

Research Article

Analysis on Meshing Characteristics and Transmission Error of Elliptic Gears

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As an important parameter to distinguish noncircular gear from cylindrical gear, eccentricity is very important for the meshing characteristics and transmission error of noncircular gear. In order to study the transmission characteristics of the elliptic gear, a pair of elliptic gear in the reversing device of a new type of drum pumping was taken as the research object. Based on the analysis of the transmission pressure angle and instantaneous contact ratio of the elliptic gear, the eccentricity error was introduced into the analysis model of transmission error. The influences of the eccentricity on the transmission pressure angle, instantaneous contact ratio, and transmission error were analyzed, and the analysis accuracy is verified by the finite element method. The results show that the eccentricity has a great influence on the transmission pressure angle, instantaneous contact ratio, and transmission error of the elliptic gear, and the eccentricity error has a significant influence on the transmission error. In order to ensure the normal meshing condition of the elliptic gear, the eccentricity should be less than 0.7071, and the maximum instantaneous contact ratio is 1.809. The research results can provide some guidance for the following noncircular spur gear transmission test and transmission error research.

1. Introduction

As a new type of gear, elliptic gear has the characteristics of strong bearing capacity, compact structure, and variable ratio transmission. It is mainly used in low-speed and large torque occasions, such as agriculture, textile, instruments, navigation ships, hydraulic pump, hydraulic motor, and flow instrument [1]. As is known to all that gear transmission transmits power by the mesh force along the mesh line, the dynamic excitation along the mesh line will then be produced in the transmission process, so that the displacement difference between the driving and driven gears in the direction of the meshing line will be generated. This is what is called transmission error, which can be divided into static and dynamic transmission error [2]. At present, there are mainly two kinds of transmission error research methods: analytical method and numerical method [3], while the numerical calculation methods are mainly divided into conventional finite element method and contact finite

element method. The former is mainly represented by gear loaded contact analysis method (LTCA). The contact finite element method is to establish a nonlinear contact element in the meshing area and simulate the dynamic meshing process of the gear and simultaneously obtains the gear deformation, transmission error, and stress [4].

For the gear transmission system, the transmission error is the main factor affecting the accuracy of motion, so it is necessary to study it. For the transmission error of cylindrical gears, a lot of research results have been accumulated. References [5–8] analyzed the transmission error of the helical gear. Hotait and Kahraman [9] studied the relationship between the dynamic transmission error and the dynamic stress parameters of the spur gear pair. Park et al. [10] used the integrated empirical mode decomposition method to analyze the transmission error of the gear experimental device and apply it to the fault diagnosis of the gear. On the basis of studying the modeling of time-varying meshing stiffness and static transmission error of gear, the

nonlinear frequency response characteristics of spur gear were analyzed by Yang et al. [11]. The contact model and transmission error of lightweight gear were studied in [12] by means of finite element analysis. The influence of assembly error on transmission error of gear was studied by Chun and Chen [13]. Park [14] analyzed the tooth surface friction and gear transmission error caused by sliding friction of spur gear under quasi-static condition. Based on the consideration of tooth profile modification, manufacturing error, and assembly error, Li et al. [15] proposed a calculation method of no-load transmission error based on measured discrete tooth surfaces. By combining with the principle of triangle intersection judgment, a method of gear tooth contact analysis is proposed to analyze the no-load transmission error, which provides a new method and idea for subsequent transmission error analysis of gear pair. Chin [16] analyzed the influence of average wear depth on gear transmission error. By simulating the meshing process of gear under dry friction condition, the change trend of transmission error was obtained and the efficiency of the proposed method was verified. Based on the establishment of load sharing ratio and quasi-static transmission error, Miguel [17] analyzed the influence on the quasi-static transmission error of symmetric long profile modifications on high contact ratio spur gears and obtained the dynamic matching relationship between the optimal tooth profile repair length and coincidence degree. References [18–22] described the calculation method of different types of gear load transmission error in detail. In [23–27], the analytical calculation model of transmission error was derived under the condition of considering eccentricity error and installation error, and the transmission error of gear was analyzed by finite element method, which provides a new method for the subsequent analysis of gear transmission error.

The above research results have certain guiding significance for the study of transmission errors of elliptic gears. At present, there are few literatures about the transmission characteristics and transmission errors of noncircular gears. Therefore, this paper takes a pair of elliptic gear pairs in the reversing device of a new type of drum pumping as the research object and introduces the eccentric error into the gear transmission error calculation model based on the analysis of the meshing characteristics of the elliptic gear. On this basis, the transmission error of the elliptic gear is studied. Figure 1 shows the elliptic gear reversing device model.

2. Theoretical Mathematical Models

2.1. Mathematical Model of Elliptic Gear. It can be seen from [28] that the elliptic gear has two focal points. In Figure 2, the equation of elliptic pitch curve with focal point O_1 as polar coordinate point is

$$r_1 = \frac{A(1 - e^2)}{1 - e \cos t_1}. \quad (1)$$

The tooth profile of elliptic gears can be divided into two parts, the point higher than the pitch curve and the point

lower than the pitch curve, and there are different methods for solving the two-part tooth profile equation.

For points above pitch curve profile, the angles between the vector \mathbf{an} and the polar axis are $t_1 - \mu + \alpha$ (right profile angle) and $\mu - t_1 + \alpha$ (left profile angle), as shown in Figure 3.

The equation of the right tooth profile is

$$\begin{cases} x_R = r \cos t_1 + an \cos(t_1 - \mu + \alpha), \\ y_R = r \sin t_1 + an \sin(t_1 - \mu + \alpha). \end{cases} \quad (2)$$

The equation of the left tooth profile is

$$\begin{cases} x_L = r \cos t_1 + a'n' \cos(\mu - t_1 + \alpha), \\ y_L = r \sin t_1 + a'n' \sin(\mu - t_1 + \alpha). \end{cases} \quad (3)$$

For points on the tooth profile below the pitch curve, the angles between the vector \mathbf{an} and the polar axis are $t_1 - \mu - \alpha$ (right profile angle) and $\mu - t_1 - \alpha$ (left profile angle), as shown in Figure 4.

The equation of the right tooth profile is

$$\begin{cases} x_R = r \cos t_1 + an \cos(t_1 - \mu - \alpha), \\ y_R = r \sin t_1 + an \sin(t_1 - \mu - \alpha). \end{cases} \quad (4)$$

The equation of the left tooth profile is

$$\begin{cases} x_L = r \cos t_1 - a'n' \cos(\mu - t_1 - \alpha), \\ y_L = r \sin t_1 - a'n' \sin(\mu - t_1 - \alpha). \end{cases} \quad (5)$$

According to equations (2) to (5), the three-dimensional tooth surface equation of elliptic gear can be obtained, and the right tooth surface equation of elliptic gear is

$$\begin{cases} x_R = r \cos t_1 \pm an \cos(t_1 - \mu + \alpha), \\ y_R = r \sin t_1 \pm an \sin(t_1 - \mu + \alpha), \\ z_R = z_i. \end{cases} \quad (6)$$

The left tooth surface equation of elliptic gear is

$$\begin{cases} x_L = r \cos t_1 \mp a'n' \cos(\mu - t_1 - \alpha), \\ y_L = r \sin t_1 \mp a'n' \sin(\mu - t_1 - \alpha), \\ z_L = z_i, \end{cases} \quad (7)$$

where z_i refers to the direction of the tooth line and is equal to the width of the tooth.

2.2. Transmission Pressure Angle of Elliptic Gear. In Figure 2, gear 1 and gear 2 are, respectively, driving gear and driven gear, and the center distance of the elliptic gear is a . The pitch curve equation of the driven gear is

$$r_2 = a - r_1 = \frac{A(1 - 2e \cos t_1 + e^2)}{1 - e \cos t_1}. \quad (8)$$

The pressure angle of the circular tooth profile at the base circle is equal to the tooth angle of the tool rack profile ($\alpha_0 = 20^\circ$), but the transmission pressure angle of the elliptic gear changes due to the time-varying pitch curve radius. Figure 5 shows the transmission pressure angles of the left

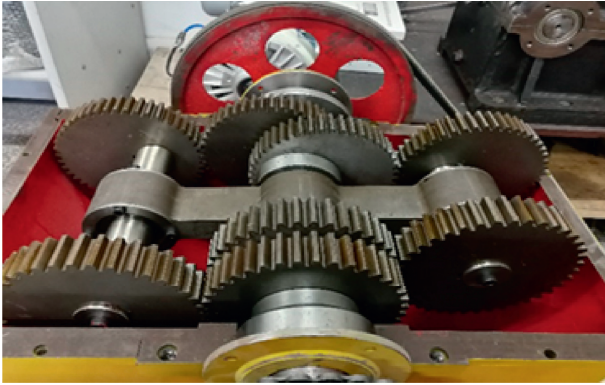


FIGURE 1: Reversing device of planetary gear train with elliptic gears.

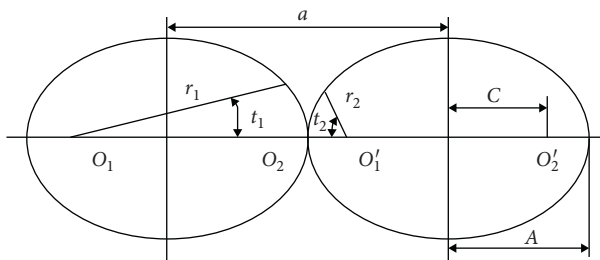


FIGURE 2: Transmission pitch curve of elliptic gear.

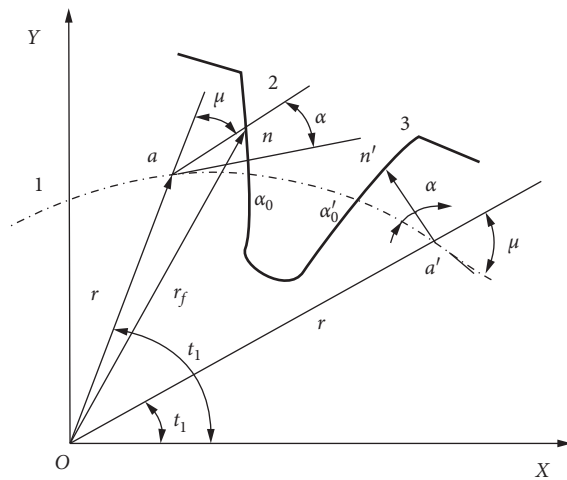


FIGURE 3: Tooth profile equation diagram above pitch curve.

and right profiles of the elliptic gears. where + and - represent left and right tooth profiles, respectively.

$$\alpha = \alpha_0 \pm \frac{\pi}{2} - \mu, \quad (9)$$

According to literature [29], the transmission ratio of elliptic gear is

$$i_{12} = \frac{1 + e^2 - 2e \cos(t_1)}{1 - e^2}, \quad (10)$$

$$\tan \mu = \frac{r_1}{(dr/dt_1)}.$$

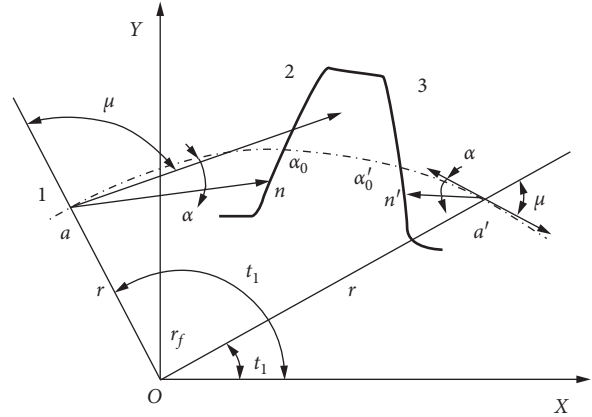


FIGURE 4: Tooth profile equation diagram below pitch curve.

By simplifying equation (3), we can get

$$\alpha = \alpha_0 \pm \frac{\pi}{2} - \arctan\left(\frac{1 - e \cos t_1}{e \sin t_1}\right). \quad (11)$$

2.3. Instantaneous Contact Ratio of Elliptic Gear. The instantaneous contact ratio of noncircular gear is the same as that of cylindrical gear, which is the ratio of the effective meshing line length to the gear base circular pitch. Figure 6 shows two curves are tangent at point P at a certain moment. The curvature radii of pitch curves of driving and driven gears at point P are, respectively, ρ_1 and ρ_2 . It can be approximated that the cylindrical gear pair with ρ_1 and ρ_2 as the pitch radius is used to calculate the contact ratio instead of the noncircular gear [29]. The curvature radius of each point on the pitch curve of the elliptic gear is different, so the contact ratio in the meshing process is also variable.

The angle between the engagement line and the tangent of the pitch curve at point P is α_0 in Figure 6. When letting the intersection of the tooth top and the meshing line of gears 1 and 2 be A and B, respectively, and setting $\overline{PA} = L_1, \overline{PB} = L_2$, we can get

$$L_1 = \sqrt{(\rho_1 + h_{a1})^2 - (\rho_1 \cos \alpha_0)^2} - \rho_1 \sin \alpha_0,$$

$$L_2 = \sqrt{(\rho_2 + h_{a2})^2 - (\rho_2 \cos \alpha_0)^2} - \rho_2 \sin \alpha_0,$$

$$\rho_1 = \frac{A \left[1 + (i'_{12} / (1 + i_{12}))^2 \right]^{(3/2)}}{1 + i_{12} + i''_{12}}, \quad (12)$$

$$\rho_2 = \frac{A i_{12} \left[1 + (i'_{12} / (1 + i_{12}))^2 \right]^{(3/2)}}{1 + i_{12} - i_{12} i''_{12} + (i''_{12})^2}.$$

Therefore, the contact ratio of elliptic gear is

$$\varepsilon = \frac{L_1 + L_2}{\pi m \cos \alpha_0}. \quad (13)$$

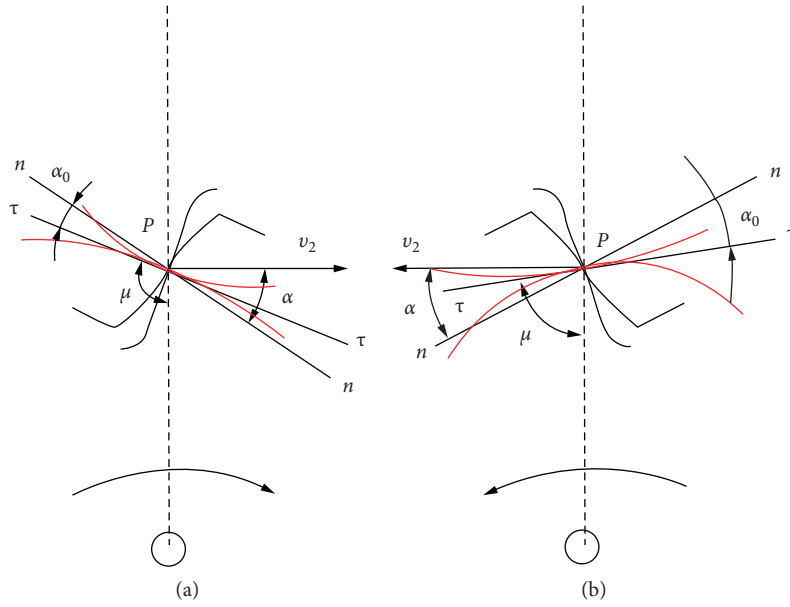


FIGURE 5: Pressure angle of elliptic gear profile. (a) Right profile and (b) left profile.

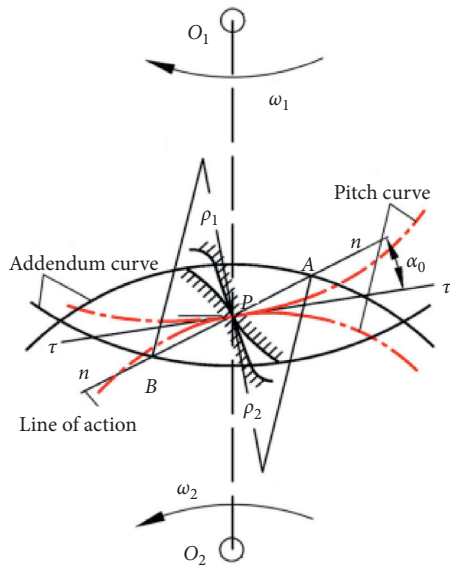


FIGURE 6: Contact ratios of elliptic gears.

2.4. Elliptic Gear Transmission Error Mathematical Model.

Figure 7 shows the meshing model of elliptic gear pairs with the same parameters. The center of rotation is at the left focal point of the elliptic curve, and gear 1 and gear 2 are, respectively, driving gear and driven gear. In Figure 7, O'_1 and O'_2 are the centers of rotation of the gears with eccentric errors, and their trajectories are circles with O_1 and O_2 as the center of the circle and eccentric errors e_1 and e_2 as the radius, respectively. If O'_1 and O'_2 are rotated to the positions of O_{10} and O_{20} , and point p_0 is the node at the time of standard meshing. At this time, R_{01} and R_{02} are the instantaneous meshing curve radii of the elliptic gear. The connection line of the two gear rotation centers is $O'_1O'_2$, and the tangent point of the two instantaneous pitch curves is P' .

The instantaneous meshing line PP' of the two gears intersects the line O_1O_2 of the theoretical center of revolution at point P . The parameters of the elliptic gear are shown in Table 1.

The coordinate system is established with the rotation center O_1 of driving gear 1 as the origin of coordinate axis and O_1O_2 as the y -axis. Then the coordinates of points O'_1 , P' , and O'_2 are, respectively,

$$\begin{cases} x_1 = -e_1 \sin(\varphi_1 + \theta_1), \\ y_1 = e_1 \cos(\varphi_1 + \theta_1) - R_{01} - R_{02} - e_1 - e_2, \\ x_2 = x_3 - \frac{R_{02}}{R_{01} + R_{02}} (x_3 - x_1), \\ y_2 = y_3 - \frac{R_{02}}{R_{01} + R_{02}} (y_3 - y_1), \\ x_3 = e_2 \sin(\varphi_2 + \theta_2), \\ y_3 = e_2 \cos(\varphi_2 + \theta_2). \end{cases} \quad (14)$$

It can be seen from Figure 8 that the instantaneous transmission ratio of the elliptic gear pair is

$$i'_{21} = \frac{R'_1}{R'_2}. \quad (15)$$

The theoretical transmission ratio is

$$i_{21} = \frac{R_{01}}{R_{02}}. \quad (16)$$

The transmission ratio error is

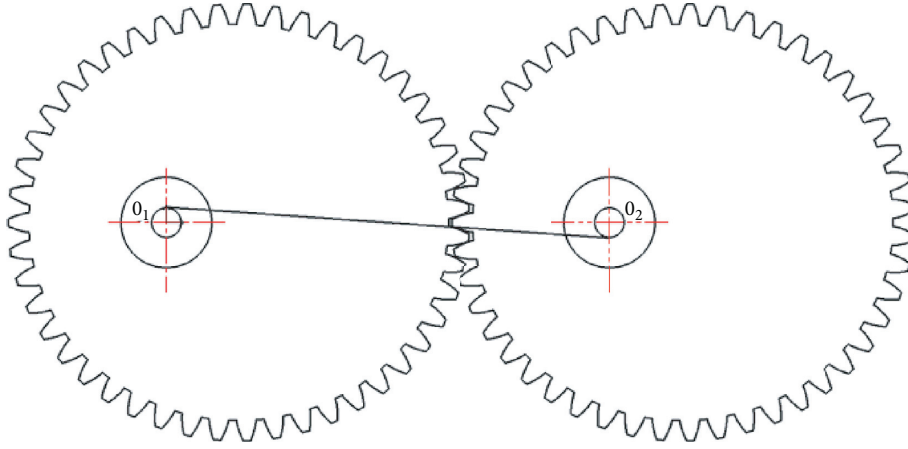


FIGURE 7: Meshing diagram of elliptic gear with installation error.

TABLE 1: Elliptic cylinder gear design parameters.

Parameter	Value
Modulus m/mm	3
Number of teeth Z	47
Center distance a/mm	150
Tooth height coefficient ha^a	1
Head clearance factor C^b	0.25
Tooth width B/mm	30
Eccentricity e	0.3287
Pitch curve equation r	$r = (64.667 / (1 \pm 0.3287 \cos \varphi))$

^aThe addendum coefficient; ^bthe backlash coefficient.

$$\Delta i_{21} = i'_{21} - i_{21} = \frac{R_{02} + R_{01} + e_1 + e_2}{R'_2} - \frac{R_{02} + R_{01}}{R_{02}}. \quad (17)$$

The slopes of line segments $O'_1O'_2$ and PP' are, respectively,

$$K_1 = \frac{y_3 - y_1}{x_3 - x_1}, \quad (18)$$

$$K_2 = \frac{1 + K_1 \tan \alpha}{\tan \alpha - K_1}.$$

Let $L_m = ((R_{01} + R_{02})/R_{01})$; the ordinate y_p of point P can be obtained as follows:

$$y_p = \frac{1}{L_m [\tan \alpha (x_1 - x_3) - (y_1 - y_3)]} \left[-y_1^2 - (L_m - 2)y_1y_3 - L_my_1y_3 \tan \alpha - x_1^2 + L_mx_1y_3 \tan \alpha - (L_m - 2)x_1x_3 + (L_m - 1)(x_3^2 + y_3^2) \right]. \quad (19)$$

At this time, the transmission ratio error is

$$\Delta i_{21} = \frac{R_{02} + R_{01} + e_1 + e_2}{|y_p|} - \frac{R_{02} + R_{01}}{R_{02}}. \quad (20)$$

For the pitch curve radius of the elliptic gear $R_{01}, R_{02} \gg e_1, e_2$, equation (20) can be simplified to obtain

$$\Delta i_{12} = \frac{1}{64.667} \frac{(1 - 0.3287 \cos t_1) (\cos(\varphi_1 + \theta_1 - \alpha))}{\cos \alpha} + \frac{(1 + 0.3287 \cos t_2)^2 \cos(\varphi_2 + \theta_2 - \alpha)}{\cos \alpha (1 - 0.3287 \cos t_1)}. \quad (21)$$

The ideal transmission ratio of a pair of elliptic gears is

$$i_{21} = \frac{d\theta_2}{d\theta_1}. \quad (22)$$

The actual transmission ratio is

$$i'_{21} = \frac{d\theta'_2}{d\theta_1}. \quad (23)$$

The transmission ratio error and the rotational angle error of driven gear are, respectively,

$$\Delta i_{21} = i'_{21} - i_{21} = \frac{d\theta'_2 - d\theta_2}{d\theta_1}, \quad (24)$$

$$\Delta \theta_2 = \int_0^{\theta_1} \Delta i_{21} d\theta_1.$$

If the parameters in Table 1 are brought in, the transmission error of the elliptic gear pair can be obtained as follows:

$$\Delta \theta_2 = \frac{1 + 0.3287 \cos t_2}{64.667 \cos \alpha} \left[e_1 [\sin(\theta_1 + \varphi_1 + \alpha) - \sin(\varphi_1 + \alpha)] + e_2 [\sin(\theta_2 + \varphi_2 + \alpha) - \sin(\varphi_2 + \alpha)] \right]. \quad (25)$$

3. Transmission Pressure Angle

According to equation (11), the variation of transmission pressure angle of elliptic gear under different eccentricity is obtained, as shown in Figure 9. When the eccentricity is 0, the transmission pressure angle is 20° , which is equivalent to the pressure angle of the spur gear. With the increasing of

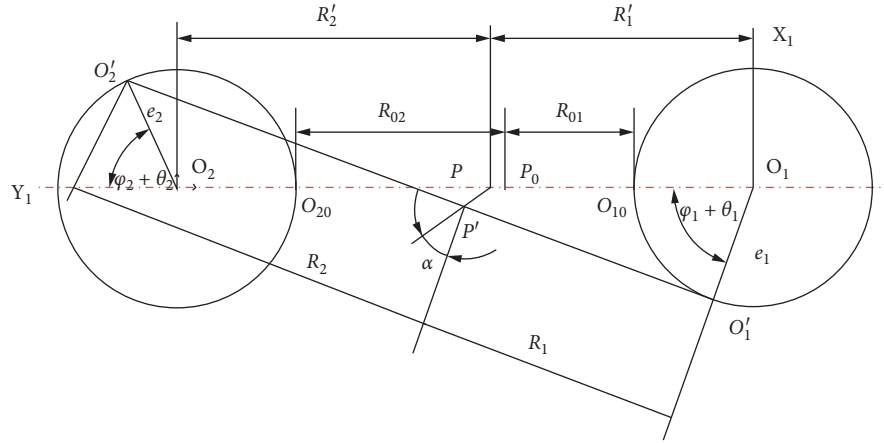


FIGURE 8: Calculation model of elliptic gear transmission error.

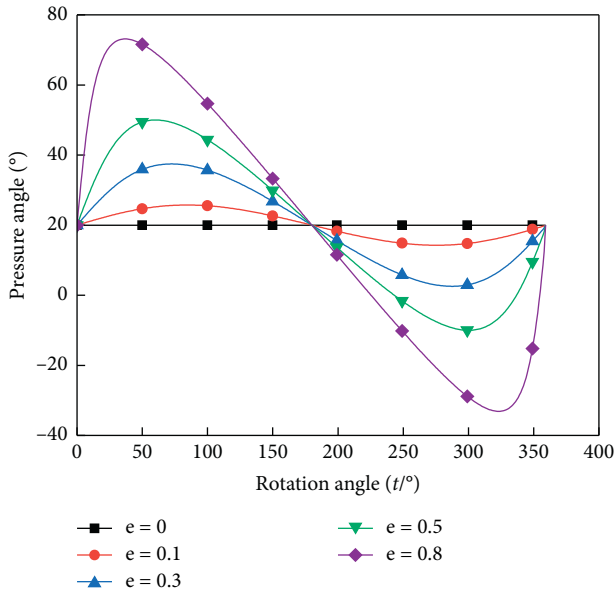


FIGURE 9: Transmission pressure angle of elliptic gear.

eccentricity, the transmission pressure angle also increases. The corresponding relationship between different eccentricity and the maximum value of transmission pressure angle is shown in Figure 10.

It is pointed out in [29] that when the pressure angle α is large, the force arm between the gear teeth is small, and the force is greater when the same torque is transmitted. When α is too large, self-locking may occur, making the gear unable to rotate. In order to avoid this phenomenon, the range of μ angle is between 45° and 135° ; at this time, the angle is less than 65° . According to the above theory, the derivative of equation (11) is obtained:

$$\begin{cases} \frac{d\alpha}{d\theta} = \frac{e^2 - e \cos \theta}{1 + e^2 - 2e \cos \theta}, \\ \frac{d\alpha}{d\theta} = 0 \implies e = \cos \theta. \end{cases} \quad (26)$$

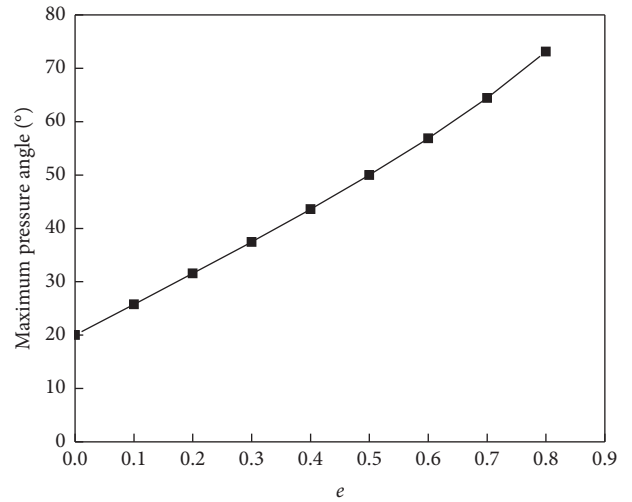


FIGURE 10: Relationships between eccentricity and maximum pressure angle.

Taking equation (26) into equation (11), we get

$$\begin{cases} \alpha_{\max} = \alpha_0 \pm \frac{\pi}{2} - \arctan\left(\frac{\sqrt{1-e^2}}{e}\right), \\ \alpha_{\max} \leq 65^\circ \implies e \leq \frac{\sqrt{2}}{2} \approx 0.7071. \end{cases} \quad (27)$$

Therefore, in order to ensure the normal transmission of the elliptic gear pair without self-locking, its eccentricity should be less than 0.7071.

4. Instantaneous Contact Ratios

The variation of the contact ratio of elliptic gears under different eccentricity is shown in Figure 11. If $e = 0$, the contact ratio is always 1.7547. When $e > 0$, the instantaneous contact ratio shows periodic change. There are two peaks in the contact ratio of the elliptic gear when it rotates for one circle. The reason is that the contact ratio of the elliptic gear

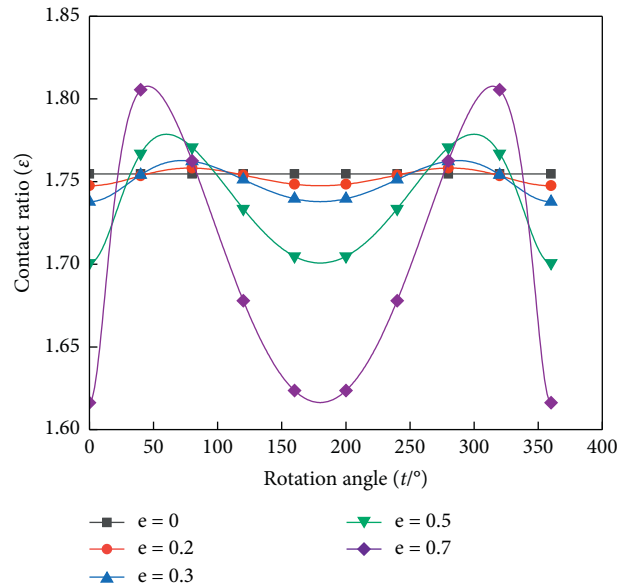


FIGURE 11: Instantaneous contact ratio of elliptic gear under different eccentricity.

is related to the curvature radius of the pitch curve. The elliptic pitch curve takes its long axis as the symmetry axis and distributes symmetrically. In this process, the variation of the curvature radius shows two-week distribution. In Figure 11, with the increase of eccentricity, the maximum contact ratio of elliptic gear increases; and when $e = 0.7$, the contact ratio is 1.8077.

It is the basic condition of gear continuous engagement that the contact ratio is not less than 1. The larger the contact ratio is, the more teeth are engaged in engagement at the same time, and the single tooth load will be reduced, which is conducive to improving the service life of gear. According to the analysis of transmission pressure angle in Chapter 3, the maximum eccentricity is 0.7071, and the maximum contact ratio of elliptic gear is 1.809 at this time.

5. Calculation of Transmission Error

5.1. Transmission Ratio Error. Due to the skew and eccentricity of the gear base circle axis relative to the rotation axis, the error of gear transmission ratio is produced. When the eccentricity is fixed, the change of rotation angle determines the transmission ratio error. When the elliptic gear takes a focal point of its elliptic pitch curve as its rotation center, it is an eccentric elliptic gear. Considering the installation error, the transmission ratio error of the elliptic gear under different eccentricity conditions is shown in Figure 12. In order to reflect the periodicity of transmission ratio error, the horizontal axis is taken as $0 \sim 720^\circ$, and the gear rotates for two cycles. With the increase of eccentricity, the fluctuation range of transmission ratio error increases gradually. The mean value and range of transmission ratio errors corresponding to different eccentricities are shown in Table 2. The mean value of transmission ratio error in Table 2 is basically about 0.015, and the range changes significantly with the increase of eccentricity. Transmission ratio error is mainly

caused by manufacturing error, gear clearance, and other factors.

5.2. Transmission Error Calculation and Simulation Verification. The variation trend of transmission error of elliptic gear with eccentricity is shown in Figure 13. With the increase of eccentricity, the transmission error also shows an increasing trend. The previous analysis shows that the eccentricity should be less than 0.7071, and the maximum transmission error of the gear is $2.201 \mu\text{m}$, which meets the design requirements. In order to verify the rationality of the analysis, the dynamic transmission error of the elliptic gear is obtained by using the finite element analysis, as shown in Figure 14. The black curve shows the no-load transmission error of the gear, and the red curve represents the dynamic transmission error of gear tooth obtained by LS-DYNA finite element software, which considered the load on the teeth.

In order to simulate the actual contact situation during the tooth engagement process, the following boundary conditions should be set: the inner ring of the rigid body shaft hole drives the gear body to rotate, the gear material is Solid-164 flexible body, and the inner hole of the shaft hole is Shell-163 rigid body. The driving and driven gears are limited to X, Y, Z three-direction moving degree of freedom and X, Y rotation degrees of freedom, and the rotation speed of the driving gear is 600 r/min. In the process of solving the tooth meshing model, the time step and the scale factor of the calculation time step are too large to interrupt the simulation, while the generation of negative volume is mostly caused by grid distortion, which is related to mesh quality and material and load conditions. Therefore, the appropriate time step should be taken to avoid the negative volume. The value of the debug time step scale factor TSSFAC is taken to be 0.5, and the time step DT2MS values

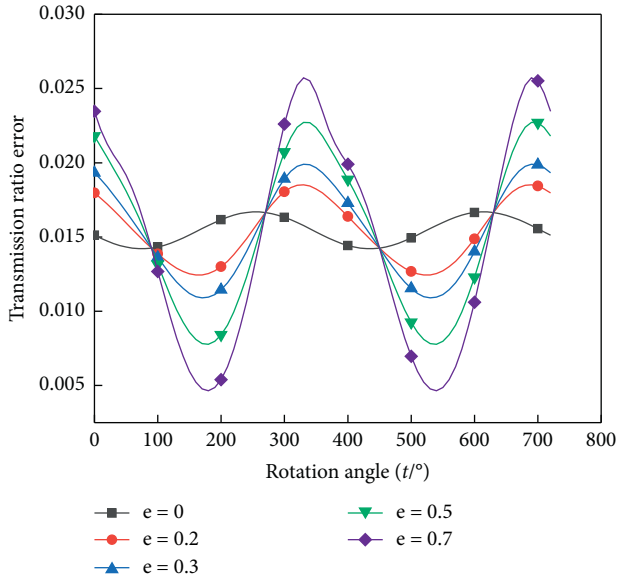


FIGURE 12: Transmission ratio error of elliptic gear under different eccentricity.

TABLE 2: Mean value and range of transmission ratio error of elliptic gear under different eccentricity.

Eccentricity	Transmission ratio error	
	Average value	Range
0	0.01545	0.00248
0.2	0.01538	0.00608
0.3	0.01534	0.00901
0.5	0.01521	0.01496
0.7	0.01496	0.02108

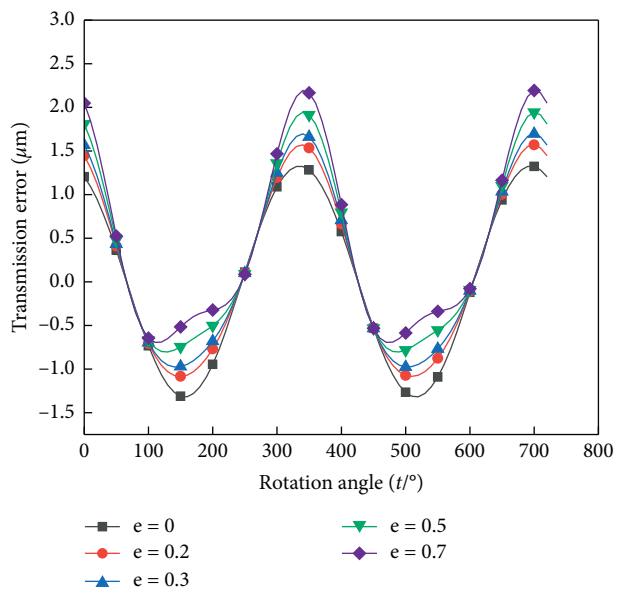


FIGURE 13: Transmission error of elliptic gear under different eccentricity.

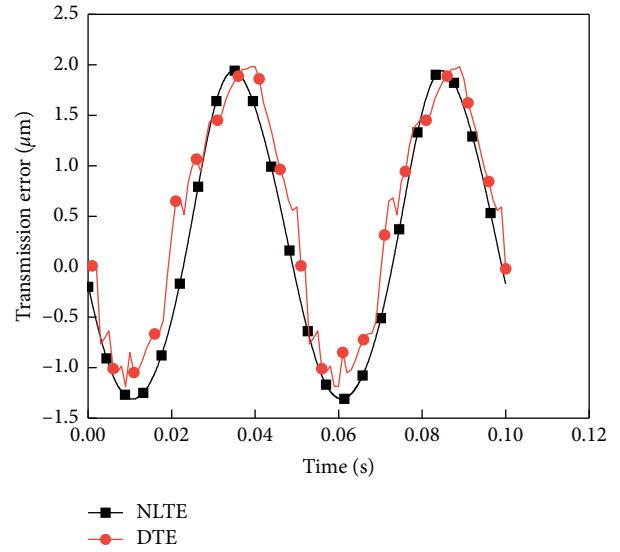


FIGURE 14: Simulation comparison of elliptic gear transmission error.

-2×10^{-7} can complete the analog tooth engagement. After gear meshing, the number of driving gear nodes is 205716, the number of units is 210211, the number of driven gear nodes is 181740, and the number of units is 186180.

The variation of the two curves is the same, but the dynamic transmission error curve has certain fluctuation. The reason is that the dynamic transmission error mainly considers the impact vibration, material damping, and other factors of the gear. In the process of analyzing gear tooth meshing, a load is applied and the penetration amount of gear tooth meshing is set. Also, it is assumed that the gear teeth have a certain rigidity, which makes the curve between the above two have a certain difference. The existence of dynamic backlash makes the curve of dynamic transmission error different from that of no-load transmission. The NLTE curve of theoretical analysis is relatively smooth, and there are violent fluctuations in the dynamic transmission error curve, which further shows that the calculation of dynamic transmission error by the finite element method truly reflects the meshing impact in the process of gear transmission.

5.3. Influence of Eccentricity Error on Gear Transmission Error. When manufacturing and installing the gear, it is inevitable to produce tooth shape error, pitch error, tooth direction error, and so forth, which will affect the transmission accuracy, transmission stability, and load distribution uniformity of the gear. The manufacturing error and installation error are reflected by the eccentric error of the gear. Figure 15 shows the effect of the eccentric error of the elliptic gear on the transmission error. With the increase of eccentricity error, the transmission error of elliptic gear is gradually increasing. Therefore, in order to ensure good transmission performance, the transmission error of gear can be reduced by improving the installation and manufacturing accuracy.

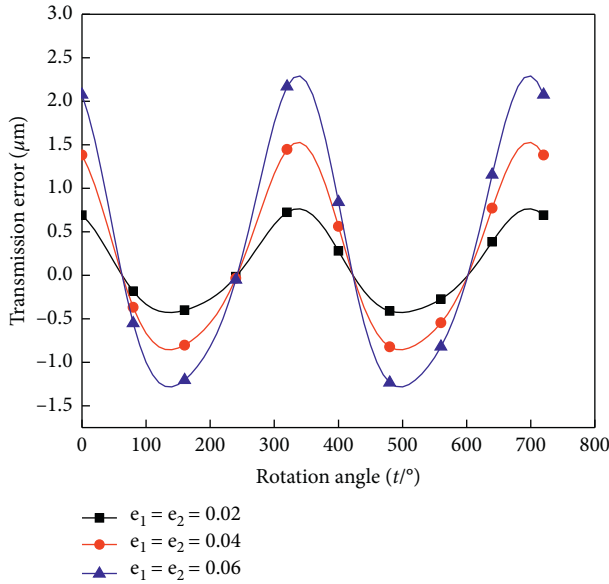


FIGURE 15: Influence of eccentricity error on transmission error of elliptic gear.

6. Conclusions

The eccentricity has a great influence on the meshing characteristics of the elliptic gear. By introducing the eccentricity error into the transmission error calculation model, the variation trend of transmission pressure angle, instantaneous contact ratio, and transmission error with eccentricity is obtained. The specific conclusions are as follows:

- (1) In order to ensure the normal meshing between the elliptic gears without self-locking, the eccentricity of the elliptic gears should be less than 0.7071.
- (2) The maximum instantaneous contact ratio of elliptic gear will increase with the increase of eccentricity. With the increase of contact ratio, the number of teeth participating in meshing at the same time will be increased, and the load of single tooth will be reduced, which is conducive to improving the service life of gears. Under the normal meshing condition, the maximum instantaneous contact ratio of the elliptic gear can reach 1.809.
- (3) The transmission ratio error and transmission error of the elliptic gear will increase with the increase of the eccentricity. In order to ensure the accuracy of the gear transmission, a small eccentricity can reduce the transmission error of the elliptic gear under the condition of satisfying the given motion law.
- (4) The eccentricity error has a significant effect on the transmission error of the elliptic gear. The transmission error will increase dramatically with the increase of the eccentricity error. The eccentricity error can be reduced by improving the installation and manufacturing accuracy of the elliptic gear, and then the transmission error of the elliptic gear can be reduced.

Symbols

- r_1, r_2 : Radii of pitch curves of driving and driven gears
- A : Long axis radius of ellipse
- c : Half focal length of ellipse
- e : Eccentricity of ellipse
- t_1, t_2 : The angle of the ellipse around the rotation center
- a : Center distance of elliptic gear
- α_0 : Profile angle of tool rack
- α : Transmission pressure angle
- $n-n$: Tangent pitch curve at point P
- s : Normal pitch curve at point P
- μ : Angle between tangent and normal
- ρ_1, ρ_2 : Curvature radius of pitch curve of driving and driven gears at point P
- ω_1, ω_2 : Rotation speed of driving and driven gears
- i_{12} : Transmission ratio
- ε : Contact ratio
- $O_1X_1Y_1$: Coordinate system built on the driving gear
- $O'_1O'_1$: The center of rotation of gear teeth under the condition of eccentricity error
- P_0 : Node in standard engagement
- R_{01}, R_{02} : The instantaneous curve radius of elliptic gear
- e_1, e_2 : Eccentricity error of driving and driven gears
- $R'_1R'_2$: The instantaneous curve radius of elliptic gear under the condition of eccentricity error
- φ_1, φ_2 : Initial phase angle of driving and driven gears
- θ_1, θ_2 : Ideal rotation angle of driving and driven gears
- x_1, y_1 : Coordinate of O'_1
- x_2, y_2 : Coordinate of P'
- x_3, y_3 : Coordinate of O'_2
- θ'_1, θ'_2 : Actual rotation angles of driving and driven gears.

Data Availability

The data used to support the findings of this study are included within the article, and the results of the article can be obtained by simply entering the various parameters into the formula derived in the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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