

## Vector-valued intensity measures for incremental dynamic analysis



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### ABSTRACT

In incremental dynamic analysis (IDA), use of an efficient intensity measure (IM) is of vital importance. Efficiency means good explanatory power of the IM in regard to specific engineering demand parameter (EDP) that can help reduce the number of records used to estimate response of structures under given accuracy. According to dimension of the parameters, intensity measures can be classified into scalar-valued IMs and vector-valued IMs. While scalar-valued IMs have been studied systematically, vector-valued IMs have not been investigated comprehensively. Besides, vector-valued IMs considering the effect of higher modes have not been proposed. Hence in order to provide a systematic investigation on vector-valued IMs, this paper proposes five vector-valued IMs with two considering higher mode effect and three incorporating period elongation effect. Then a low-rise and a middle-rise reinforced concrete frames are modeled and analysed by PERFORM-3D and IDA under fifteen suites ground motion records for each structure. To evaluate efficiency of the proposed IMs, residual sum of squares (RSS) and  $R^2$  were calculated by logistic regression of IDA data. Results verified the better efficiency of vector-valued IMs than scalar-valued IMs. It is also proved that the relationship between IM and EDP can be expressed as a linear regression of logarithm for both scalar-valued IMs and vector-valued IMs. It turns out that a desirable IM should be selected based on the features of the specific structure. For structures dominated by the first mode, the impact of nonlinearity is of vital importance and should be mainly considered when choosing desirable IMs. Furthermore, for IMs considering nonlinearity effect, efficiency is relevant to the number of spectral acceleration incorporated; for IMs incorporating higher mode effect, the proposed multi-valued IM is more efficient than the two-valued type.

### 1. Introduction

When assessing the seismic response of a structure using dynamic analysis, it is important to identify ground motion properties that are related to the resulting structural response. These properties are often referred to as ground motion intensity measures, or IMs [1]. The most important characteristic of a desirable IM is efficiency, which means good explanatory power of the IM in regard to specific engineering demand parameter that can help reduce the number of records used to estimate the response of structures under given accuracy. According to the dimension of the parameters, intensity measures can be classified into two generic groups: scalar-valued IMs and vector-valued IMs.

Scalar-valued IMs represent ground motion intensity using one parameter, implying that the relationship between IM and engineering demand parameter (EDP) can be expressed in a two-dimensional coordinate system. Due to their superiority in succinct expression, scalar-valued IMs have been comprehensively investigated by scholars all around the world. In past years, the time-domain peak ground

acceleration (PGA) and the frequency-domain elastic spectral acceleration at the fundamental period of a structure ( $S_a(T_1)$ ) [2–4] were common scalar IMs. The  $S_a(T_1)$  was proved to have the strongest correlation with the common damage measures - the maximum and average inter-story drifts (Kostinakis et al. [5]). What is more, the pseudo spectral acceleration was also used as the IM by Bradley [6] to investigate modification factors for his model of probabilistic seismic hazard analysis to develop revised design response, particularly for long vibration periods. The PGA is not relevant to the characteristics of a specific structure and as a result cannot reflect the force acting on the structure directly, while  $S_a(T_1)$  fails to perform well when the effect of nonlinearity or that of higher modes cannot be ignored. To make up for these defects, scalar IMs incorporating nonlinearity or higher mode effects have been studied extensively.

Scalar-valued IMs considering the period elongation of a structure derived from nonlinearity development include IMs depending on two, three or multi-values of elastic spectral acceleration, IMs based on certain area zone of elastic response spectrum, and IMs derived from

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inelastic spectrum. Based on two values of spectral acceleration, Cordova et al. [7] proposed a scalar-valued IM expressed as  $S^* = S_a(T_1)^{1-\alpha} \cdot S_a(T_f)^\alpha$ , in which  $T_f$  is recommended to be twice of the fundamental period  $T_1$  and  $\alpha$  equals to 0.5, while Mehanny et al. [8] as  $S^* = S_a(T_1)^{1-\alpha} \cdot S_a(\sqrt{R} T_1)^\alpha$ , in which  $R$  a design parameter relying on the demand of nonlinearity. Nevertheless, such IMs cannot present structural response precisely at all period values because they are determined by spectral acceleration values at two fixed periods. Therefore,  $S_{a,avg}$ , an IM depending on multi-valued spectral acceleration was introduced by Bojórquez and Iervolino [9]. The  $S_{a,avg}$  is suggested to be comprised of 10 spectral acceleration values from  $S_a(T_1)$  to  $S_a(2T_1)$ . Kostinakis and Athanatopoulou [10] studied the effectiveness of the two-valued IMs, multi-valued IMs mentioned above as well as seven IMs composed of spectral velocity for three-dimensional (3D) reinforced concrete (RC) buildings and proved that the effectiveness of the IMs depends on the features of the buildings, the degree of nonlinearity and the seismic incident angle. An area-based IM,  $A(T_1)$ , which can take more spectral values into consideration, was proposed by Zhou and Li [11], where  $A(T_1)$  is the area at the interval  $[T_1, 2T_1]$  under the acceleration spectrum. Yang et al. [12] put forward  $NS_a(T_1, 5\%)$ , the spectral acceleration of inelastic spectrum as a scalar-valued IM, to consider the influence of nonlinearity in place of elastic spectral value and proved the discreteness reduction of the incremental dynamic analysis (IDA). Tothong and Luco [13] used the inelastic spectral displacement  $S_{di}$  to be the IM considering nonlinearity.

As structural height increases, the effect of higher modes cannot be ignored anymore. Consequently, two, three and multi-valued IMs incorporating higher mode spectral acceleration have been put forward by scholars. Asgarian et al. [14] put forward  $S_a(T_n, 5\%) = S_a(\tau_a, 5\%)^{1-\beta-\gamma} S_a(\tau_b, 5\%)^\beta S_a(\tau_c, 5\%)^\gamma$  as a scalar-valued IM, based on the first three periods of elastic response spectrum. Zhou et al. [15] proposed two IMs:  $S_{12}$  and  $S_{123}$  considering the first two modes and the first three modes, respectively, while Lu et al. [16] presented an IM  $\bar{S}_a = \sqrt[n]{\prod_{i=1}^n S_a(T_i)}$  to incorporate multi-values of spectral acceleration in which  $n$  is determined by the period of a given structure. Tothong and Luco [13] used the  $IM_{I \& 2E} = [S_{di}^I(T_1, \xi_1, d_y) / S_d(T_1, \xi_1)] \sqrt{[PF_1^{[2]} |S_d(T_1, \xi_1)|^2 + [PF_2^{[2]} |S_d(T_2, \xi_2)|^2]}$  to be the IM incorporating the influence of higher modes.

While scalar-valued IMs have been studied thoroughly and systematically, vector-valued IMs have not been given enough investigation. Baker and Cornell [17] proposed the first vector-valued IM of the form  $\langle S_a(T_1), \varepsilon \rangle$  which takes the effect of  $\varepsilon$  into consideration.  $\varepsilon$  is defined as a measure of the difference between the spectral acceleration of a ground motion record and the mean of a ground motion prediction equation at the given period. Then based on scalar-valued IMs which consider the effect of nonlinearity depending on two or multi-valued spectral acceleration, Baker and Cornell [1] proposed the  $\langle S_a(T_1), R_{T_1, T_2} \rangle$  and Bojórquez and Iervolino [18] the  $\langle S_a(T_1), N_p \rangle$  as vector-valued IMs, respectively. Bojórquez et al. [19] classified vector-valued IMs into three categories: IMs based on a combination of peak parameters of ground motion, IMs based on peak and cumulative damage potential parameters and IMs based on the spectral shape and then proved the advantages of the IMs based on the spectral shape over the others. Lately, Li et al. [20] proved the efficiency of the vector-valued IM  $\langle S_a(T_1), S_a(T_2) \rangle$  for seismic vulnerability analysis of bridge structures, while Yakhchalian et al. [21] presented  $\langle S_a(T_1), S_a(T_2) / DSI \rangle$  as an optimal intensity measure for seismic collapse assessment of structures.

Based on the literature review mentioned above, it is clearly identified that the vector-valued IMs have not been investigated as comprehensively and systematically as scalar-valued IMs. Besides, vector-valued IMs considering the effect of higher modes have not been proposed. Thus, in order to provide a systematic investigation on vector-valued IMs as well as to fill the gap of vector-valued IMs considering higher modes effect, this paper

**Table 1**  
Intensity measures considered.

Factors considered	Combination form	Vector-valued IM	Corresponding scalar-valued IM
Higher mode	Two-valued form	$\langle S_a(T_1), S_{12}/S_a(T_1) \rangle$	$S_{12}$
	Multi-valued form	$\langle S_a(T_1), S_{G1}/S_a(T_1) \rangle$	$S_{G1}$
	Two-valued form	$\langle S_a(T_1), S_{N1}/S_a(T_1) \rangle$	$S_{N1}$
Nonlinearity	Multi-valued form	$\langle S_a(T_1), S_{a,avg}/S_a(T_1) \rangle$	$S_{a,avg}$
	Area-based form	$\langle S_a(T_1), A(T_1)_2/S_a(T_1) \rangle$	$A(T_1)_2$

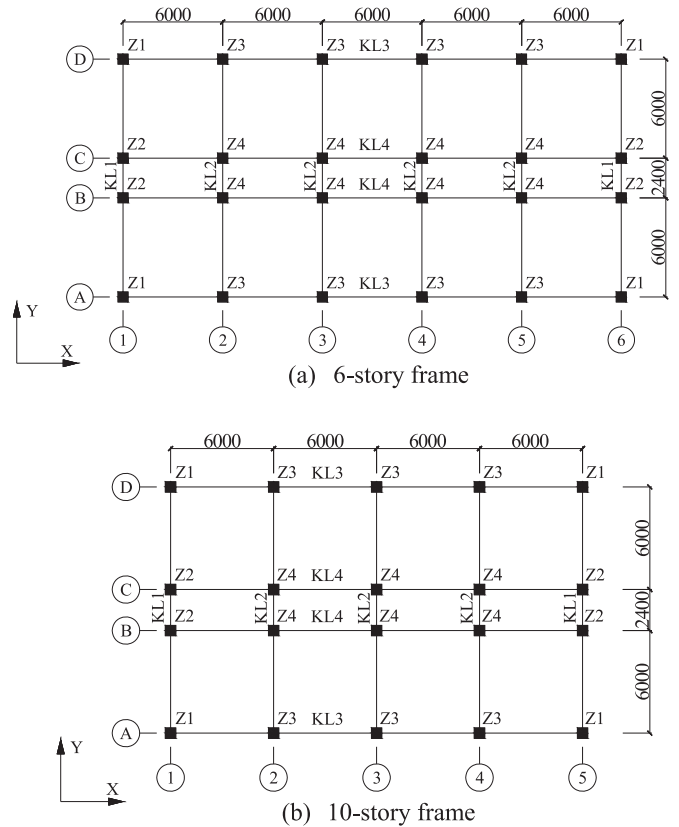


Fig. 1. Plan layout of the building (mm).

**Table 2**  
Periods (modal mass participation) for frames.

Direction	Nature Period	6-story frame	10-story frame
X	$T_1$	0.892(84%)	1.461(81%)
	$T_4$	0.282(10%)	0.473(10%)
	$T_7$	0.155(4%)	0.268(4%)

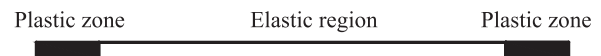


Fig. 2. Plastic zone model for beam-column model.

- (1) puts forward the new scalar-valued  $S_{G1}$  and vector-valued  $\langle S_a(T_1), S_{G1}/S_a(T_1) \rangle$  to consider higher modes effect.
- (2) puts forward and studies the vector-valued IMs  $\langle S_a(T_1), A(T_1)_2/S_a(T_1) \rangle$  and  $\langle S_a(T_1), S_{12}/S_a(T_1) \rangle$ , of which the scalar  $A(T_1)_2$  [11] and  $S_{12}$  [15] have been proposed and investigated by the authors.
- (3) divides IMs into IMs considering higher modes effect and nonlinearity development for the first time in order to take advantage that these two effects when acting together one of them is predominated.

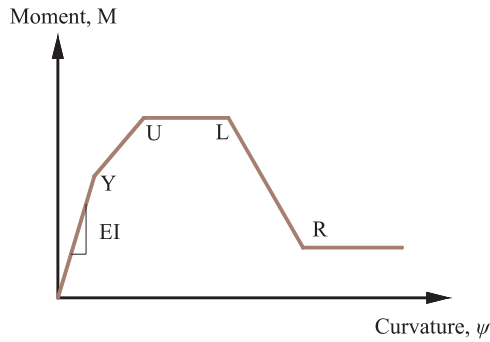


Fig. 3. The relationship between the moment and curvature of the plastic zone.

**Table 3**  
Reinforcement information of the 6-story frame.

Story	1	2	3	4	5	6
Z1	12Φ28	12Φ25	12Φ22	12Φ18	12Φ18	12Φ18
Z2	12Φ28	12Φ25	12Φ22	12Φ18	12Φ18	12Φ18
Z3	12Φ28	12Φ25	12Φ22	12Φ18	12Φ18	12Φ18
Z4	12Φ25	12Φ22	12Φ20	12Φ18	12Φ18	12Φ18
KL1	8Φ25	8Φ25	8Φ24	8Φ24	8Φ18	8Φ18
KL2	8Φ25	8Φ25	8Φ24	8Φ24	8Φ18	8Φ18
KL3	8Φ27	8Φ27	8Φ25	8Φ24	8Φ21	8Φ18
KL4	8Φ27	8Φ27	8Φ25	8Φ24	8Φ21	8Φ18

**2. Engineering demand parameters and proposed vector-valued intensity measures**

Incremental dynamic analysis (IDA) is a useful nonlinear analysis method in Performance-Based Earthquake Engineering for evaluating the demands on structures under earthquake excitations. The basic principle of IDA is performing nonlinear dynamic analysis of a structural model to a suite of ground motion records, each scaled to multiple levels of IM and recording the response [4]. Results of IDA are shown by curves combining IM of a site-specific ground motion and engineering demand parameter (EDP) of a given structure.

Engineering demand parameter, which is also called the structural state variable, should not only be easily derived from nonlinear analysis results but also demonstrate the desired dynamic performance. In framed building structures, the maximum inter-story drift ratio, the top displacement and the base shear, all have been used as EDP based on structure features and research objectives. Generally,  $\theta_{max}$ , the maximum inter-story drift ratio, is the most frequently used EDP for investigation of seismic vulnerability of buildings due to its fine correlation with structural damage [22]. Based on the purpose of the research and characteristics of structures considered in this paper,  $\theta_{max}$  is chosen as the EDP.

IDA curves display significant record-to-record variability due to the randomness of earthquake. Different IMs can lead to different variability. For many years, scholars have been struggling to seek desirable IMs efficient enough to reduce the discreteness of different records.

**Table 4**  
Reinforcement information of the 10-story frame.

Story	1	2	3	4	5	6	7	8	9	10
Z1	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25
Z2	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25
Z3	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25
Z4	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25	8Φ25
KL1	8Φ22	8Φ22	8Φ22	6Φ22	6Φ22	6Φ22	6Φ20	6Φ20	6Φ20	6Φ20
KL2	8Φ22	8Φ22	8Φ22	6Φ22	6Φ22	6Φ22	6Φ20	6Φ20	6Φ20	6Φ20
KL3	8Φ22	8Φ22	8Φ22	6Φ22	6Φ22	6Φ22	6Φ20	6Φ20	6Φ20	6Φ20
KL4	8Φ22	8Φ22	8Φ22	6Φ22	6Φ22	6Φ22	6Φ20	6Φ20	6Φ20	6Φ20

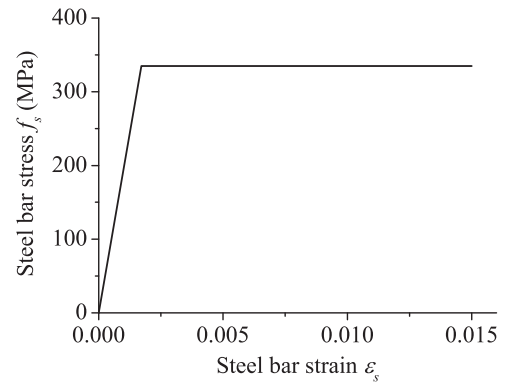


Fig. 4. The stress-strain curve of a steel bar.

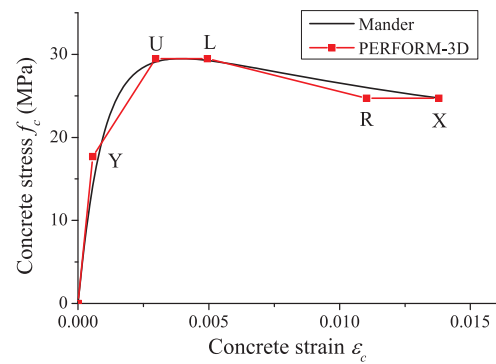


Fig. 5. The stress-strain curve of confined concrete.

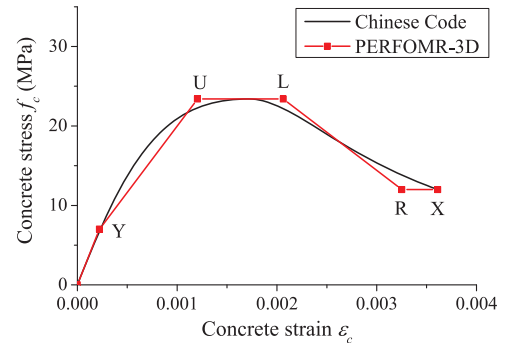


Fig. 6. The stress-strain curve of unconfined concrete.

Based on the literature review and thorough analysis in the introduction, this paper investigates five vector-valued IMs with two of them considering higher modes effect and three of them incorporating period elongation effect as follows:

1. The vector-valued IM considering higher modes effect, which incorporates two values of spectral acceleration at two periods  $T_1$  and  $T_2$  and read as

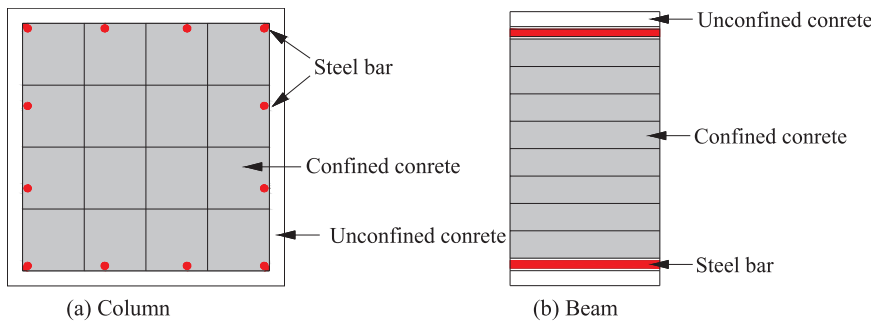


Fig. 7. Section division of the columns and beams in the first floor of the six-story frame in Perform-3D. (a) Column (b) Beam.

Table 5  
Information of ground motions for the 6-story frame.

No.	Event	Station	Year
GM-1	Borrego Mtn	San Onofre-SO Cal Edson	1968
GM-2	Imperial Valley-06	El Centro Array #12	1979
GM-3	Chalfant Valley-04	Zack Brothers Ranch	1986
GM-4	Whittier Narrows-01	Covina-S Grand Ave	1987
GM-5	Whittier Narrows-02	LA-Baldwin Hills	1987
GM-6	Loma Prieta	Coyote Lake Dam (Downst)	1989
GM-7	Loma Prieta	Fremont-Emerson Court	1989
GM-8	Northridge-01	Jensen Filter Plant Generator	1994
GM-9	Northridge-01	LA-Pico & Sentous	1994
GM-10	Northridge-01	LA-Wonderland Ave	1994
GM-11	Northridge-01	Lakewood-Del Amo Blvd	1994
GM-12	Chi-Chi, Taiwan-02	TCU119	1999
GM-13	Chi-Chi, Taiwan-05	CHY088	1999
GM-14	Chi-Chi, Taiwan-05	HWA024	1999
GM-15	Chi-Chi, Taiwan-05	TCU102	1999

Table 6  
Information of ground motions for the 10-story frame.

No.	Event	Station	Year
GM-1	San Fernando	Buena Vista - Taft	1971
GM-2	Friuli, Italy-02	Buia	1976
GM-3	Imperial Valley-06	El Centro Array #6	1979
GM-4	Irpinia, Italy-01	Auletta	1980
GM-5	Chalfant Valley-02	Zack Brothers Ranch	1986
GM-6	Superstition Hills-01	Wildlife Liquef. Array	1987
GM-7	Loma Prieta	Agnews State Hospital	1989
GM-8	Loma Prieta	Fremont - Mission San Jose	1989
GM-9	Landers	LA - N Westmoreland	1992
GM-10	Big Bear-01	Joshua Tree	1992
GM-11	Northridge-01	El Monte - Fairview Av	1994
GM-12	Hector Mine	Lake Hughes #1	1999
GM-13	Chi-Chi, Taiwan-02	HWA048	1999
GM-14	Chi-Chi, Taiwan-04	CHY058	1999
GM-15	Chi-Chi, Taiwan-05	ILA041	1999

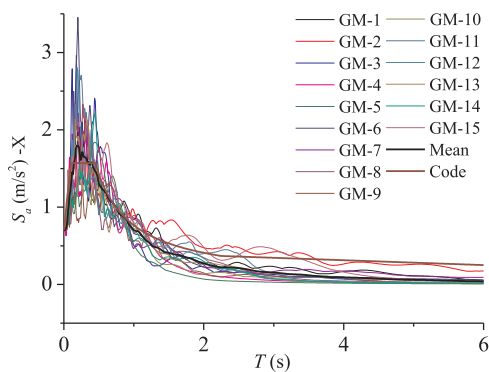


Fig. 8. Acceleration spectra of ground motions for the 6-story frame.

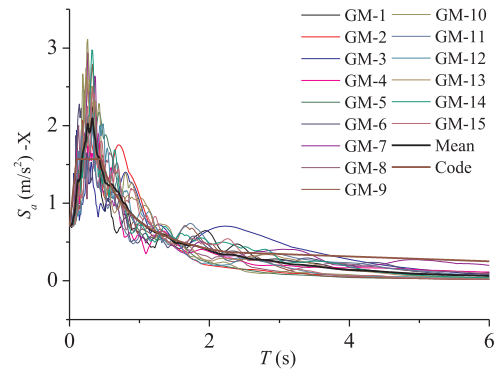


Fig. 9. Acceleration spectra of ground motions for the 10-story frame.

$$\langle S_a(T_1), S_{12}/S_a(T_1) \rangle \tag{1}$$

where

$$S_{12} = [S_a(T_1, \xi)]^\alpha \cdot [S_a(T_2, \xi)]^\beta \tag{2}$$

$$\alpha = m_1/(m_1 + m_2), \quad \beta = m_2/(m_1 + m_2) \tag{3}$$

with  $m_i$  being the modal participating mass ratio of the  $i^{\text{th}}$  mode,  $S_a(T_i)$  the spectral acceleration of the  $i^{\text{th}}$  mode and  $\xi$  the damping ratio.

- The vector-valued IM considering higher modes effect, which incorporates multi values of spectral acceleration at different periods  $T_i$  and read as

$$\langle S_a(T_1), S_{G1}/S_a(T_1) \rangle \tag{4}$$

where

$$S_{G1} = \sum_{i=1}^n m_i \cdot S_a(T_i) \tag{5}$$

with  $m_i$  being the modal participating mass ratio of the  $i^{\text{th}}$  mode,  $n$  the mode in which the cumulative modal participation exceeds 80% plus other modes whose modal participation exceeds 5% and  $S_a(T_i)$  the spectral acceleration of the  $i^{\text{th}}$  mode.

- The vector-valued IM considering nonlinearity effect, which uses two values of spectral acceleration and read as

$$\langle S_a(T_1), S_{N1}/S_a(T_1) \rangle \tag{6}$$

where

$$S_{N1} = S_a(T_1)^\alpha \cdot S_a(CT_1)^{1-\alpha} \tag{7}$$

$$\alpha = 0.5, \quad C = 2 \tag{8}$$

- The vector-valued IM considering nonlinearity effect, which uses multi values of spectral acceleration and read as

$$\langle S_a(T_1), S_{a,avg}/S_a(T_1) \rangle \tag{9}$$

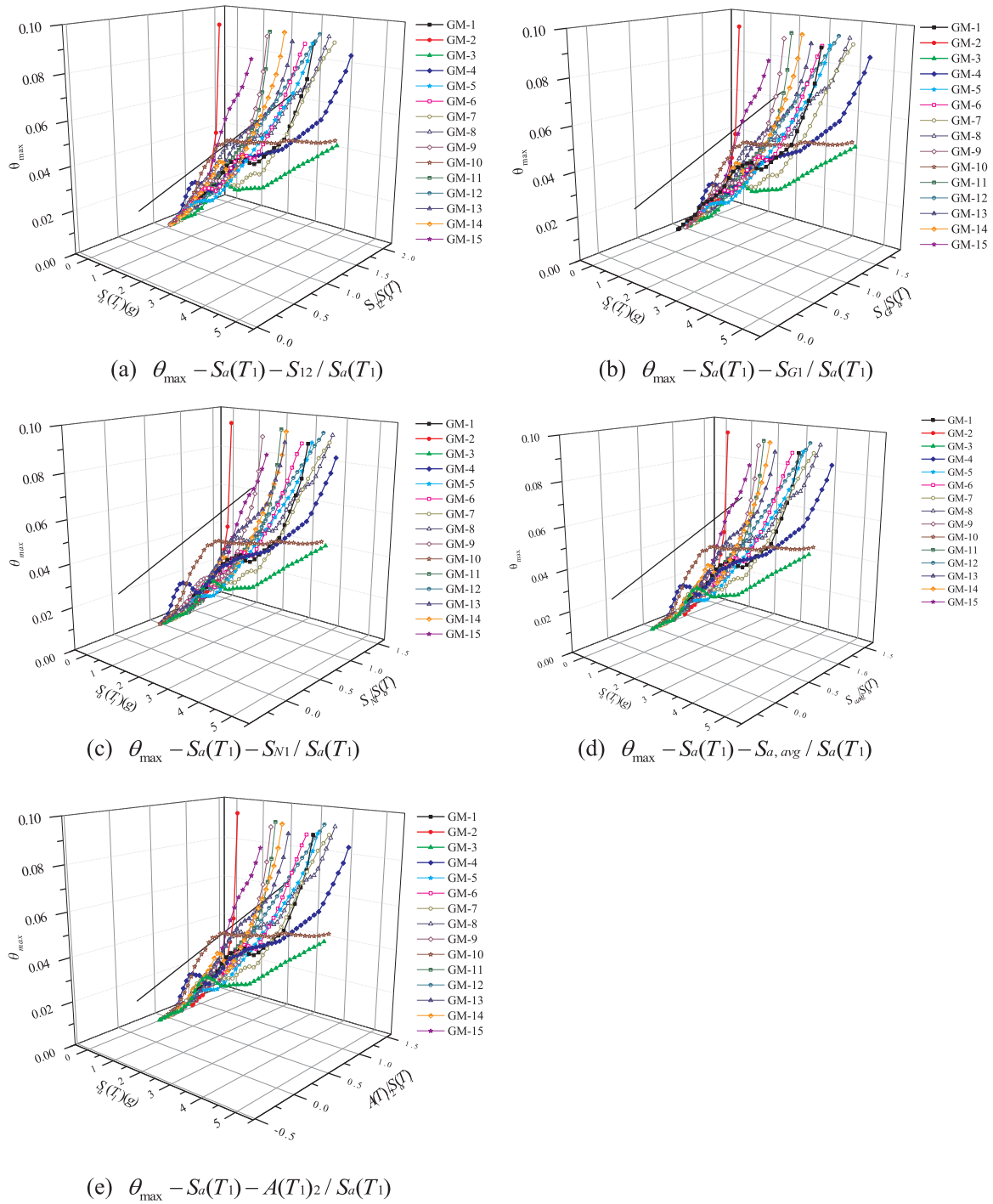


Fig. 10.  $\theta_{\max} - IM_1 - IM_2$  curves for the 6-story frame under different vector IMs.

where

$$S_{a, avg} = \left( \prod_{i=1}^n S_a(T_i) \right)^{1/n} \tag{10}$$

$$n = 10, T_n = 2T_1 \tag{11}$$

5. The vector-valued IM considering nonlinearity effect, which employs the area-based scalar IM and read as

$$\langle S_a(T_1), A(T_1)_2 / S_a(T_1) \rangle \tag{12}$$

where  $A(T_1)$  is the area at the interval  $[T_1, 2T_1]$  under the acceleration spectrum.

Table 1 provides a concise description of the above intensity measures considered in this work.

### 3. Structural analytical models

To explore the IMs mentioned above, two reinforced concrete space frames were analysed under seismic excitations: one low-rise six story and one middle-rise ten story. The plan layouts of the 6-story frame and 10-story frame are shown in Fig. 1(a)-(b). For the 6-story RC frame, the height is 22.2 m and dimensions for the frame columns and beams are

550 mm × 550 mm and 300 mm × 550 mm, respectively. For the 10-story RC frame, the height is 39.3 m with dimensions of 600 mm × 600 mm and 300 mm × 600 mm for columns and beams. The strength grade for concrete, longitudinal bars and stirrup bars is C35 (compressive strength  $f_{ck} = 23.4$  MPa), HRB335 (tensile strength  $f_{tk} = 455$  MPa and the elastic modulus  $E = 2.0 \times 10^5$  MPa) and HPB300 (tensile strength  $f_{tk} = 420$  MPa and the elastic modulus  $E = 2.0 \times 10^5$  MPa), respectively.

Periods and corresponding modal mass participation along X axis are shown in Table 2 according to modal analysis results from PERFORM-3D software [23,24].

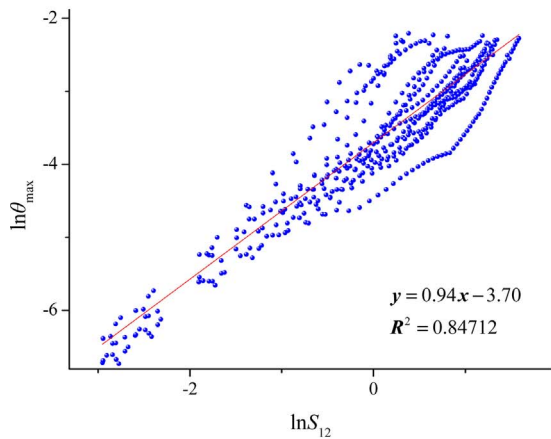
The aforementioned building frames were analysed by the PERFORM-3D software [23,24] using inelastic beam-column elements with plastic zones at their two ends and an elastic zone in their middle, as shown in Fig. 2. The relationship of moment-curvature for the plastic zone model is shown in Fig. 3. The plastic zone at each end has a length of 0.5 times the member depth and is composed of two kinds of materials: steel and concrete, of which the dimension and position are based on the reinforcement information calculated by the PKPM software [25] and shown in Tables 3 and 4. A bilinear stress-strain model is used for the steel material (Fig. 4). Two models with one used for unconfined concrete at the edge of a cross section (Fig. 5) and one used for confined concrete in the core area (Fig. 6) are adopted for the concrete material based on the Chinese Code for Design of Concrete Structures [26] and Mander et al. [27].

Take the beams and columns in the first floor of the six story frame

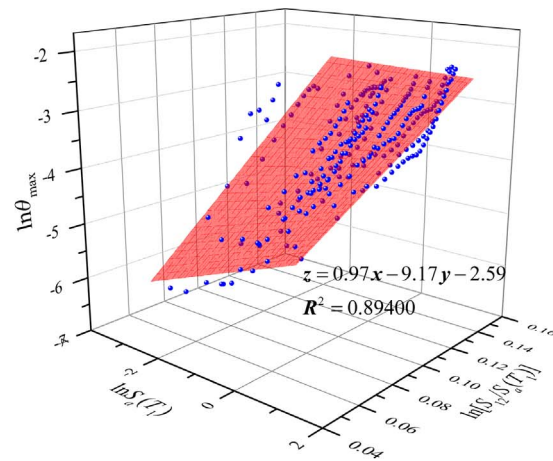
as an example to illustrate the element modeling in PERFORM-3D. As shown in Table 3, the Z1 columns in the first floor of the six story frame have 12 bars with a diameter of 28. Reinforcement is distributed at the edge of the column with the same space and the section is divided into 16 fiber units (Fig. 7(a)). The dimension and position of the fiber units can be determined by inputting area and two-dimensional coordinates of the element in the software. The entire section can be divided into two parts: the peripheral protection part which can be simulated by the stress-strain model of the unconfined concrete and the core part which can be modeled by the stress-strain model of the confined concrete. As for beams, the dimension and position of the fiber unit can be determined by area and one-dimensional coordinates. The KL1 beams in the first floor of the six story frame has 8 bars with a diameter of 25 as shown in Table 3 and can be divided into the units shown in Fig. 7(b) with the reinforcement fiber layer distributed at two ends of the core concrete. The unconfined concrete is also simulated by the constitutive relationship in Fig. 6, while the confined concrete is modeled by the stress-strain curve in Fig. 5.

#### 4. Selection of ground motion records

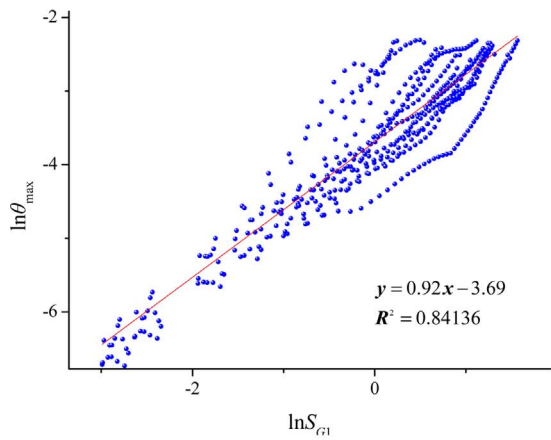
According to the Recommended Seismic Design Criteria for New Steel Moment-Frame Buildings [28], 10–20 ground motions are required for IDA of a specific structure. The Chinese Code for Design of Concrete Structures [26] requires the difference between the spectral acceleration



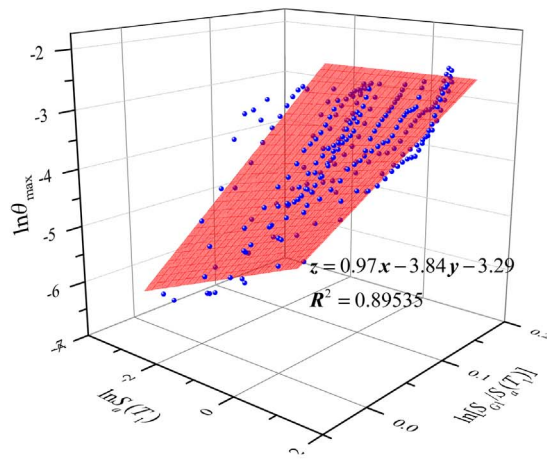
(a)  $\ln \theta_{\max} - \ln S_{12}$



(b)  $\ln \theta_{\max} - \ln S_a(T_1) - \ln[S_{12} / S_a(T_1)]$



(c)  $\ln \theta_{\max} - \ln S_{G1}$



(d)  $\ln \theta_{\max} - \ln S_a(T_1) - \ln[S_{G1} / S_a(T_1)]$

Fig. 11. Fitting results for the 10-story frame under different scalar and vector IMs.,.

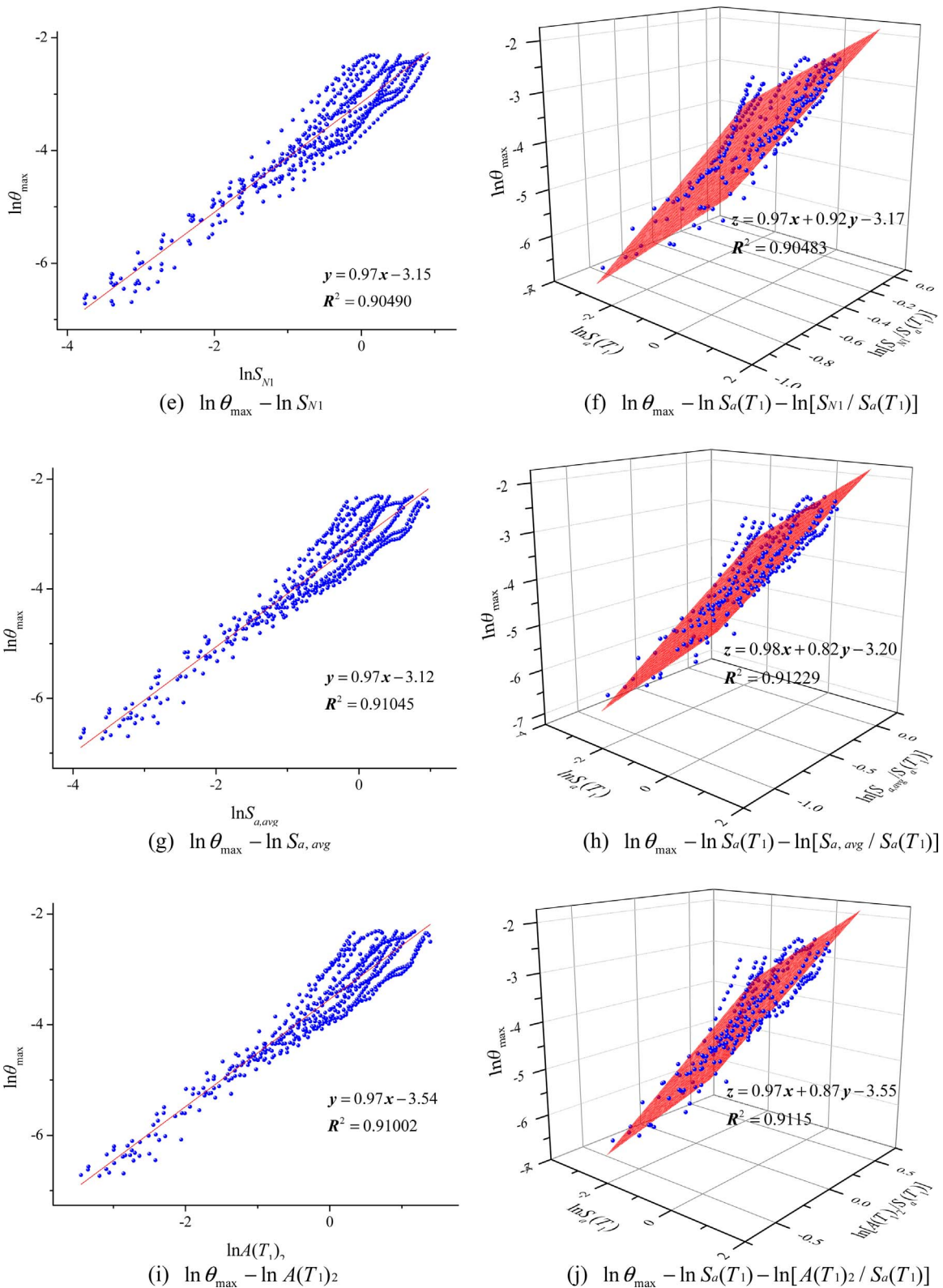


Fig. 11. (continued)

of the selected ground motions at main periods of a structure be statistically consistent with the standard response spectrum. In this work, the main periods of the two structures are different as it can be seen from Table 2, thus 15 sets of ground motion records for each frame were selected from the strong motion database of Pacific Earthquake Engineering Research Center (PEER). The difference between the

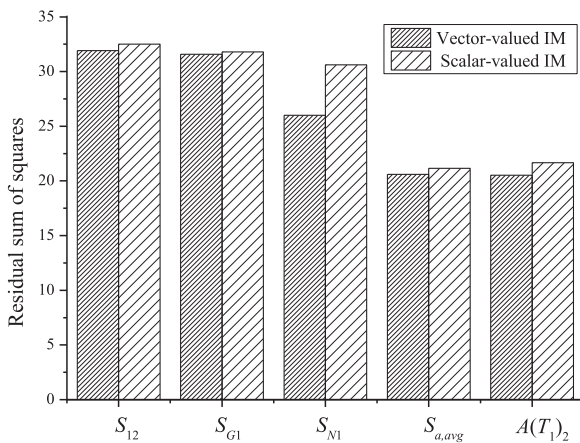
spectral acceleration at the first three periods of the structure under the selected ground motions and standard response spectrum is less than 20%. Details of the selected ground motions are shown in Tables 5 and 6. Compared curves of the selected ground motion acceleration spectra and design response spectra are shown in Figs. 8 and 9. Since the problem of how to combine the IMs in different directions when ground

**Table 7**  
Results of linear fitting for 6-story frame.

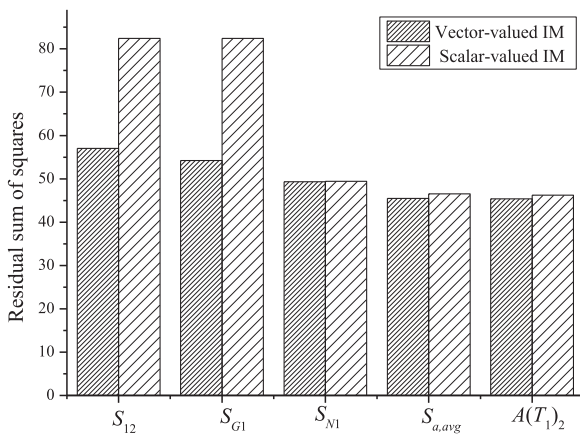
IM	RSS	R <sup>2</sup>
$\langle S_a(T_1), S_{12}/S_a(T_1) \rangle$	31.91	0.94596
$S_{12}$	32.51	0.94504
$\langle S_a(T_1), S_{G1}/S_a(T_1) \rangle$	31.59	0.94650
$S_{G1}$	31.80	0.94624
$\langle S_a(T_1), S_{N1}/S_a(T_1) \rangle$	26.01	0.95595
$S_{N1}$	30.61	0.94825
$\langle S_a(T_1), S_{a,avg}/S_a(T_1) \rangle$	20.60	0.96511
$S_{a,avg}$	21.15	0.96424
$\langle S_a(T_1), A(T_1)_2/S_a(T_1) \rangle$	20.52	0.96525
$A(T_1)_2$	21.66	0.96338

**Table 8**  
Results of linear fitting for 10-story frame.

IM	RSS	R <sup>2</sup>
$\langle S_a(T_1), S_{12}/S_a(T_1) \rangle$	57.03	0.89400
$S_{12}$	82.42	0.84712
$\langle S_a(T_1), S_{G1}/S_a(T_1) \rangle$	54.25	0.89535
$S_{G1}$	82.41	0.84136
$\langle S_a(T_1), S_{N1}/S_a(T_1) \rangle$	49.34	0.90483
$S_{N1}$	49.41	0.90490
$\langle S_a(T_1), S_{a,avg}/S_a(T_1) \rangle$	45.47	0.91229
$S_{a,avg}$	46.52	0.91045
$\langle S_a(T_1), A(T_1)_2/S_a(T_1) \rangle$	45.38	0.91150
$A(T_1)_2$	46.22	0.91102



**Fig. 12.** Residual sum of squares for IMs of 6-story frame.



**Fig. 13.** Residual sum of squares for IMs of 10-story frame.

motions are input in two or three directions has not been solved, the record is input along the X direction when nonlinear time-history analysis is carried out for the structure. Therefore, according to Table 4, for the 6-story frame,  $m_1$  and  $m_2$  of  $S_{12}$  are 0.8936 and 0.1064, respectively, and  $n$  for  $S_{G1}$  is 2; for the 10-story frame,  $m_1$  and  $m_2$  of  $S_{12}$  are 0.8901 and 0.1099, respectively, and  $n$  for  $S_{G1}$  is 2.

**5. Analysis of vector-valued intensity measures**

To perform IDA, the selected ground motion records were scaled up one by one, with PGA increasing from 0.7  $m/s^2$  to the value that causes collapse of the structure. The increment of each scaling step is 1  $m/s^2$ . Collapse of structure occurs when the maximum inter-story drift ratio  $\theta_{max}$  reaches 0.1 or the tangent slope equals 20% of the elastic slope. Nonlinear time-history is carried out for each model by PERFORM-3D [23,24] using the 15 suites scaled ground motions. Based on the results of IDA,  $\theta_{max} - IM_1 - IM_2$  curves under  $\theta_{max}$  and different IMs are obtained. Figs. 10(a)–(e) show  $\theta_{max} - IM_1 - IM_2$  curves of the 6-story under above mentioned vector-valued IMs.

As mentioned above, efficiency is the most important property of an IM, which can help to reduce the discreteness derived from randomness of ground motions and thus help to decrease the number of records needed to estimate response of structures under given accuracy. According to Cornell et al. [29], the relationship between IM and EDP can be expressed as a linear regression of logarithms. On the other hand, according to theory of statistics, the residual sum of squares (RSS) is a good indicator to measure discreteness of regression and thus is chose to be the first indicator to evaluate efficiency of different IMs. The RSS in a model with two explanatory variables is expressed as

$$RSS = \sum_{i=1}^n (z_i - f(x_i, y_i))^2 \tag{13}$$

where  $z_i$  is the  $i^{th}$  value of the variable to be predicted,  $x_i$  and  $y_i$  are the  $i^{th}$  value of the explanatory variables, and  $f(x_i, y_i)$  is the predicted value of  $z_i$ . The smaller the RSS value is, the more efficient the IM is. In addition,  $R^2$ , a statistical measure of how well a regression line approximates real data points, is used as the second indicator to evaluate efficiency. The closer to 1 the  $R^2$  value is, the more efficient the IM is.

The fitting results of the 10-story frame based on ten IMs (five scalar IMs and five vector IMs) mentioned above are shown in Fig. 11(a)–(j). Summary of RSS and  $R^2$  of the 6-story frame as well as the 10-story frame are presented in Tables 7, 8.

As is shown in Tables 7 and 8 as well as Figs. 12 and 13, among all IMs, the efficiency of IMs considering nonlinearity is better than that of IMs incorporating higher modes influence. It demonstrates that, for structures dominated by the first mode, as it is usually the case with framed building structures, the impact of nonlinearity is of vital importance and should be taken into account when choosing IMs. On the contrary, consideration of higher modes effect does not contribute significantly to the efficiency and can be ignored when electing IMs for such structures. Therefore, a desirable IM is the one selected according to the features of a specific structure and should include the predominant information of the structure as much as possible.

Besides, vector-valued IMs are more efficient than corresponding scalar-valued IMs as shown in Figs. 12 and 13. It is observed that the vector-valued IMs can represent more information of specific ground motion and have an advantage over scalar-valued IMs requiring the same computational efforts. What is more, vector-valued IMs can represent two aspects of the seismic records with one parameter for each aspect while scalar IMs couple two aspects of ground motions by one parameter. For instance, if there is a need to consider the effect of nonlinearity and higher modes at the same time, then the best parameter incorporating period elongation effect  $A(T_1)_2$  and the best parameter considering higher modes influence  $S_{G1}$  can be combined into an optimal vector-valued  $\langle A(T_1)_2, S_{G1} \rangle$ , which can reflect the



impact of the two aspects simultaneously, while avoiding coupling them together.

Furthermore, whether considering scalar IMs or vector IMs, the area-based type is the most efficient IM, while the two-valued type is the least efficient IM among all IMs considering the effect of non-linearity. This means that the more spectral acceleration is incorporated, the more efficient the IM is. However, the value of RSS and  $R^2$  for the 10-valued IM and area-based IM are basically the same and thus, in order to avoid integral evaluation and simplify calculations, the 10-valued IM can be used to substitute the area-based IM.

Of all IMs taking the higher modes effect into account, compared to the two-valued IM combining spectral acceleration under different periods by power exponent, the new multi-valued IM is more efficient and can be calculated more easily. Hence this new type is suggested to be used as a desirable IM when higher modes effect needs to be considered.

Finally, as it can be seen from the fitting formula in Fig. 11(a)–(j) as well as the range of the RSS and  $R^2$  values in Tables 7 and 8, the assumption of the relationship between IM and EDP is rational.

## 6. Conclusions

In order to provide a systematic investigation of vector-valued IMs as well as to fill the gap of vector-valued IMs considering higher modes effect, this paper proposed five vector-valued intensity measures (IM), with two IMs (a power exponent type and an addition type) incorporating higher modes effect and three IMs (a two-valued type, a multi-valued type and an area-based type) considering the influence of nonlinearity. Then two reinforced concrete space frames were modeled by PERFORM-3D and subjected to incremental dynamic analyses (IDA) by using 15 suites ground motion records for each structure. To evaluate efficiency of the IMs, residual sum of squares (RSS) and  $R^2$  were calculated using the results of IDA. The main conclusions reached are as follows:

The assumption of the relationship between IM and EDP, which is expressed as a linear regression of logarithm, is valid for both scalar-valued IMs and vector-valued IMs as the fitting process has shown.

A desirable IM should be selected according to features of a specific structure and should include the predominant information of the structure as much as possible. For structures dominated by the first mode in this paper, the impact of nonlinearity is of vital importance, whereas higher modes effect has little influence. Hence, it is rational that IMs considering the influence of nonlinearity are more efficient than IMs incorporating higher modes influence.

Vector-valued IMs are more efficient than corresponding scalar-valued IMs because vector-valued IMs can incorporate more information of specific ground motions. Besides, vector-valued IMs can represent two aspects of the seismic records with one parameter for each aspect while scalar IMs couple two aspects of ground motions by one parameter. All these demonstrate that vector-valued IMs have an advantage over scalar-valued IMs and deserve further investigation.

Efficiency of IMs considering nonlinearity effect is relevant to the number of spectral acceleration included. The more spectral accelerations are considered, the more efficient the IM is. Thus, in general, area-based IMs are more efficient than multi-valued IMs and multi-valued IMs are more efficient than two-valued IMs. However, when the number of spectral accelerations reaches a certain level, the efficiency of IMs does not improve significantly anymore. So it is reasonable to use multi-valued IMs as a substitute for area-based IMs to avoid integral evaluation and simplify the calculation.

With comparison to the two-valued IM considering higher modes effect by combining spectral acceleration under different periods using power exponent, the new multi-valued IM based on addition is more

efficient and can be calculated more easily. Hence this new type is suggested to be used as an IM when higher modes need to be considered.

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