$,$c = 2.3$.

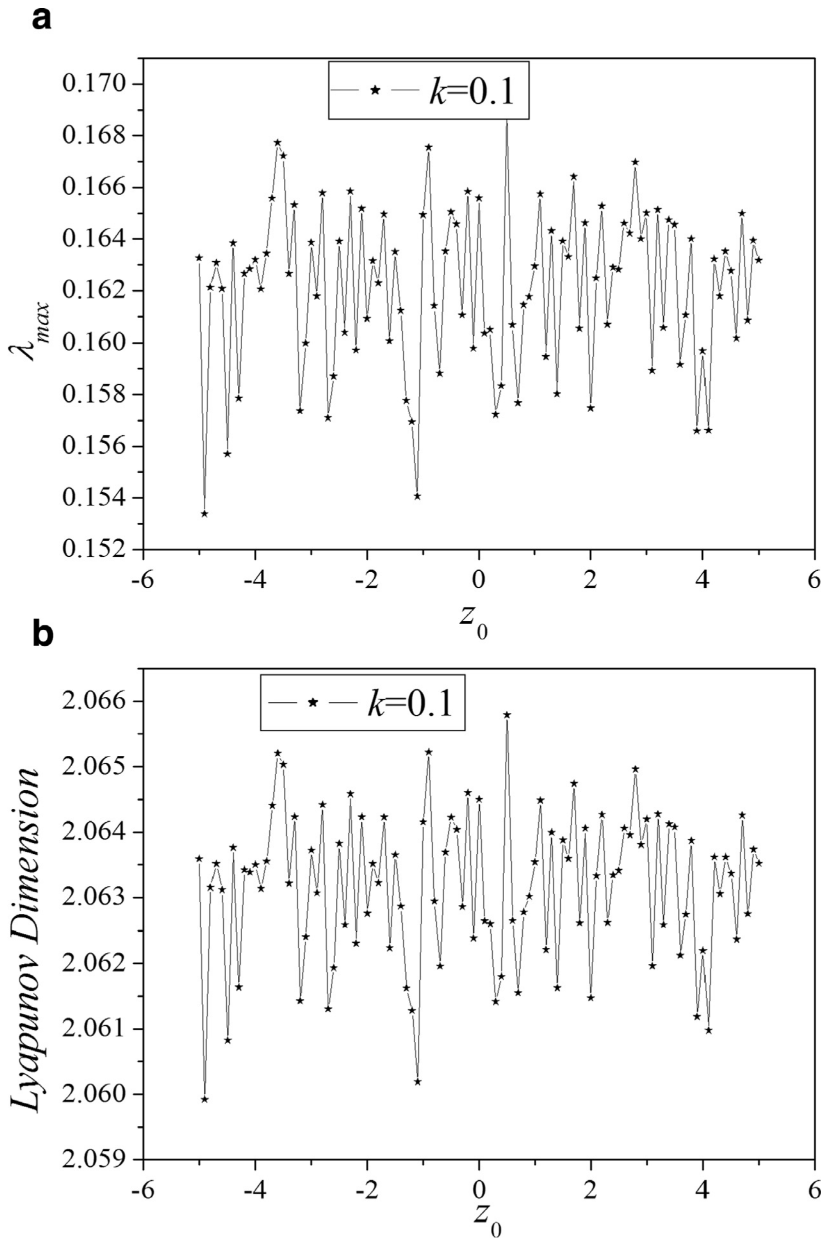

Fig. 11. The maximal Lyapunov exponent (a) and Lyapunov dimension (b) is respectively calculated under different initials z_0. The other initials are selected as $x_0 = 0.0$, $y_0 = 0.0$, and parameters are given with $a = 0.4, b = 0.2, c = 2.3, k = 0.1$.

The results in Fig. 10 found that the largest Lyapunov exponent is beyond zero but it is also close to zero greatly and the York dimension is close to 1, which indicates the chaotic state is much weak and possible periodic state is approached. The Lyapnov exponent can reach a peak in the curve when the initial value z_0 is set zero, in this case, initial selection can trigger a chaotic orbit in the begging. Extensive numerical results confirmed that Lyapunov exponent is switched to negative value by further increasing the initial value z_0. Furthermore, chaotic state can be enhanced by adjusting another parameter, for example, $a = 0.4$, and the distribution for largest Lyapunov exponents is calculated in Fig. 11.

It is found in Fig. 11 that different largest Lyapunov exponents can be approached under appropriate initials selection, and the chaotic attractors show some difference as well. As a result, the chaotic attractors and basin can be switched by applying different initials for the third variable. Similar results can be confirmed by adding nonlinearity as $-kz^2y$ on the second variable under appropriate parameter selection. In fact, this scheme can be extended to construct more initial-dependent dynamical system, for example, a nonlinear term as kx^2y, kx^2z can make the system is dependent on the initials selection x_0, a nonlinear term as ky^2x, ky^2z can create a dynamical system to be dependent on the initial selection y_0.

3. Open problems and further investigation

There is some difference between the dissipative system (e.g., memristor coupled oscillator) and emanative system (e.g., Rössler oscillator) when an initial-dependent system is designed. For the emanative system, initials selection can produce different periodic states or stable states or different chaotic states (state region is not continuous), while dissipative system can select different periodic, chaotic, and even stable states (state region is continuous). As a result, these improved dynamical system with higher nonlinearity (quadratic terms multiply other variable) makes the system be dependent on the parameter region and the initial selection as well, it could be helpful to enhance the security in communication because the attack could be blocked. For chaos control, we can switch the initial-dependent variable to certain value thus chaotic state can be suppressed and chaotic attractors can be switched if necessary. It is interesting to clarify the realization on general dimensionless dynamical systems presented with the following dynamical equations

$$\begin{cases} \dfrac{dx_1}{dt} = f_1(x_1, x_2, x_3, ...) \\ \dfrac{dx_2}{dt} = f_2(x_1, x_2, x_3, \ldots) \\ \dfrac{dx_3}{dt} = f_3(x_1, x_2, x_3, \ldots) \\ \vdots \\ \dfrac{dx_i}{dt} = f_i(x_1, x_2, x_3, \ldots) \end{cases} \tag{7}$$

where x_1, x_2,..., x_i represents the variable and this dimensionless system can be mapped into nonlinear circuit with same order. These observable variables can be detected from the equivalent circuit, surely, memristor and capacitor will be used. Most of the variables will be described by the outputs for voltages from the capacitors via scale transformation. For example, $x_1 = V_1/V_0$, $x_1 = V_2 / V_0$, $x_3 = V_3 / V_0$,..., $x_\mathrm{i} = \varphi/\varphi_0$, $t = \tau / (LC)^{1/2}$, where L,C is the inductance, capacitance value, φ_0,V_0 is standard unit for magnetic flux and voltage, respectively. To produce a dynamical system dependent on the variable x_i, a nonlinear term $kx_i^2x_j$ is formed, where x_j is the output variable from the system ($i \neq j$), and k is the feedback gain. When the variable x_i is controlled by a memristor, the dynamical system will be dependent on the initials selection by adding the nonlinear term $kx_i^2x_j$ on any formula of the systems and setting appropriate feedback gain. Furthermore, it is interesting to discuss potential application for this initial-dependent chaotic system, for example, it is suggested that this system is dependent on the ith variable. As a result, intermittent disturbance on the ith variable by applying feedback in the following form

$$\begin{cases} \dfrac{dx_1}{dt} = f_1(x_1, x_2, x_3, ...) \\ \dfrac{dx_2}{dt} = f_2(x_1, x_2, x_3, \ldots) \\ \dfrac{dx_3}{dt} = f_3(x_1, x_2, x_3, \ldots) - kx_i^2x_3 \\ \vdots \\ \dfrac{dx_i}{dt} = f_i(x_1, x_2, x_3, \ldots) + k_1(S_{random} - x_i) \end{cases} \tag{8}$$

where k_1 is feedback coefficient, S_{random} is stochastic or random values. By applying feedback coefficient k_1, the ith variable will be reset for new values to trigger a new phase portrait thus the reconstruction of phase space become more difficult. As a result, the outputs from the system (8) can be used for generating secure keys and masked waves. On the other hand, carefully selection for S_{random} with intermittent period can also control the system to reach arbitrary orbit. When the initials are reset, the controller can be removed and the system can develop to step into the desired target in its own way. In the case of network, the initial-dependent oscillators can be used to describe local kinetics for the node of the network. As a result, the network becomes dependent on the initial selection. For example, readers can design a chain network, regular network or small-world network for patter selection and control. Due to its initial-dependent properties, multi-channels inputs on the network will be helpful to check the stability of pattern and memory robustness of network. The authors of this paper will feel much pleasure to find more extensive works can be carried out on these initial-dependent dynamical systems.

4. Conclusions

In this paper, the state dependence on initials selection in dynamical system is investigated. The distribution for largest Lyapunov exponent and maximal phase size for attractors is calculated respectively by changing the initials under fixed parameter region. It is found in the oscillator composed memristor and improved Rössler oscillator, which quadratic nonlinearity z^2 is used for generating new nonlinear terms, periodic and chaotic state can be controlled and selected by applying appropriate initials. The potential mechanism could be that quadratic nonlinearity plays as more sensitive bifurcation parameter, and bifurcation can be induced during the switch of time-varying parameters. Furthermore, intermittent switch for one variable (initial-dependent) is used to control the nonlinear dynamical system, and the switch between periodical and chaotic states can be selected arbitrarily. This property could be useful to design some chaotic systems for possible secure communication thus the safety can be enhanced because there construction of phase space becomes more difficult.

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References

[1] F. Takens, Detecting strange attractor in turbulence, Mathematics 898 (1981) 366–381.
[2] L.O. Chua, G.N. Lin, Canonical realization of Chua Circuit family, IEEE Trans. Circ. Sys. 37 (1990) 885–902.
[3] G.A. Gottwald, I. Melbourne, Testing for chaos in deterministic systems with noise, Physica D 212 (2005) 100–110.
[4] G.A. Leonov, N.V. Kuznetsov, V.I. Vagaitsev, Hidden attractor in smooth Chua systems, Physica D 241 (2012) 1482–1486.
[5] J.P. Crutchfield, Between order and chaos, Nat. Phys. 8 (2012) 17–24.
[6] Y.X. Hao, L.H. Chen, W. Zhang, et al., Nonlinear oscillations, bifurcations and chaos of functionally graded materials plate, J. Sound Vib. 312 (2008) 862–892.
[7] D.X. Tran, D. Sato, A. Yochelis, et al., Bifurcation and chaos in a model of cardiac early after depolarizations, Phys. Rev. Lett. 102 (2009) 258103.
[8] G.A. Gottwald, I. Melbourne, On the implementation of the 0-1 test for chaos, SIAM J. App. Dyn. Syst. 8 (2009) 129–145.
[9] R.B. Karabalin, M.C. Cross, M.L. Roukes, Nonlinear dynamics and chaos in two coupled nanomechanical resonators, Phys. Rev. B 79 (2009) 165309.
[10] K.J. Painter, T. Hillen, Spatio-temporal chaos in a chemotaxis model, Physica D 240 (2011) 363–375.
[11] G.C. Wu, D. Baleanu, Discrete fractional logistic map and its chaos, Nonlinear Dyn. 75 (2014) 283–287.
[12] M. Peil, M. Jacquot, Y.K. Chembo, et al., Routes to chaos and multiple time scale dynamics in broadband bandpass nonlinear delay electro-optic oscillators, Phys. Rev. E 79 (2009) 026208.
[13] S. Boccaletti, C. Grebogi, Y.C. Lai, H. Mancini, D. Maza, The control of chaos: theory and applications, Phys. Rep. 329 (2000) 103–197.
[14] S. Boccaletti, J. Kurths, G. Osipov, D.L. Valladares, C.S. Zhou, The synchronization of chaotic systems, Phys. Rep. 366 (2002) 1–101.
[15] H. Hsagawa, Responses of a Hodgkin–Huxley neuron to various types of spike-train inputs, Phys. Rev. E 61 (2000) 718726.
[16] H.G. Gu, B.B. Pan, G.R. Chen, et al., Biological experimental demonstration of bifurcations from bursting to spiking predicted by theoretical models, Nonlinear Dyn. 78 (2014) 391–407.
[17] H.G. Gu, S.G. Chen, Potassium-induced bifurcations and chaos of firing patterns observed from biological experiment on a neural pace-maker, Sci. China Technol. Sci. 57 (2014) 854–871.
[18] H.G. Gu, B.B. Pan, A four-dimensional neuronal model to describe the complex nonlinear dynamics observed in the firing patterns of a sciatic nerve chronic constriction injury model, Nonlinear Dyn. 81 (2015) 2107–2126.
[19] P. Zhou, K. Huang, A new 4-D non-equilibrium fractional-order chaotic system and its circuit implementation, Commun. Nonlin. Sci. Numer. Simul. 19 (2014) 2005–2011.
[20] M.S. Azzaz, C. Tanougast, S. Sadoudi, et al., A new auto-switched chaotic system and its FPGA implementation, Commun. Nonlin. Sci. Numer. Simul. 18 (2013) 1792–1804.
[21] R. Trejo-Guerra, E. Tlelo-Cuautle, V.H. Carbajal-Gómez, et al., A survey on the integrated design of chaotic oscillators, Appl. Math. Comput. 219 (2013) 5113–5122.
[22] Z. Arama, S. Jafaria, J. Ma, et al., Using chaotic artificial neural networks to model memory in the brain, Commun. Nonlin. Sci. Numer. Simulat. 44 (2016) 449–459.
[23] O.I. Tacha, C.K. Volos, I.M. Kyprianidis, et al., Analysis, adaptive control and circuit simulation of a novel nonlinear finance system, Appl. Math. Comput. 276 (2016) 200–217.
[24] Y. Xu, H. Wang, Y.G. Li, et al., Image encryption based on synchronization of fractional chaotic systems, Commun. Nonlin. Sci. Numer. Simul. 19 (2014) 3735–3744.
[25] J.L. Mata-Machuca, R. Martinez-Guerra, R. Aguilar-Lpezb, et al., A chaotic system in synchronization and secure communications, Commun. Nonlin. Sci. Numer. Simul. 17 (2012) 1706–1713.
[26] D. Ghosh, A.R. Chowdhury, Parameter estimation of delay dynamical system from a scalar time series under external noise, Appl. Math. Comput. 216 (2010) 2069–2076.
[27] C.N Wang, Y.J. He, J. Ma, et al., Parameters estimation, mixed synchronization, and antisynchronization in chaotic systems, Complexity 20 (2014) 64–73.
[28] C.N. Wang, R.T. Chu, J. Ma, Controlling a chaotic resonator by means of dynamic track control, Complexity 21 (2015) 370–378.
[29] J. Ma, A.H. Zhang, Y.F. Xia, et al., Optimize design of adaptive synchronization controllers and parameter observers in different hyper-chaotic systems, Appl. Math. Comput. 215 (2010) 3318–3326.
[30] J. Ma, F. Li, L. Huang, et al., Complete synchronization, phase synchronization and parameters estimation in a realistic chaotic system, Commun. Nonlin. Sci. Numer. Simul. 16 (2011) 3770–3785.
[31] J.D. Cao, R. Sivasamy, R. Rakkiyappan, Sampled-Data H-infinity synchronization of chaotic Lur'e systems with time delay, Circ. Syst. Sign. Proc. 35 (2016) 811–835.
[32] B. Nana, P. Woafo, Chaotic masking of communication in an emitter-relay-receiver electronic setup, Nonlinear Dyn. 82 (2015) 899–908.
[33] R. Rakkiyappan, R. Sivasamy, X.D. Li, Synchronization of identical and nonidentical memristor-based chaotic systems via active backstepping control technique, Circ. Syst. Sign. Proc. 34 (2015) 763–778.
[34] A. Khanzadeh, M. Pourgholi, A novel continuous time-varying sliding mode controller for robustly synchronizing non-identical fractional-order chaotic systems precisely at any arbitrary pre-specified time, Nonlinear Dyn. 86 (2016) 543–558.
[35] A. Soukkou, A. Boukabou, S. Leulmi, Prediction-based feedback control and synchronization algorithm of fractional-order chaotic systems, Nonlinear Dyn. 85 (2016) 2183–2206.
[36] H. Handa, B.B. Sharma, Synchronization of a set of coupled chaotic FitzHugh–Nagumo and Hindmarsh–Rose neurons with external electrical stimulation, Nonlinear Dyn. 85 (2016) 1517–1532.
[37] P. Zhou, R.J. Bai, J.M. Zheng, Stabilization of a fractional-order chaotic brushless DC motor via a single input, Nonlinear Dyn. 82 (2015) 519–525.
[38] P. Zhou, R.J. Bai, The adaptive synchronization of fractional-order chaotic system with fractional-order $1 < q < 2$ via linear parameter update law, Nonlinear Dyn. 80 (2015) 753–765.
[39] M.E. Yalcin, Multi-scroll and hypercube attractors from a general jerk circuit using Josephson junctions, Chaos Solitons Fractals 34 (2007) 1659–1666.
[40] J. Ma, X.Y. Wu, R.T. Chu, et al., Selection of multi-scroll attractors in Jerk circuits and their verification using Pspice, Nonlinear Dyn. 76 (2014) 1951–1962.
[41] W.K.S. Tang, G.Q. Zhong, G.R. Chen, Generation of n-scroll attractors via sine function, IEEE Trans. Circ. Syst. I 48 (2001) 1369–1372.

[42] C. Sarasola, F.J. Torrealdea, A. d'Anjou, et al., Energy balance in feedback synchronization of chaotic systems, Phys. Rev. E 69 (2004) 011606.
[43] F. Li, C.G. Yao, The infinite-scroll attractor and energy transition in chaotic circuit, Nonlinear Dyn. 84 (2016) 2305–2315.
[44] A. Chandrasekar, R. Rakkiyappan, F.A. Rihan, S. Lakshmanan, Exponential synchronization of Markovian jumping neural networks with partly unknown transition probabilities via stochastic sampled-data control, Neurocomput 133 (2014) 385–398.
[45] A. Chandrasekar, R. Rakkiyappan, J.D. Cao, Impulsive synchronization of Markovian jumping randomly coupled neural networks with partly unknown transition probabilities via multiple integral approach, Neural Netw. 70 (2014) 27–38.
[46] A. Chandrasekar, R. Rakkiyappan, J.D. Cao, S. Lakshmanan, Synchronization of memristor-based recurrent neural networks with two delay components based on second-order reciprocally convex approach, Neural Netw. 57 (2014) 79–93.
[47] H.B. Bao, J.H. Park, J.D. Cao, Matrix measure strategies for exponential synchronization and anti-synchronization of memristor-based neural networks with time-varying delays, Appl. Math. Comput. 270 (2015) 543–556.
[48] K. Mathiyalagan, J.H. Park, R. Sakthivel, Synchronization for delayed memristive BAM neural networks using impulsive control with random nonlinearities, Appl. Math. Comput. 259 (2015) 967–979.
[49] C. Sanchez-López, R. Trejo-Guerra, J.M. Munoz-Pacheco, N-scroll chaotic attractors from saturated function series employing CCII+s, Nonlinear Dyn. 61 (2010) 331–341.
[50] L. Zhou, C.H. Wang, L.L. Zhou, Generating hyperchaotic multi-wing attractor in a 4D memristive circuit, Nonlinear Dyn. 85 (2016) 2653–2663.
[51] T.Q. Luo, Z. Wang, Dynamics and SC-CNN circuit implementation of a periodically forced non-smooth mechanical system, Nonlinear Dyn. 85 (2016) 87–96.
[52] M. Kountchou, P. Louodop, S. Bowong, et al., Analog circuit design and optimal synchronization of a modified Rayleigh system, Nonlinear Dyn. 85 (2016) 399–414.
[53] X.Y. Hu, C.X. Liu, L. Liu, et al., An electronic implementation for Morris–Lecar neuron model, Nonlinear Dyn. 84 (2016) 2317–2332.
[54] O. Mirzaei, M. Yaghoobi, H. Irani, A new image encryption method: parallel sub-image encryption with hyper chaos, Nonlinear Dyn. 67 (2012) 557–566.
[55] X.J. Tong, M. Zhang, Z. Wang, et al., A joint color image encryption and compression scheme based on hyper-chaotic system, Nonlinear Dyn. 84 (2016) 2333–2356.
[56] S. Zhang, T.G. Gao, A coding and substitution frame based on hyper-chaotic systems for secure communication, Nonlinear Dyn. 84 (2016) 833–849.
[57] M.F. Hassan, Synchronization of uncertain constrained hyperchaotic systems and chaos-based secure communications via a novel de-composed nonlinear stochastic estimator, Nonlinear Dyn. 83 (2016) 2183–2211.
[58] L.O. Chua, Memristor-missing circuit element, IEEE Trans. Circ. Theor. CT-18 (1971) 507.
[59] F. Corinto, A. Ascoli, M. Gilli, Nonlinear dynamics of memristor oscillators, IEEE Trans. Circ. Syst. I 58 (2011) 1323–1336.
[60] C.K. Volos, I.M. Kyprianidis, I.N. Stouboulos, et al., Memristor: a new concept in synchronization of coupled neuromorphic circuits, J. Eng. Sci. Technol. Rev. 8 (2015) 157–173.
[61] B.C. Bao, Z. Liu, J.P. Xu, Transient chaos in smooth memristor oscillator, Chinese Phys. B 19 (2010) 030510.
[62] M. Itoh, L.O. Chua, Memristor oscillators, Int. J. Bifur. Chaos 18 (2008) 3183.
[63] B. Muthuswamy, P.P. Kokate, Memristor-based chaotic circuits, IEEE Technol. Rev. 26 (2009) 415–426.
[64] J. Ma, Q.Y. Wang, W.Y. Jin, et al., Control chaos in Hindmarsh–Rose neuron by using intermittent feedback with one variable, Chinese Phys. Lett. 25 (2008) 3582–3585.
[65] C.N. Wang, J. Ma, Y. Liu, et al., Chaos control, spiral wave formation, and the emergence of spatiotemporal chaos in networked Chua circuits, Nonlinear Dyn. 67 (2012) 139–146.