

Article

Two-Stage Adaptive Relay Selection and Power Allocation Strategy for Cooperative CR-NOMA Networks in Underlay Spectrum Sharing

Suoping Li ^{1,2,*} , Wenwu Liang ^{1,*}, Vicent Pla ³ , Nana Yang ¹ and Sa Yang ²¹ School of Sciences, Lanzhou University of Technology, Lanzhou 730050, China; xjyangnana@126.com² School of Electrical and Information Engineering, Lanzhou University of Technology, Lanzhou 730050, China; m18338530896@163.com³ Department of Communications, Universidad Politécnica de Valencia, 46022 Valencia, Spain; vpla@upv.es

* Correspondence: lsuop@163.com (S.L.); lwwlut@163.com (W.L.)

Abstract: In this paper, we consider a novel cooperative underlay cognitive radio network based on non-orthogonal multiple access (CR-NOMA) with adaptive relay selection and power allocation. In secondary networks, dedicated relay assistance and user assistance are used to achieve communication between the base station and the far (and near) user. Here, a two-stage adaptive relay selection and power allocation strategy is proposed to maximize the achievable data rate of the far user while ensuring the service quality of near user. Furthermore, the closed-form expressions of outage probability of two secondary users are derived, respectively, under interference power constraints, revealing the impact of transmit power, number of relays, interference threshold and target data rate on system outage probability. Numerical results and simulations validate the advantages of the established cooperation and show that the proposed adaptive relay selection and power allocation strategy has better outage performance.

Keywords: cognitive radio networks; non-orthogonal multiple access; underlay spectrum sharing; relay selection; power allocation



Citation: Li, S.; Liang, W.; Pla, V.; Yang, N.; Yang, S. Two-Stage Adaptive Relay Selection and Power Allocation Strategy for Cooperative CR-NOMA Networks in Underlay Spectrum Sharing. *Appl. Sci.* **2021**, *11*, 10433. <https://doi.org/10.3390/app112110433>

Academic Editor:
Subhas Mukhopadhyay

Received: 25 September 2021
Accepted: 4 November 2021
Published: 6 November 2021

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.



Copyright: © 2021 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Industrial Internet is a significant breakthrough to accelerate the commercial deployment of 5G [1]. However, the unprecedented increase of spectrum occupancy makes the improvement of spectrum efficiency become a research hotspot in academia and industry. As one of the key technologies to implement 5G wireless communication, non-orthogonal multiple access (NOMA) has received much attention due to the characteristics of high spectrum utilization, low latency, and large-scale connectivity [2–4]. Cognitive radio networks (CRNs) can effectively alleviate the problem of spectrum shortage and spectrum underutilization by dynamically reusing the frequency assigned to the primary user [5]. Combined with the above advantages, cognitive radio networks based on non-orthogonal multiple access (CR-NOMA) are expected to further improve spectrum utilization efficiency and communication reliability.

Power-domain NOMA transmits multiple information streams with different powers over the overlapping channel in time/frequency/code domain. Classical cooperative NOMA systems are generally divided into two categories: one uses the dedicated relay to assist communication between the source and the user, and the other uses the user as a relay [6]. The cooperative NOMA scheme was proposed in [7] for the first time. Authors in [8] investigated a downlink cooperative NOMA scenario with two users, in which the near user acted as a full-duplex (FD) relay for the far user. NOMA systems with dedicated relay have achieved a large number of research achievements [9–13]. In multi-relay networks, relay selection is a low-complexity approach that can achieve the

desired full diversity gain [14,15]. In [9], a two-stage max-min relay selection strategy based on the cooperative NOMA system was proposed to ensure the quality of service (QoS) requirements of users. However, the outage performance obtained by using the fixed power allocation scheme is not optimal. Authors in [10] developed a two-stage relay selection strategy based on decode-and-forward (DF) and amplify-and-forward (AF) relays with adaptive power allocation. This scheme not only obtained full diversity gain, but also reduced the outage probability of the cooperative NOMA system. Two relay selection strategies named two-stage weighted-max-min (WMM) with fixed power allocation and max-weighted-harmonic-mean (MWHM) with adaptive power allocation were proposed in [11], and research results confirmed that the reasonable power allocation coefficient has an important effect on system performance. To extend the work of [10,11], researchers in [16] established an optimization model by taking rate fairness and imperfect channel into consideration.

However, the above cooperative NOMA systems considered user cooperation and relay cooperation separately. It is obvious that if both user cooperation and dedicated relay cooperation are adopted, diversity gain and overall system performance will be improved. Coordinated direct and relay transmission into NOMA systems was introduced by [17], where the far user obtained the side information from the relay or near user. Compared with non-coordinated direct and relay transmission, the scheme in [17] achieved greater capacity gain. An AF relay selection strategy was proposed for a two-user downlink NOMA system in [18], where the near user was likely to be the optimal relay if it could decode the superimposed signal successfully; otherwise, an optimal relay was selected to serve both users. However, power adaptation was not considered in [17,18]. Therefore, the authors in [6] extended the model of [18] and proposed a two-stage relay selection scheme with power adaptation to obtain $N + 1$ full diversity gain for users, where N is the number of relays.

In addition, dynamic spectrum or bandwidth allocation effectively improves the utilization of network resources by rationalizing their allocation [19]. For example, cognitive radio is a classical technology to solve the problem of spectrum scarcity. The integration of CRNs and NOMA could further improve spectrum efficiency, and the core idea of CR-NOMA is to provide opportunistic services for secondary users while ensuring the QoS of primary users. Recently, many works have been carried out to evaluate and analyze the performance of cooperative CR-NOMA networks, in which relay selection and power allocation are the focus. Overlay CR-NOMA networks with user cooperation were considered in [20,21]. In [20], the signals of primary users and secondary users were relayed by secondary transmitters; while in [21], the optimal decoded secondary user was selected to relay the signals of undecoded users. A different overlay relay cooperation was proposed in [22] in which a dedicated NOMA-based relay was used to forward assistant signals to primary users and secondary users. However, the above research do not consider the cooperative coexistence of user cooperation and dedicated relay cooperation, nor do they consider the power adaption.

Unlike the overlay mode, the underlay mode of CRNs pays more attention to ensuring the performance of secondary networks under interference constraints. In an underlay CR-NOMA system [23], signals of two secondary users were forwarded using the dedicated NOMA-based relay. In [24], authors investigated the performance of two-hop underlay CR-NOMA networks under interference constraints, but using a single relay could not ensure reliable communication at the user side. Thus, extending the model to multi-relay cooperation is necessary. In [25], authors analyzed the outage performance of CR-NOMA networks with AF relays by using the partial relay selection method, and results indicated that the adoption of adaptive power allocation strategy helps to improve system performance. In [26], authors studied the impact of multiple antennas and cooperative NOMA users on the performance of CR-NOMA, in which the multi-antenna base station selected the user with strong channel gain as a relay to help another user with poor channel gain. Results showed that the cell edge user with poor channel gain can benefit from

cooperative NOMA and opportunistic relay transmission [26]. Different from the above research, authors in [27,28] considered the coexistence of direct link and relay-cooperative links between the far user and base station, in which relays existed to compensate for the reduction of reliability and coverage of cognitive networks due to power constraints. However, user cooperation is not considered in [23–25,27,28]. The scenario developed in [29] considered the link from near user to far user in addition to links from relays to far user, but the transmit power constraints were not fully taken into account.

However, cooperative CR-NOMA in underlay spectrum sharing mode still has the following problems: (i) most CR-NOMA systems only consider partial interference links; (ii) partial relay selection strategy based on the first hop may not result in the optimal outage performance; (iii) fixed power allocation coefficient cannot guarantee optimal system performance; (iv) the links between near user and relays, near user and far user should be considered, as this can effectively improve communication efficiency.

Inspired by [6,28], this paper investigates adaptive relay selection and power allocation in cooperative underlay CR-NOMA networks by considering mutual interference and constraints between primary network and secondary network. In order to guarantee the reliable transmission of the primary network, transmit powers of secondary source and the selected relay are restricted. The main contributions are summarized as follows:

- A novel cooperation scheme is established for underlay CR-NOMA networks, where dedicated relay cooperation and user cooperation are used in secondary network to achieve communication between the base station and the far (and near) user.
- A two-stage adaptive relay selection and power allocation strategy is proposed to maximize the achievable data rate of the far user and to obtain the optimal power allocation coefficient on the premise of guaranteeing the QoS of near user. Through the proposed strategy, the challenging issue of whether only the far user needs assistance or both the near and the far users need assistance is addressed.
- A novel decoding order for successive interference cancellation (SIC) is introduced to the secondary users. Both secondary users can adaptively adjust their decoding order in the second time slot according to the decoding status of the near user in the first time slot, which is different from the existing works that mostly depend on the channel quality.
- The closed-form expressions of outage probabilities of secondary users are obtained by deriving the corresponding distribution functions, and the impact of transmit power, number of relays, interference threshold and target data rate on the system outage probability is revealed.
- Numerical results and simulations validate the advantages of the established cooperation, and the proposed adaptive relay selection and power allocation strategy shows better outage performance compared to the existing schemes.

This paper is organized as follows. In Section 2, we describe the system model and transmission process. Then, a two-stage adaptive relay selection and power allocation strategy is introduced in Section 3. Section 4 investigates the outage probabilities of two secondary users and the numerical results are presented in Section 5. Finally, Section 6 concludes this paper.

2. System Model and Transmission Process

Firstly, we develop a cooperative underlay CR-NOMA network model. Then, by describing the signal model, we propose the transmission process and analyze the achievable data rate at the receiver. Four possible decoding results at U_1 are given, and the transmit powers of the secondary and the candidate relay sets are expressed, respectively.

2.1. System Model

As shown in Figure 1, we consider a communication scenario working in underlay mode. The primary network consists of a pair of primary users, the primary transmitter (PT) and the primary receiver (PR). The secondary network is a downlink NOMA system,

consisting of a base station (BS), multiple relays ($R_n, n = 1, 2, \dots, N$), a near user (U_1) and a far user (U_2). While PT sends a message to PR, BS broadcasts a superimposed signal composed of U_1 and U_2 messages. U_1 can receive the signal via the direct link $BS \rightarrow U_1$. However, due to serious shadow or path loss, U_2 obtains the signal via relay nodes. The characteristic of the system model is that if U_1 fails to decode the signal from the direct link, relays are first sought for help. Otherwise, U_1 has a chance to become a candidate to assist U_2 . The system model is also based on the following assumptions:

- All communication links and interference links are quasi-static Rayleigh fading channels, i.e., $h_{XY} \sim CN(0, \lambda_{XY})$ where XY denotes the link $X \rightarrow Y$ between the transmitting node X and the receiving node Y ;
- Each channel is affected by additive white Gaussian noise (AWGN) with the mean of 0 and the variance of σ^2 ;
- All nodes (including transmitters, receivers and DF-based relays) with single antenna work in half-duplex (HD) mode and can perfectly receive channel state information (CSI);
- BS uses fixed power allocation while relays can adaptively adjust their power allocation coefficient;
- The transmit power of BS, R_n and U_1 is adaptive constrained in underlay mode.

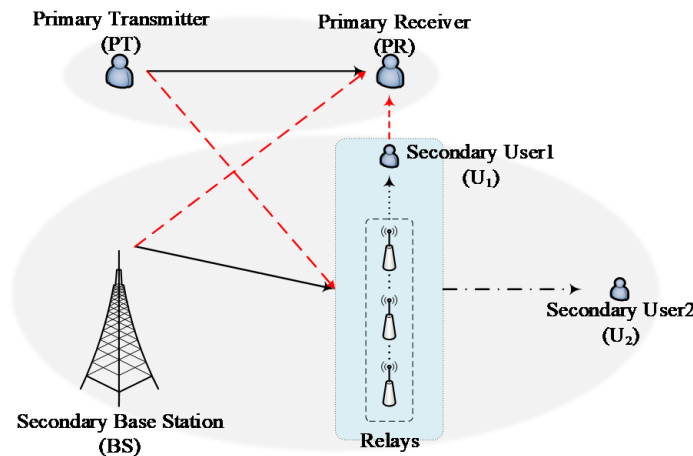


Figure 1. System model.

2.2. Transmission Process

In underlay CR-NOMA networks, the primary and secondary networks transmit messages simultaneously within the same spectrum. In the primary network, PT sends a signal, x_p , to PR with the transmit power, P_T , where $E\{|x_p|^2\} = 1$. In the secondary network, the transmission process consists of direct transmission stage and cooperative transmission stage, which are completed in two time slots as follows.

2.2.1. Direct Transmission Stage of the First Time Slot

In the first time slot, BS broadcasts the superimposed signals of U_1 and U_2 via the superimposed coding (SC). After receiving the signal, decoding result from U_1 will be sent to all relays, and the decodable relays will be divided into two candidate sets. The details are as follows.

BS broadcasts a superimposed signal, $x_S = \sqrt{\alpha_1}x_1 + \sqrt{\alpha_2}x_2$, which can be received by U_1 and the potential relays with the transmit power, P_S . Here, x_1 and x_2 are the signals for U_1 and U_2 with $E\{|x_1|^2\} = E\{|x_2|^2\} = 1$, and α_1 and α_2 denote the power allocation coefficients at BS for messages x_1 and x_2 with $\alpha_1 + \alpha_2 = 1$ and $\alpha_2 \geq \alpha_1 \geq 0$, respectively. Therefore, the received signal at the n -th relay can be expressed as

$$y_{SR_n} = \sqrt{P_T}h_{TR_n}x_p + \sqrt{P_S}h_{SR_n}x_S + \eta_{R_n} \tag{1}$$

where η_{R_n} is AWGN at the n -th relay R_n .

Then, SIC technique is used to try to decode the received superimposed signal. The signal with high power (or poor channel quality) is first decoded, i.e., at the n -th relay, x_2 is first decoded and x_1 is treated as interference, and then x_1 can be decoded only after x_2 successfully decodes and cancels. Therefore, the achievable rates of x_2 and x_1 at the relay R_n are given by

$$R_{SR_n}^{x_2} = \frac{1}{2} \log_2 \left(1 + \frac{\alpha_2 \rho_S |h_{SR_n}|^2}{\rho_T |h_{TR_n}|^2 + \alpha_1 \rho_S |h_{SR_n}|^2 + 1} \right) \text{ and } R_{SR_n}^{x_1} = \frac{1}{2} \log_2 \left(1 + \frac{\alpha_1 \rho_S |h_{SR_n}|^2}{\rho_T |h_{TR_n}|^2 + 1} \right) \quad (2)$$

respectively, where $\rho_T = P_T/\sigma^2$ and $\rho_S = P_S/\sigma^2$ are the transmit signal-to-noise ratio (SNR). Relays that can successfully decode x_1 and x_2 are divided into two candidate sets, $S_1 = \{n | n \in N, R_{SR_n}^{x_2} \geq R_2\}$ and $S_2 = \{n | n \in N, R_{SR_n}^{x_2} \geq R_2, R_{SR_n}^{x_1} \geq R_1\}$, respectively, where R_1 and R_2 denote the target data rates of x_1 and x_2 at the receiving nodes, respectively.

The received signal at U_1 can be expressed as

$$y_{SU_1} = \sqrt{P_T} h_{TU_1} x_P + \sqrt{P_S} h_{SU_1} x_S + \eta_{U_1} \quad (3)$$

where η_{U_1} is AWGN at U_1 . After using SIC, the achievable rates of x_2 and x_1 at U_1 are given, respectively, as

$$R_{SU_1}^{x_2} = \frac{1}{2} \log_2 \left(1 + \frac{\alpha_2 \rho_S |h_{SU_1}|^2}{\rho_T |h_{TU_1}|^2 + \alpha_1 \rho_S |h_{SU_1}|^2 + 1} \right) \text{ and } R_{SU_1}^{x_1} = \frac{1}{2} \log_2 \left(1 + \frac{\alpha_1 \rho_S |h_{SU_1}|^2}{\rho_T |h_{TU_1}|^2 + 1} \right) \quad (4)$$

Note that the prerequisite for a receiving node to decode x_1 is that x_2 has been successfully detected and cancelled, i.e., $R_{SR_n}^{x_1}$ and $R_{SU_1}^{x_1}$ are achievable under the conditions that $R_{SR_n}^{x_2} \geq R_2$ and $R_{SU_1}^{x_2} \geq R_2$ in Equation (2) and Equation (4), respectively.

At the end of the first time slot, the decoding result is sent to relays as a signaling. There are two possible cases: (1) U_1 fails to get the desired message in the first time slot. Then, it will send a signaling to relays and an optimal relay from the candidate set will be selected to forward the superimposed signal to U_1 and U_2 in the next time slot; (2) U_1 successfully decodes the desired message. Then, a signaling will be sent by U_1 and an optimal relay from the candidate set or U_1 will be selected to forward the signal that U_2 expects. Note that U_1 sends the signaling to relays instead of BS [30–32].

Then, we can obtain the signaling sent by U_1 at the end of the first slot. Let Θ ($\Theta \in \{11, 10, 01, 00\}$) denote the signaling sent by U_1 . As shown in Table 1, $\Theta = '11'$ indicates that U_1 successfully decodes x_2 and x_1 and does not need the help of relays; $\Theta = '10'$ means U_1 correctly decodes x_2 but cannot detect x_1 , so it needs the help of relays; $\Theta = '01'$ indicates that U_1 fails to decode x_2 but successfully detects x_1 , so if U_1 can receive x_2 in the next time slot, it can successfully extract x_1 from the superimposed signal obtained in the first slot; $\Theta = '00'$ is that U_1 fails to decode x_2 and x_1 even if x_2 is obtained in the next slot.

Table 1. The signaling sent by U_1 .

Θ	The First Time Slot		The Second Time Slot	
	Descriptions	Candidate Relay Sets	$x_{R_n}^\Theta$	Decoding Order
11	$R_{SU_1}^{x_2} \geq R_2, R_{SU_1}^{x_1} \geq R_1$	$S_1 = \{n n \in N, R_{SR_n}^{x_2} \geq R_2\}$ and U_1	x_2	$x_2 \rightarrow x_1$
10	$R_{SU_1}^{x_2} \geq R_2, R_{SU_1}^{x_1} < R_1$	$S_2 = \{n n \in N, R_{SR_n}^{x_2} \geq R_2, R_{SR_n}^{x_1} \geq R_1\}$	$\sqrt{\beta_{n1}^{10}} x_1 + \sqrt{\beta_{n2}^{10}} x_2$	$x_1 \rightarrow x_2$
01	$R_{SU_1}^{x_2} < R_2, R_{SU_1}^{x_1} \geq R_1$	$S_1 = \{n n \in N, R_{SR_n}^{x_2} \geq R_2\}$	x_2	$x_2 \rightarrow x_1$
00	$R_{SU_1}^{x_2} < R_2, R_{SU_1}^{x_1} < R_1$	$S_2 = \{n n \in N, R_{SR_n}^{x_2} \geq R_2, R_{SR_n}^{x_1} \geq R_1\}$	$\sqrt{\beta_{n1}^{00}} x_1 + \sqrt{\beta_{n2}^{00}} x_2$	$x_1 \rightarrow x_2$

2.2.2. Cooperative Transmission Stage of the Second Time Slot

In the second time slot, the cooperative links are activated. An optimal relay is selected to forward the signal from BS by using adaptive power allocation coefficients. There are two possible scenarios:

- (1) Relay R_n is selected as the optimal relay. R_n will re-encode the decoded signal and generate a new superimposed signal, $x_{R_n}^\ominus = \sqrt{\beta_{n1}^\ominus}x_1 + \sqrt{\beta_{n2}^\ominus}x_2$, according to SC. Here β_{n1}^\ominus and β_{n2}^\ominus denote the power allocation coefficients at the optimal relay for messages x_1 and x_2 with $\beta_{n1}^\ominus \geq 0$, $\beta_{n2}^\ominus \geq 0$ and $\beta_{n1}^\ominus + \beta_{n2}^\ominus = 1$, respectively. U_2 is not affected by PT because it is far from PT. Therefore, the received signal at U_2 and U_1 can be expressed as

$$y_{R_n U_2} = \sqrt{P_R}h_{R_n U_2}x_{R_n}^\ominus + \eta_{U_2} \tag{5}$$

and

$$y_{R_n U_1} = \sqrt{P_T}h_{T U_1}x_P + \sqrt{P_R}h_{R_n U_1}x_{R_n}^\ominus + \eta_{U_1} \tag{6}$$

respectively, where P_R is the transmit power of relays and U_1 .

- (2) U_1 is selected as the optimal relay. Then, U_1 will forward x_2 with full power because it has successfully decoded both x_1 and x_2 . The received signal at U_2 is

$$y_{U_1 U_2} = \sqrt{P_R}h_{U_1 U_2}x_2 + \eta_{U_2} \tag{7}$$

and the achievable rate of x_2 at U_2 is given by

$$R_{U_1 U_2}^{x_2} = \frac{1}{2} \log_2 \left(1 + \rho_R |h_{U_1 U_2}|^2 \right) \tag{8}$$

where $\rho_R = P_R/\sigma^2$ is the transmit SNR for relays and U_1 .

2.3. Transmit Power Constraints in Secondary Networks

In underlay mode, primary and secondary users in CRNs can work within the same spectrum simultaneously. However, if the secondary network has no transmit power constraint, it will cause serious signal interference to the primary network. Therefore, in order to ensure the performance of the primary network, in this paper, BS and the selected relay adaptively adjust their transmit powers.

Next, I_{th} is defined as a threshold and represents the maximum tolerable interference level of primary network, at which reliable communication can be guaranteed. Assuming that the maximum permissible transmit powers of BS and relays are P_1 and P_2 , respectively, the transmit powers of BS and relays are constrained as [33].

$$P_S = \min \left\{ \frac{I_{th}}{|h_{SR}|^2}, P_1 \right\} \text{ and } P_R = \min \left\{ \frac{I_{th}}{|h_{R_n R}|^2}, P_2 \right\}. \tag{9}$$

Especially, when U_1 is used as an optimal relay, $h_{R_n R}$ in (9) will be replaced by $h_{U_1 R}$.

3. Two-Stage Adaptive Relay Selection and Power Allocation Strategy

In this section, a two-stage adaptive relay selection and power allocation strategy is proposed to maximize the achievable data rate of x_2 at U_2 while guaranteeing the QoS of U_1 . The selection criteria of the optimal relay change adaptively according to the signal sent by U_1 at the end of the first time slot (i.e., the decoding result of U_1). After all relays receive the signal, the candidate sets will be determined, and the optimal relay will be selected from the candidate sets or U_1 to transmit the user's desired signal. There are two stages:

- In the first stage, the relays are selected from the candidate sets to ensure that the achievable rate of the transmitted signal is not lower than the target data rate of U_1 ;
- In the second stage, a relay that maximizes the achievable rate of x_2 at U_2 is selected from the relays determined in the first stage.

Power allocation of the optimal relay is adaptive. That means the power allocation needs to ensure the QoS of U_1 first, then the remaining power is allocated to U_2 . After sending the signaling, U_1 will adjust its decoding order in the second time slot according to decoding result. In particular, if U_1 fails to decode its own signal in the first time slot, it will be considered as a “weak user” in the second time slot. Therefore, according to the signaling sent by U_1 , four relay selection criteria with power allocation are designed as follows.

Case 1: U_1 successfully decodes x_2 and x_1 ($\Theta = '11'$)

In this case, U_1 successfully decodes x_2 and x_1 in the first time slot and does not need the assistance from relays in the second time slot. The optimal relay from S_1 or U_1 is selected to transmit x_2 to U_2 .

If $R_n, n \in S_1$ is selected, the power allocation coefficient will be set as $\beta_{n2}^{11} = 1$. It means that R_n will send $x_{R_n}^{11} = x_2$ with full power to U_2 . Based on Equation (5), the achievable rate of x_2 at U_2 is given by

$$R_{R_n U_2}^{x_2, 11} = \frac{1}{2} \log_2 \left(1 + \rho_R |h_{R_n U_2}|^2 \right) \tag{10}$$

If all the relays cannot meet the target data rate requirements of U_2 , U_1 is used as the optimal relay. The criterion for determining the optimal relay R_{n^*} is given by

$$n^* = \begin{cases} 0, & \max_{n \in S_1} \{ R_{R_n U_2}^{x_2, 11} \} < R_2, \\ \operatorname{argmax}_{n \in S_1} \{ R_{R_n U_2}^{x_2, 11} \}, & \text{otherwise.} \end{cases} \tag{11}$$

where $n^* = 0$ suggests that U_1 is selected as the optimal relay. Different from the selection strategy in [29], relay participation is preferred in this paper.

Case 2: U_1 successfully decodes x_2 but fails to decode x_1 ($\Theta = '10'$)

In this case, both U_1 and U_2 need the assistance from relays in the second time slot. The optimal relay is selected from S_2 . Assuming that $R_n, n \in S_2$ is selected, it will generate a new superimposed signal $x_{R_n}^{10} = \sqrt{\beta_{n1}^{10}} x_1 + \sqrt{\beta_{n2}^{10}} x_2$ based on SC to serve U_1 and U_2 . Note that U_1 is considered as the weak user in the second slot because it fails to decode its own signal in the first slot. Based on SIC, x_1 is decoded first after U_2 receives the superimposed signal. Based on Equation (5), the achievable rates of x_1 and x_2 at U_2 can be expressed as

$$R_{R_n U_2}^{x_1, 10} = \frac{1}{2} \log_2 \left(1 + \frac{\beta_{n1}^{10} \rho_R |h_{R_n U_2}|^2}{\beta_{n2}^{10} \rho_R |h_{R_n U_2}|^2 + 1} \right) \text{ and } R_{R_n U_2}^{x_2, 10} = \frac{1}{2} \log_2 \left(1 + \beta_{n2}^{10} \rho_R |h_{R_n U_2}|^2 \right) \tag{12}$$

Since U_1 has successfully decoded x_2 in the first time slot, it can remove x_2 from the superimposed signal received in the second time slot. Thus, U_1 can decode x_1 without interference from x_2 . Therefore, based on Equation (6), the achievable rate of x_1 at U_1 is given by

$$R_{R_n U_1}^{x_1, 10} = \frac{1}{2} \log_2 \left(1 + \frac{\beta_{n1}^{10} \rho_R |h_{R_n U_1}|^2}{\rho_T |h_{T U_1}|^2 + 1} \right) \tag{13}$$

The prerequisite for U_2 to decode x_2 is that x_1 is successfully decoded and canceled, i.e., U_2 must satisfy the condition of $R_{R_n U_2}^{x_1, 10} \geq R_1$. In addition, in order to ensure the QoS of U_1 , the signal x_1 at U_1 should satisfy the decoding requirements of $R_{R_n U_1}^{x_1, 10} \geq R_1$. Therefore, in this case, the criterion for determining the optimal relay R_{n^*} is given by

$$n^* = \operatorname{argmax}_{n \in S_2} \{ R_{R_n U_2}^{x_2, 10} \} \text{ subject to } R_{R_n U_1}^{x_1, 10} \geq R_1 \text{ and } R_{R_n U_2}^{x_1, 10} \geq R_1 \tag{14}$$

From Equations (12)–(14), the optimal power allocation coefficient can be obtained by

$$\beta_{n2}^{10} = \min \left\{ \frac{[\rho_R |h_{R_n U_2}|^2 - \gamma_1]^+}{\rho_R |h_{R_n U_2}|^2 (1 + \gamma_1)}, \frac{[\rho_R |h_{R_n U_1}|^2 - \gamma_1 (\rho_T |h_{T U_1}|^2 + 1)]^+}{\rho_R |h_{R_n U_1}|^2} \right\}, \quad (15)$$

where $\gamma_1 = 2^{2R_1} - 1$, $\gamma_2 = 2^{2R_2} - 1$, and $[x]^+ = \max\{0, x\}$.

Case 3: U_1 fails to decode x_2 but successfully detects x_1 ($\Theta = '01'$)

In this case, U_1 fails to decode x_2 but successfully detects x_1 in the first time slot. Thus, if U_1 obtains x_2 in the next slot, it can successfully extract x_1 from the superimposed signal obtained in the first slot. The optimal relay is selected from S_1 . Suppose that $R_n, n \in S_1$ is selected, the power allocation coefficient is set as $\beta_{n2}^{01} = 1$. This means that R_n will send $x_{R_n}^{01} = x_2$ with full power to serve U_1 and U_2 . Based on Equations (5) and (6), the achievable rate of x_2 at U_2 and U_1 can be given as

$$R_{R_n U_2}^{x_2, 01} = \frac{1}{2} \log_2 (1 + \rho_R |h_{R_n U_2}|^2) \text{ and } R_{R_n U_1}^{x_2, 01} = \frac{1}{2} \log_2 \left(1 + \frac{\rho_R |h_{R_n U_1}|^2}{\rho_T |h_{T U_1}|^2 + 1} \right) \quad (16)$$

Then, the criterion for determining the optimal relay R_{n^*} is given by

$$n^* = \underset{n \in S_1}{\operatorname{argmax}} \{ R_{R_n U_2}^{x_2, 01} \} \text{ subject to } R_{R_n U_1}^{x_2, 01} \geq R_2 \quad (17)$$

Case 4: U_1 fails to decode x_2 and x_1 ($\Theta = '00'$)

In this case, both U_1 and U_2 need the assistance from relays in the second slot. The optimal relay is selected from S_2 . Assuming that $R_n, n \in S_2$ is selected, it will generate a new superimposed signal $x_{R_n}^{00} = \sqrt{\beta_{n1}^{00}} x_1 + \sqrt{\beta_{n2}^{00}} x_2$ based on SC to serve U_1 and U_2 . U_1 is also considered as the weak user in this case. Based on SIC, x_1 is decoded firstly after U_1 and U_2 receive the superimposed signal. Based on Equation (5), the achievable rates of x_1 and x_2 at U_2 are given as

$$R_{R_n U_2}^{x_1, 00} = \frac{1}{2} \log_2 \left(1 + \frac{\beta_{n1}^{00} \rho_R |h_{R_n U_2}|^2}{\beta_{n2}^{00} \rho_R |h_{R_n U_2}|^2 + 1} \right) \text{ and } R_{R_n U_2}^{x_2, 00} = \frac{1}{2} \log_2 (1 + \beta_{n2}^{00} \rho_R |h_{R_n U_2}|^2) \quad (18)$$

Unlike case 2, U_1 does not get any signal in the first time slot. The achievable rate of x_1 at U_1 in the second time slot is given by

$$R_{R_n U_1}^{x_1, 00} = \frac{1}{2} \log_2 \left(1 + \frac{\beta_{n1}^{00} \rho_R |h_{R_n U_1}|^2}{\rho_T |h_{T U_1}|^2 + \beta_{n2}^{00} \rho_R |h_{R_n U_1}|^2 + 1} \right) \quad (19)$$

Similarly, the criterion for determining the optimal relay R_{n^*} is given by

$$n^* = \underset{n \in S_2}{\operatorname{argmax}} \{ R_{R_n U_2}^{x_2, 00} \} \text{ subject to } R_{R_n U_1}^{x_1, 00} \geq R_1 \text{ and } R_{R_n U_2}^{x_1, 00} \geq R_1. \quad (20)$$

Then, the optimal power allocation coefficient can be obtained by

$$\beta_{n2}^{00} = \min \left\{ \frac{[\rho_R |h_{R_n U_2}|^2 - \gamma_1]^+}{\rho_R |h_{R_n U_2}|^2 (1 + \gamma_1)}, \frac{[\rho_R |h_{R_n U_1}|^2 - \gamma_1 (\rho_T |h_{T U_1}|^2 + 1)]^+}{\rho_R |h_{R_n U_1}|^2 (1 + \gamma_1)} \right\}. \quad (21)$$

The above four cases correspond to four decoding results of U_1 at the end of the first time slot. The adaptability of relay selection is reflected in that each decoding result of U_1 has its own relay selection criteria. Based on the proposed two-stage adaptive relay

selection and power allocation strategy, we evaluate the performance of the cooperative underlay CR-NOMA network.

4. Outage Performance

Outage probability reflects the ability of the user in the system to receive and decode data correctly. The outage probabilities of U_1 and U_2 are obtained based on the cumulative distribution functions (CDFs) and probability density function (PDF).

A characteristics function is defined, i.e., $I_{\{A\}} = 1$ for the event A occurs and $I_{\{A\}} = 0$ for otherwise, which will be used in the following. In order to facilitate the derivation of outage probability expressions, the following propositions are presented.

Proposition 1. Defining a random variable $\rho|h|^2$ where $\rho = P/\sigma^2$, $P = \min\{I_{th}/|h_0|^2, P_0\}$, $h_0 \sim CN(0, \lambda_0)$, $h \sim CN(0, \lambda)$ and σ^2, I_{th}, P_0 are constants, the CDF of $\rho|h|^2$ is given by

$$F_{\rho|h|^2}(x) = 1 - \exp\left(-\frac{\sigma^2 x}{\lambda P_0}\right) \left(1 - \frac{\lambda_0 \sigma^2 x}{\lambda I_{th} + \lambda_0 \sigma^2 x} \exp\left(-\frac{I_{th}}{\lambda_0 P_0}\right)\right). \quad (22)$$

Proof. Noting that $F_{|h_0|^2}(x) = 1 - \exp(-x/\lambda_0)$, since $h_0 \sim CN(0, \lambda_0)$, the random variable $P = \min\{I_{th}/|h_0|^2, P_0\}$ can be written as

$$P = \begin{cases} P_0, & |h_0|^2 \leq I_{th}/P_0, \\ I_{th}/|h_0|^2, & |h_0|^2 \geq I_{th}/P_0. \end{cases} \quad (23)$$

Therefore, for a given $|h_0|^2$, the CDF of the random variable $\rho|h|^2$ can be given by

$$F_{\rho|h|^2}(x) = \int_0^\infty Pr\{\rho|h|^2 \leq x \mid |h_0|^2 = y\} f_{|h_0|^2}(y) dy \quad (24)$$

$$\begin{aligned} & \text{where } Pr\{\rho|h|^2 \leq x \mid |h_0|^2 = y\} \\ & = Pr\{\rho|h|^2 \leq x \mid |h_0|^2 = y, y \leq \frac{I_{th}}{P_0}\} I_{\{y \leq \frac{I_{th}}{P_0}\}} + Pr\{\rho|h|^2 \leq x \mid |h_0|^2 = y, y > \frac{I_{th}}{P_0}\} I_{\{y > \frac{I_{th}}{P_0}\}} \\ & = \left(1 - \exp\left(-\frac{\sigma^2 x}{\lambda P_0}\right)\right) I_{\{y \leq \frac{I_{th}}{P_0}\}} + \left(1 - \exp\left(-\frac{\sigma^2 xy}{\lambda I_{th}}\right)\right) I_{\{y > \frac{I_{th}}{P_0}\}}; \end{aligned} \quad (25)$$

$$f_{|h_0|^2}(y) = 1/\lambda_0 \exp(-y/\lambda_0) \quad (26)$$

Substituting Equations (25) and (26) into (24), and after some algebraic manipulations, we can obtain Equation (22), which completes the proof for Proposition 1. \square

Proposition 2. Defining a random variable $\rho'|h|^2 + 1$ where $\rho' = P'/\sigma^2$, $h \sim CN(0, \lambda)$ $h \sim CN(0, \lambda)$ and σ^2, P' are constants, the CDF and PDF of $\rho'|h|^2 + 1$ are given, respectively, as

$$F_{\rho'|h|^2+1}(x) = \left[1 - \exp\left(-\frac{\sigma^2(x-1)}{\lambda P'}\right)\right] I_{\{x \geq 1\}} \text{ and } f_{\rho'|h|^2+1}(x) = \left[\frac{\sigma^2}{\lambda P'} \exp\left(-\frac{\sigma^2(x-1)}{\lambda P'}\right)\right] I_{\{x \geq 1\}} \quad (27)$$

Proof obviously.

The following two theorems provide the closed-form expressions for the outage probabilities achieved by the two-stage adaptive relay selection and power allocation of two users in the proposed cooperative underlay CR-NOMA network.

Theorem 1. The outage probability for U_2 in cooperative underlay CR-NOMA networks with the proposed two-stage adaptive relay selection and power allocation strategy is given by

$$\begin{aligned}
 P_2 &= \frac{\sigma^2}{\lambda_{TU_1}P_T} \exp\left(\frac{\sigma^2}{\lambda_{TU_1}P_T}\right) (Q_1 - Q_2) \left[1 - \exp\left(-\frac{\sigma^2\gamma_2}{\lambda_{U_1U_2}P_2}\right) \left(1 - \frac{\lambda_{U_1R}\sigma^2\gamma_2}{\lambda_{U_1U_2}I_{th} + \lambda_{U_1R}\sigma^2\gamma_2} \exp\left(-\frac{I_{th}}{\lambda_{U_1R}P_2}\right)\right)\right] \\
 &\times \prod_{n=1}^N \left[\frac{\sigma^2}{\lambda_{TR_n}P_T} \exp\left(\frac{\sigma^2}{\lambda_{TR_n}P_T}\right) (Q_5 - Q_6) \left(1 - \exp\left(-\frac{\sigma^2\gamma_2}{\lambda_{R_nU_2}P_2}\right) \left(1 - \frac{\lambda_{R_nR}\sigma^2\gamma_2}{\lambda_{R_nU_2}I_{th} + \lambda_{R_nR}\sigma^2\gamma_2} \exp\left(-\frac{I_{th}}{\lambda_{R_nR}P_2}\right)\right)\right)\right] \\
 &+ \left[1 - \frac{\sigma^2}{\lambda_{TU_1}P_T} \exp\left(\frac{\sigma^2}{\lambda_{TU_1}P_T}\right) (Q_3 - Q_4)\right] \\
 &\times \prod_{n=1}^N \left[\begin{aligned} &1 - \exp\left(-\frac{\sigma^2\gamma_0}{\lambda_{R_nU_2}P_2}\right) \left(1 - \frac{\lambda_{R_nR}\sigma^2\gamma_0}{\lambda_{R_nU_2}I_{th} + \lambda_{R_nR}\sigma^2\gamma_0} \exp\left(-\frac{I_{th}}{\lambda_{R_nR}P_2}\right)\right) \\ &\times \exp\left(-\frac{\sigma^2(1+\gamma_1)\gamma_2}{\lambda_{R_nU_2}P_2}\right) \left(1 - \frac{\lambda_{R_nR}\sigma^2(1+\gamma_1)\gamma_2}{\lambda_{R_nU_2}I_{th} + \lambda_{R_nR}\sigma^2(1+\gamma_1)\gamma_2} \exp\left(-\frac{I_{th}}{\lambda_{R_nR}P_2}\right)\right) \\ &\times \frac{\sigma^4}{\lambda_{TR_n}P_T\lambda_{TU_1}P_T} \exp\left(\frac{\sigma^2}{\lambda_{TR_n}P_T} + \frac{\sigma^2}{\lambda_{TU_1}P_T}\right) (Q_7 - Q_8)(Q_9 - Q_{10}) \end{aligned} \right] \\
 &+ \frac{\sigma^2}{\lambda_{TU_1}P_T} \exp\left(\frac{\sigma^2}{\lambda_{TU_1}P_T}\right) \cdot [(Q_2 - Q_1) - (Q_4 - Q_3)] \\
 &\times \begin{cases} \prod_{n=1}^N \left[\begin{aligned} &1 - \exp\left(-\frac{\sigma^2\gamma_2}{\lambda_{R_nU_2}P_2}\right) \left(1 - \frac{\lambda_{R_nR}\sigma^2\gamma_2}{\lambda_{R_nU_2}I_{th} + \lambda_{R_nR}\sigma^2\gamma_2} \exp\left(-\frac{I_{th}}{\lambda_{R_nR}P_2}\right)\right) \\ &\times \frac{\sigma^4}{\lambda_{TR_n}P_T\lambda_{TU_1}P_T} \exp\left(\frac{\sigma^2}{\lambda_{TR_n}P_T} + \frac{\sigma^2}{\lambda_{TU_1}P_T}\right) (Q_5 - Q_6)(Q_{11} - Q_{12}) \end{aligned} \right], & \alpha_2 \leq 1 - \frac{\gamma_1}{\gamma_0}, \\ \prod_{n=1}^N \left[\begin{aligned} &1 - \exp\left(-\frac{\sigma^2\gamma_2}{\lambda_{R_nU_2}P_2}\right) \left(1 - \frac{\lambda_{R_nR}\sigma^2\gamma_2}{\lambda_{R_nU_2}I_{th} + \lambda_{R_nR}\sigma^2\gamma_2} \exp\left(-\frac{I_{th}}{\lambda_{R_nR}P_2}\right)\right) \\ &\times \exp\left(-\frac{\sigma^2\gamma_2}{\lambda_{R_nU_2}P_2}\right) \left(1 - \frac{\lambda_{R_nR}\sigma^2\gamma_2}{\lambda_{R_nU_2}I_{th} + \lambda_{R_nR}\sigma^2\gamma_2} \exp\left(-\frac{I_{th}}{\lambda_{R_nR}P_2}\right)\right) \\ &\times \frac{\sigma^4}{\lambda_{TR_n}P_T\lambda_{TU_1}P_T} \exp\left(\frac{\sigma^2}{\lambda_{TR_n}P_T} + \frac{\sigma^2}{\lambda_{TU_1}P_T}\right) (Q_7 - Q_8)(Q_9 - Q_{10}) \end{aligned} \right], & \alpha_2 > 1 - \frac{\gamma_1}{\gamma_0}. \end{cases} \tag{28}
 \end{aligned}$$

where $\gamma_0 = \gamma_1 + \gamma_2 + \gamma_1\gamma_2$, $\gamma_\alpha = \gamma_2/(\alpha_2 - \alpha_1\gamma_2)$ and $Q_1 \sim Q_{12}$ can be found in Appendix A.

Proof. See Appendix A. □

Theorem 2. The outage probability for U_1 in cooperative underlay CR-NOMA networks with the proposed two-stage adaptive relay selection and power allocation strategy is given by

$$\begin{aligned}
 P_1 &= \left[1 - \frac{\sigma^2}{\lambda_{TU_1}P_T} \exp\left(\frac{\sigma^2}{\lambda_{TU_1}P_T}\right) (Q_3 - Q_4)\right] \\
 &\times \prod_{n=1}^N \left[\frac{\sigma^2}{\lambda_{TR_n}P_T} \exp\left(\frac{\sigma^2}{\lambda_{TR_n}P_T} + \frac{\sigma^2}{\lambda_{TU_1}P_T}\right) (Q_7 - Q_8)(Q_9 - Q_{10}) \right] \\
 &+ \frac{\sigma^2}{\lambda_{TU_1}P_T} \exp\left(\frac{\sigma^2}{\lambda_{TU_1}P_T}\right) [(Q_2 - Q_1) - (Q_4 - Q_3)] \\
 &\times \begin{cases} \prod_{n=1}^N \left[1 - \frac{\sigma^4}{\lambda_{TR_n}P_T\lambda_{TU_1}P_T} \exp\left(\frac{\sigma^2}{\lambda_{TR_n}P_T} + \frac{\sigma^2}{\lambda_{TU_1}P_T}\right) (Q_5 - Q_6)(Q_{11} - Q_{12})\right], & \alpha_2 \leq 1 - \frac{\gamma_1}{\gamma_0}, \\ \prod_{n=1}^N \left[1 - \frac{\sigma^4}{\lambda_{TR_n}P_T\lambda_{TU_1}P_T} \exp\left(\frac{\sigma^2}{\lambda_{TR_n}P_T} + \frac{\sigma^2}{\lambda_{TU_1}P_T}\right) (Q_7 - Q_8)(Q_9 - Q_{10})\right], & \alpha_2 > 1 - \frac{\gamma_1}{\gamma_0}. \end{cases} \tag{29}
 \end{aligned}$$

Proof. See Appendix B. □

5. Numerical Simulations

This section provides computer simulations to evaluate the outage performance for the proposed cooperative underlay CR-NOMA networks. Monte Carlo simulations over 10^6 are also provided to validate the correctness of the mathematical derivations. Suppose that $\lambda_{XY} = d_{XY}^{-\zeta}$, where d_{XY} denotes the distance between the transmitting node X and the receiving node Y , and ζ is the path loss exponent. Without loss of generality, we set $d_{TU_1} = d_{TR_n} = d_{SR_n} = d_{R_nU_2} = d_{R_nR} = d_{U_1U_2} = d_{U_1R} = d$, $d_{SU_1} = 2d$, $d_{SR} = 1.5d$,

$d_{R_n U_1} = 0.5d$, $\zeta = 3$, $\sigma^2 = 1$ and $P_1 = P_2 = P_0$, where it is assumed that $d = 10$ for simplicity and all relays have the same average channel gain.

Figure 2 illustrates the outage probability of the secondary users (U_1 and U_2) versus the maximum transmit power P_0 of the secondary for different number of relays. Firstly, it can be seen that both U_1 and U_2 have better outage performance as the increase of P_0 in secondary. Secondly, one can see that the proposed two-stage adaptive relay selection strategy can achieve better outage performance as the number of relays increases, since the more relays there are, the more decodable relays will be included in the candidate set. Furthermore, it can be observed that the outage probability of U_1 is always lower than that of U_2 , mainly because U_1 has both a direct link from the BS and a cooperative link from the optimal relay, while U_2 has only a link from the optimal relay. In addition, we compare the proposed cooperative scheme with the one without the link $U_1 \rightarrow U_2$, and the results show that the former has better outage performance. With the increase of the number of relays, the gap between the outage probability of U_2 caused by these two schemes gradually becomes smaller, because the existence of the link $U_1 \rightarrow U_2$ is harmonized by the excessive number of relays.

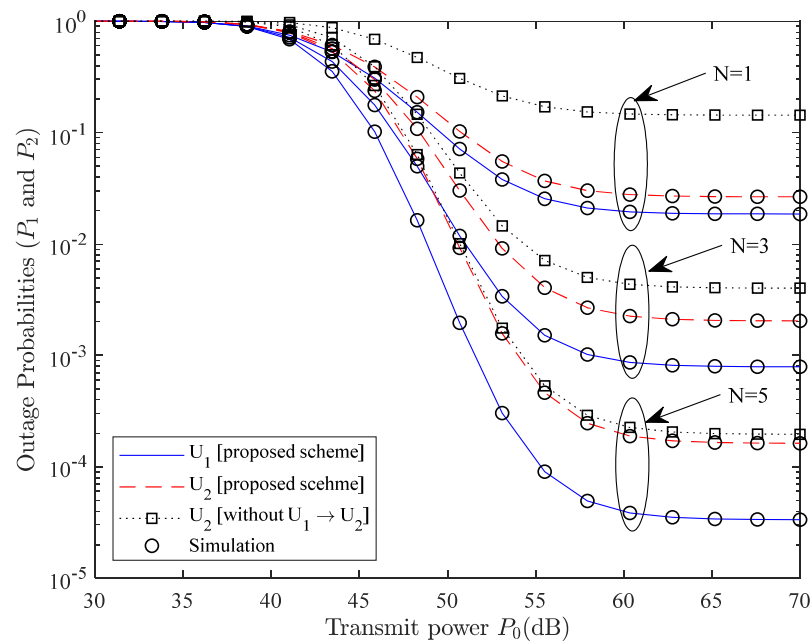


Figure 2. Outage probability of U_1 and U_2 versus P_0 with different number of relays, where $P_t = 30$ dB, $I_{th} = 25$ dB, $\alpha_2 = 0.8$, $R_2 = 1.2$ bps/Hz and $R_1 = 1.0$ bps/Hz.

Figures 3 and 4 show the impact of the transmit power of the primary and the interference threshold on the secondary outage performance for the different number of relays, respectively. Obviously, the impact of the transmit power constraint and interference constraint imposed by the primary on the secondary cannot be ignored. On the one hand, too high transmit power of the primary can significantly degrade the signal quality of the secondary network; on the other hand, a lower interference threshold, while ensuring QoS in the primary, may cause the secondary to adjust to a suitable (lower) transmit power. However, if the interference threshold constraint is relaxed, the outage probability of U_1 and U_2 will stabilize when $I_{th} > 35$ dB. This result shows that the outage performance improves until a certain lower limit of the outage probability determined by the interference constraint of the primary network.

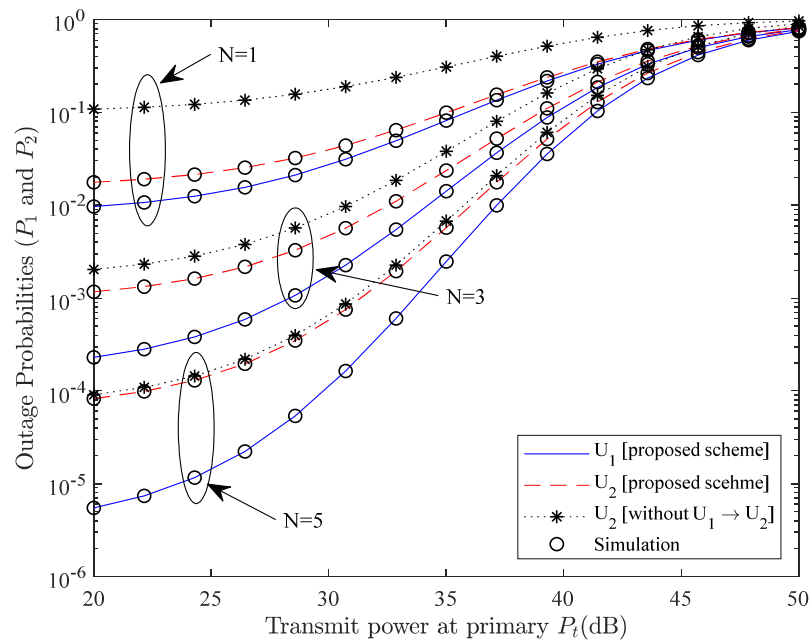


Figure 3. Outage probability of U_1 and U_2 versus P_t with different number of relays, where $P_0 = 55$ dB, $I_{th} = 25$ dB, $\alpha_2 = 0.8$, $R_2 = 1.2$ bps/Hz and $R_1 = 1.0$ bps/Hz.

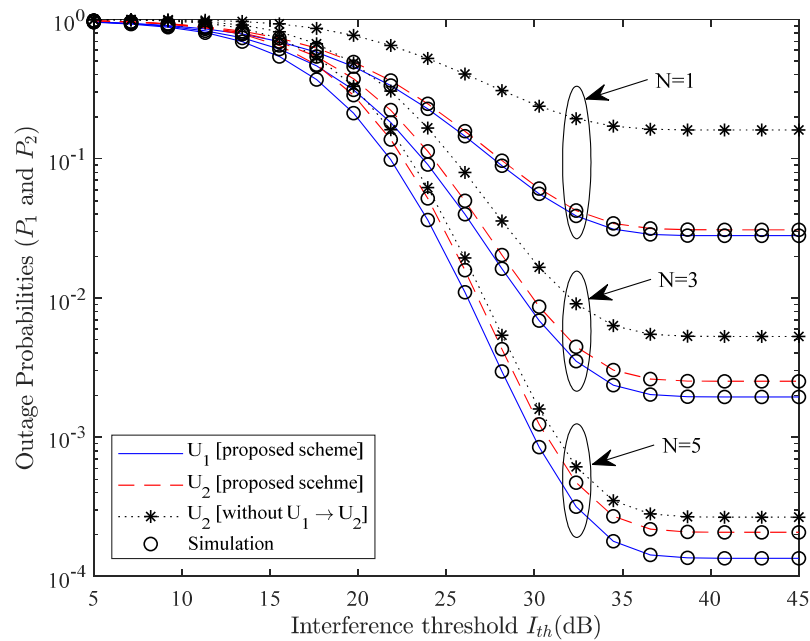


Figure 4. Outage probability of U_1 and U_2 versus I_{th} with different number of relays, where $P_0 = 60$ dB, $P_t = 40$ dB, $\alpha_2 = 0.8$, $R_2 = 1.2$ bps/Hz and $R_1 = 1.0$ bps/Hz.

Figure 5 plots the outage probability of U_2 versus the maximum transmit power P_0 for different signaling. At the end of the first slot, U_1 sends four kinds of signaling to the relays according to its decoding results. When $\Theta \in \{11, 01\}$, the optimal relay assists U_2 with full power. In particular, when $\Theta = '11'$, U_1 will give aid to U_2 as much as possible if there is no relay that satisfies the condition. However, when $\Theta \in \{10, 00\}$, the optimal relay allocates power to U_1 and U_2 with adaptive power allocation strategy, and opportunistically serves U_2 while satisfying the QoS of U_1 . Therefore, the outage performance of $\Theta \in \{11, 01\}$ is better than that of $\Theta \in \{10, 00\}$, and U_2 has the best outage performance when $\Theta = '11'$. As shown in Figure 3, The outage probability is the lowest when $\Theta = '11'$, followed by

$\Theta = '01'$, and similar when $\Theta = '10'$ and $\Theta = '00'$, which just verifies the correctness of the above analysis.

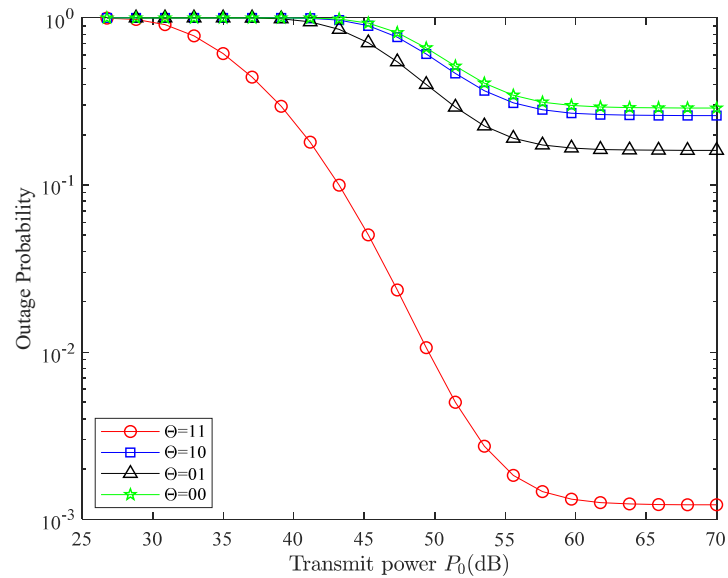


Figure 5. Outage probability of U_2 versus P_0 in different cases Θ , where $P_t = 30$ dB, $I_{th} = 25$ dB, $\alpha_2 = 0.8$, $R_2 = 1.2$ bps/Hz and $R_1 = 1.0$ bps/Hz.

In Figure 6, we investigate how the power allocation factor α_2 at BS affects the outage probability of the secondary users for different target rates. As α_2 increases, the outage probability decreases and then increases for both U_1 and U_2 , which implies that an optimal α_2 can be found to optimize the outage performance of the secondary. Compared to the cooperative scheme without the link $U_1 \rightarrow U_2$, the proposed cooperation achieves optimal outage performance by allocating less power to U_2 since the presence of the link $U_1 \rightarrow U_2$ improves the diversity gain. It can also be observed that the outage performance is enhanced significantly with low target rates, indicating that the high target data rate of the user side will adversely affect the system outage performance. When decreasing the target rate, the gap between the outage probability of two users gradually decreases, because the achievable rate at the far user is maximized through adaptive power allocation.

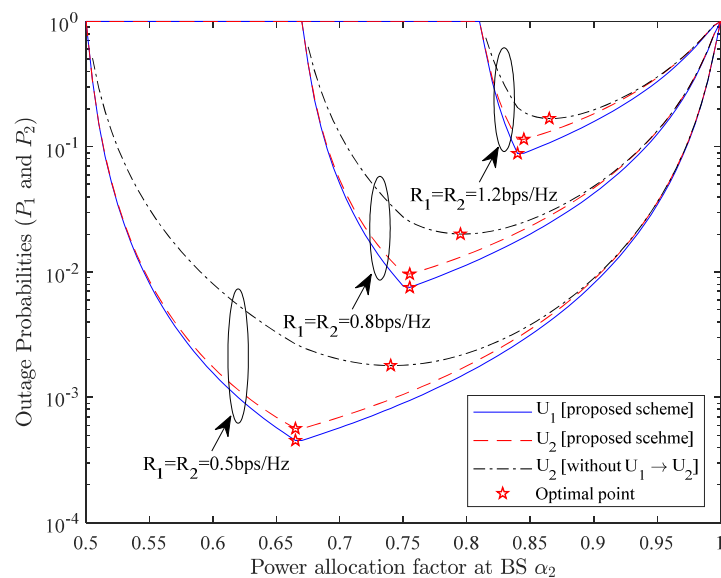


Figure 6. Outage probability of U_1 and U_2 versus α_2 with different target data rates, where $P_0 = 60$ dB, $P_t = 40$ dB, $I_{th} = 25$ dB/Hz and $N = 3$.

Figure 7 shows the impact of target rate R_2 on the secondary outage performance for different R_1 . With the increase of R_2 , the outage probability of two users first remains stable and then increases rapidly in the higher R_2 range. For a given R_2 , a higher R_1 leads to a higher outage probability. This result further confirms that it is not advisable to set too high a data rate at the user side, especially for the far user. Because the QoS of the near user is guaranteed first, and then the far user is opportunistically assisted, it is difficult to achieve a high data rate at the far user.

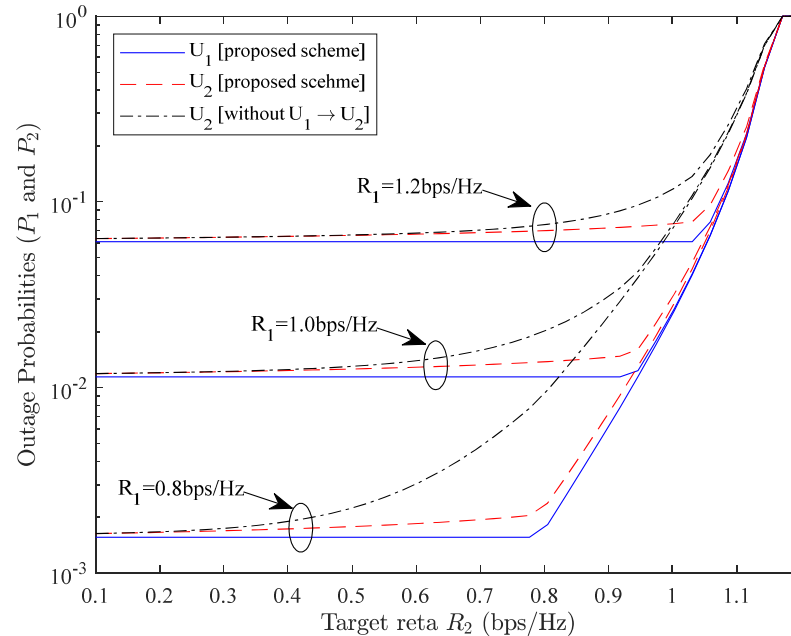


Figure 7. Outage probability of U_1 and U_2 versus R_2 with different R_1 , where $P_0 = 60$ dB, $P_t = 40$ dB, $I_{th} = 25$ bps/Hz, $\alpha_2 = 0.8$ and $N = 3$.

6. Conclusions

In this paper, we analyzed the outage performance of cooperative underlay CR-NOMA networks with a novel cooperation involving dedicated relay cooperation and user cooperation. At the end of the first time slot, four possible decoding results were derived at the near user. Accordingly, a two-stage adaptive relay selection and power allocation strategy in the secondary time slot was presented. By discussing the decoding results, four relay selection criteria and the corresponding power allocation coefficients were obtained. Finally, we derived the closed-form expressions of the outage probability for the two secondary users, and revealed the impacts of essential parameters on outage performance of the secondary network, such as transmit power, number of relays, interference threshold, and target data rate. The Monte Carlo simulations verified the correctness of the theoretical analysis.

Compared to the existing cooperative scheme without the link $U_1 \rightarrow U_2$, the proposed cooperation with a two-stage adaptive relay selection and power allocation strategy can significantly improve the system outage performance when the number of relays is small. Moreover, the proposed strategy improves the rate fairness of users and achieves the optimal value of outage performance for all users by allocating less power to the far user. Furthermore, we also verify that a low target data rate at the receiver side is more conducive to achieving a lower outage probability, which is consistent with the results for the NOMA system presented in [34].

Author Contributions: W.L. researched the literatures, conceived the study concepts, designed the algorithm, and took charge of the original draft preparation; S.L. improved the mathematical models, provided the systematic research and analysis methodology, and edited the manuscript, and supervised the completion of the refinement of the paper; V.P. gave valuable suggestions for revision;

N.Y. and S.Y. completed some numerical simulations, and checked formula deducing and English grammar. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded in part by the National Natural Science Foundation of China under Grant 61663024, in part by the Hongliu First Class Discipline Development Project of Lanzhou University of Technology, and in part by the Excellent Postgraduate “Innovation Star” Project of Gansu Province under Grant 2021CXZX-537.

Data Availability Statement: The data supporting this article are from previously reported studies and datasets, which have been cited.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Proof of Theorem 1. The outage probability of cooperative networks with DF relays is defined as the probability that the achievable data rates of relays or users are less than the target data rates [10]. This means that the outage event occurs at U_2 when all relays in the system do not help U_2 achieve its target data rate. From the Law of Total Probability, the outage probability for U_2 can be written as [6,10]

$$P_2 = \sum_{\Theta \in \{11,10,01,00\}} P^\Theta P_{out,2}^\Theta \tag{A1}$$

where P^Θ denotes the probability of the event that a signaling sent by U_1 is Θ , and $P_{out,2}^\Theta$ represents the outage probability for U_2 in the case of Θ .

Now let us focus on the derivation of P^Θ . Based on Equation (4), the probability of the event that $\Theta = '11'$ is given by

$$\begin{aligned} P^{11} &= Pr\left\{R_{SU_1}^{x_2} \geq R_2, R_{SU_1}^{x_1} \geq R_1\right\} \\ &= Pr\left\{\frac{\alpha_2 \rho_S |h_{SU_1}|^2}{\rho_T |h_{TU_1}|^2 + \alpha_1 \rho_S |h_{SU_1}|^2 + 1} \geq \gamma_2, \frac{\alpha_1 \rho_S |h_{SU_1}|^2}{\rho_T |h_{TU_1}|^2 + 1} \geq \gamma_1\right\} \\ &= Pr\left\{\rho_S |h_{SU_1}|^2 \geq (\rho_T |h_{TU_1}|^2 + 1) \varphi_1\right\} \\ &= \int_0^\infty \left[1 - F_{\rho_S |h_{SU_1}|^2}(\varphi_1 y)\right] \cdot f_{\rho_T |h_{TU_1}|^2 + 1}(y) dy \\ &= \frac{\sigma^2}{\lambda_{TU_1} P_T} \exp\left(\frac{\sigma^2}{\lambda_{TU_1} P_T}\right) \cdot (Q_1 - Q_2). \end{aligned} \tag{A2}$$

where Q_1 and Q_2 are given, respectively, as

$$\begin{aligned} Q_1 &= \int_1^\infty \exp(-a_1 y) dy = \frac{1}{a_1} \exp(-a_1), \\ Q_2 &= \exp\left(-\frac{I_{th}}{\lambda_{SR} P_1}\right) \int_1^\infty \frac{y}{b_1 + y} \exp(-a_1 y) dy \\ &\stackrel{t=b_1+y}{=} \exp\left(-\frac{I_{th}}{\lambda_{SR} P_1}\right) \int_{1+b_1}^\infty \frac{t-b_1}{t} \exp(-a_1(t-b_1)) dt \\ &= \exp\left(-\frac{I_{th}}{\lambda_{SR} P_1}\right) \exp(a_1 b_1) \left[\int_{1+b_1}^\infty \exp(-a_1 t) dt - \int_{1+b_1}^\infty \frac{b_1}{t} \exp(-a_1 t) dt \right] \\ &= \exp\left(-\frac{I_{th}}{\lambda_{SR} P_1}\right) \exp(a_1 b_1) \left[\frac{1}{a_1} \exp(-a_1 - a_1 b_1) - b_1 E_1(a_1 + a_1 b_1) \right]. \end{aligned}$$

In Q_1 and Q_2 , $a_1 = \frac{\sigma^2(\varphi_1 \lambda_{TU_1} P_T + \lambda_{SU_1} P_1)}{(\lambda_{SU_1} P_1 \lambda_{TU_1} P_T)}$, $b_1 = \frac{(\lambda_{SU_1} I_{th})}{(\lambda_{SR} \sigma^2 \varphi_1)}$, $\varphi_1 = \max\left\{\gamma_\alpha, \frac{\gamma_1}{\alpha_1}\right\}$, $\gamma_\alpha = \frac{\gamma_2}{(\alpha_2 - \alpha_1 \gamma_2)}$ and q is obtained by exponential integral function $E_1(x) = -E_i(-x) = \int_1^\infty \frac{1}{t} \exp(-t) dt$ and Equation (3.352.2) in [35].

The probability of the event that $\Theta = '10'$ is given by

$$\begin{aligned}
 P^{10} &= Pr\left\{R_{SU_1}^{x_2} \geq R_2, R_{SU_1}^{x_1} < R_1\right\} \\
 &= Pr\left\{\frac{\alpha_2 \rho_S |h_{SU_1}|^2}{\rho_T |h_{TU_1}|^2 + \alpha_1 \rho_S |h_{SU_1}|^2 + 1} \geq \gamma_2, \frac{\alpha_1 \rho_S |h_{SU_1}|^2}{\rho_T |h_{TU_1}|^2 + 1} < \gamma_1\right\} \\
 &= \left[F_{\rho_S |h_{SU_1}|^2} \left((\rho_T |h_{TU_1}|^2 + 1) \varphi_1 \right) - F_{\rho_S |h_{SU_1}|^2} \left((\rho_T |h_{TU_1}|^2 + 1) \varphi_2 \right) \right] I_{\{\gamma_\alpha < \frac{\gamma_2}{\alpha_1}\}} \\
 &= \frac{\sigma^2}{\lambda_{TU_1} P_T} \exp\left(\frac{\sigma^2}{\lambda_{TU_1} P_T}\right) \cdot [(Q_2 - Q_1) - (Q_4 - Q_3)] I_{\{\alpha_2 > 1 - \frac{\gamma_1}{\gamma_0}\}},
 \end{aligned} \tag{A3}$$

where $Q_3 = 1/a_2 \exp(-a_2)$, $Q_4 = \exp\left(-\frac{I_{th}}{(\lambda_{SR} P_1)}\right) \exp(a_2 b_2) \left[\frac{1}{a_2} \exp(-a_2 - a_2 b_2) - b_2 E_1(a_2 + a_2 b_2)\right]$.

In Q_3 and Q_4 , $a_2 = \frac{\sigma^2 (\varphi_2 \lambda_{TU_1} P_T + \lambda_{SU_1} P_1)}{(\lambda_{SU_1} P_1 \lambda_{TU_1} P_T)}$, $b_2 = \frac{(\lambda_{SU_1} I_{th})}{(\lambda_{SR} \sigma^2 \varphi_2)}$, $\varphi_2 = \min\{\gamma_\alpha, \gamma_1/\alpha_1\}$ and $\gamma_0 = \gamma_1 + \gamma_2 + \gamma_1 \gamma_2$.

The probability of the event that $\Theta = '01'$ is given by

$$\begin{aligned}
 P^{01} &= Pr\left\{R_{SU_1}^{x_2} < R_2, R_{SU_1}^{x_1} \geq R_1\right\} \\
 &= \frac{\sigma^2}{\lambda_{TU_1} P_T} \exp\left(\frac{\sigma^2}{\lambda_{TU_1} P_T}\right) \cdot [(Q_2 - Q_1) - (Q_4 - Q_3)] I_{\{\alpha_2 \leq 1 - \frac{\gamma_1}{\gamma_0}\}}
 \end{aligned} \tag{A4}$$

The probability of the event that $\Theta = '00'$ is given by

$$P^{00} = Pr\left\{R_{SU_1}^{x_2} < R_2, R_{SU_1}^{x_1} < R_1\right\} = 1 + \frac{\sigma^2}{\lambda_{TU_1} P_T} \exp\left(\frac{\sigma^2}{\lambda_{TU_1} P_T}\right) \cdot (Q_4 - Q_3). \tag{A5}$$

which completes the derivation of P^Θ . Then, let us focus on the derivation of $P_{out,2}^\Theta$, which can be expressed as

$$P_{out,2}^\Theta = \begin{cases} Pr\left\{R_{U_1 U_2}^{x_2} < R_2\right\} \cdot \prod_{n=1}^N \left[1 - P_{n,2}^\Theta\right], & \Theta = 11, \\ \prod_{n=1}^N \left[1 - P_{n,2}^\Theta\right], & \Theta \in \{10, 01, 00\}, \end{cases} \tag{A6}$$

where $Pr\left\{R_{U_1 U_2}^{x_2} < R_2\right\}$ is the probability that U_1 fails to help U_2 decode x_2 . Based on Equation (8), we have

$$\begin{aligned}
 Pr\left\{R_{U_1 U_2}^{x_2} < R_2\right\} &= Pr\left\{\rho_R |h_{U_1 U_2}|^2 < \gamma_2\right\} = F_{\rho_R |h_{U_1 U_2}|^2}(\gamma_2) \\
 &= 1 - \exp\left(-\frac{\sigma^2 \gamma_2}{\lambda_{U_1 U_2} P_2}\right) \left(1 - \frac{\lambda_{U_1 R} \sigma^2 \gamma_2}{\lambda_{U_1 U_2} I_{th} + \lambda_{U_1 R} \sigma^2 \gamma_2} \exp\left(-\frac{I_{th}}{\lambda_{U_1 R} P_2}\right)\right).
 \end{aligned} \tag{A7}$$

and $P_{n,2}^\Theta$ in (A6) denotes the probability that the optimal relay successfully assists U_2 to decode x_2 in the case of Θ ; that is, all relays in the candidate set have ability to make their achievable data rate reach the target data rate, i.e.,

$$P_{n,2}^\Theta = \begin{cases} Pr\left\{R_{SR_n}^{x_2} \geq R_2\right\} Pr\left\{R_{R_n U_2}^{x_2, \Theta} \geq R_2\right\}, & \Theta \in \{11, 01\}, \\ Pr\left\{R_{SR_n}^{x_2} \geq R_2, R_{SR_n}^{x_1} \geq R_1\right\} Pr\left\{R_{R_n U_2}^{x_2, \Theta} \geq R_2\right\}, & \Theta \in \{10, 00\}. \end{cases} \tag{A8}$$

Based on Equation (2), Proposition 1 and Proposition 2, we can get

$$\begin{aligned}
 Pr\left\{R_{SR_n}^{x_2} \geq R_2\right\} &= Pr\left\{\frac{\alpha_2 \rho_S |h_{SR_n}|^2}{\rho_T |h_{TR_n}|^2 + \alpha_1 \rho_S |h_{SR_n}|^2 + 1} \geq \gamma_2\right\} \\
 &= Pr\left\{\rho_S |h_{SR_n}|^2 \geq (\rho_T |h_{TR_n}|^2 + 1) \gamma_\alpha\right\} \\
 &= \int_0^\infty \left[1 - F_{\rho_S |h_{SR_n}|^2}(\gamma_\alpha y)\right] \cdot f_{\rho_T |h_{TR_n}|^2 + 1}(y) dy \\
 &= \frac{\sigma^2}{\lambda_{TR_n} P_T} \exp\left(\frac{\sigma^2}{\lambda_{TR_n} P_T}\right) \cdot (Q_5 - Q_6).
 \end{aligned} \tag{A9}$$

where $Q_5 = 1/a_3 \exp(-a_3)$, $Q_6 = \exp\left(-\frac{I_{th}}{\lambda_{SR} P_1}\right) \exp(a_3 b_3) \left[\frac{1}{a_3} \exp(-a_3 - a_3 b_3) - b_3 E_1(a_3 + a_3 b_3)\right]$.
 In Q_5 and Q_6 , $a_3 = \frac{\sigma^2(\gamma_\alpha \lambda_{TR_n} P_T + \lambda_{SR_n} P_1)}{(\lambda_{SR_n} P_1 \lambda_{TR_n} P_T)}$ and $b_3 = \frac{(\lambda_{SR_n} I_{th})}{(\lambda_{SR} \sigma^2 \gamma_\alpha)}$. On the other hand, we can obtain

$$\begin{aligned} Pr\left\{R_{SR_n}^{x_2} \geq R_2, R_{SR_n}^{x_1} \geq R_1\right\} &= Pr\left\{\frac{\alpha_2 \rho_S |h_{SR_n}|^2}{\rho_T |h_{TR_n}|^2 + \alpha_1 \rho_S |h_{SR_n}|^2 + 1} \geq \gamma_2, \frac{\alpha_1 \rho_S |h_{SR_n}|^2}{\rho_T |h_{TR_n}|^2 + 1} \geq \gamma_1\right\} \\ &= Pr\left\{\rho_S |h_{SR_n}|^2 \geq (\rho_T |h_{TR_n}|^2 + 1) \varphi_1\right\} \\ &= \int_0^\infty \left[1 - F_{\rho_S |h_{SR_n}|^2}(\varphi_1 y)\right] \cdot f_{\rho_T |h_{TR_n}|^2 + 1}(y) dy \\ &= \frac{\sigma^2}{\lambda_{TR_n} P_T} \exp\left(\frac{\sigma^2}{\lambda_{TR_n} P_T}\right) \cdot (Q_7 - Q_8), \end{aligned} \tag{A10}$$

where $Q_7 = 1/a_4 \exp(-a_4)$, $Q_8 = \exp\left(-\frac{I_{th}}{\lambda_{SR} P_1}\right) \exp(a_4 b_4) \left[\frac{1}{a_4} \exp(-a_4 - a_4 b_4) - b_4 E_1(a_4 + a_4 b_4)\right]$.
 In Q_7 and Q_8 , $a_4 = \frac{\sigma^2(\varphi_1 \lambda_{TR_n} P_T + \lambda_{SR_n} P_1)}{(\lambda_{SR_n} P_1 \lambda_{TR_n} P_T)}$ and $b_4 = \frac{(\lambda_{SR_n} I_{th})}{(\lambda_{SR} \sigma^2 \varphi_1)}$.

Based on the results of $R_{R_n U_2}^{x_2, \Theta}$ given in Equations (10), (12), (16) and (18), we can obtain
(a) $\Theta = '11'$:

$$\begin{aligned} Pr\left\{R_{R_n U_2}^{x_2, 11} \geq R_2\right\} &= Pr\left\{\rho_R |h_{R_n U_2}|^2 \geq \gamma_2\right\} = 1 - F_{\rho_R |h_{R_n U_2}|^2}(\gamma_2) \\ &= \exp\left(-\frac{\sigma^2 \gamma_2}{\lambda_{R_n U_2} P_2}\right) \left(1 - \frac{\lambda_{R_n R} \sigma^2 \gamma_2}{\lambda_{R_n U_2} I_{th} + \lambda_{R_n R} \sigma^2 \gamma_2} \exp\left(-\frac{I_{th}}{\lambda_{R_n R} P_2}\right)\right). \end{aligned} \tag{A11}$$

(b) $\Theta = '10'$:

$$\begin{aligned} Pr\left\{R_{R_n U_2}^{x_2, 10} \geq R_2\right\} &= Pr\left\{\beta_{n2}^{10} \cdot \rho_R |h_{R_n U_2}|^2 \geq \gamma_2\right\} \\ &= Pr\left\{\rho_R |h_{R_n U_2}|^2 \cdot \min\left\{\frac{[\rho_R |h_{R_n U_2}|^2 - \gamma_1]^+}{\rho_R |h_{R_n U_2}|^2 (1 + \gamma_1)}, \frac{[\rho_R |h_{R_n U_1}|^2 - \gamma_1 (\rho_T |h_{TU_1}|^2 + 1)]^+}{\rho_R |h_{R_n U_1}|^2}\right\} \geq \gamma_2\right\} \\ &= Pr\left\{\underbrace{\frac{[\rho_R |h_{R_n U_2}|^2 - \gamma_1]^+}{1 + \gamma_1} \geq \gamma_2}_{I_1} \cdot \underbrace{Pr\left\{\frac{\rho_R |h_{R_n U_2}|^2 \cdot [\rho_R |h_{R_n U_1}|^2 - \gamma_1 (\rho_T |h_{TU_1}|^2 + 1)]^+}{\rho_R |h_{R_n U_1}|^2} \geq \gamma_2\right\}}_{I_2}\right\} \end{aligned} \tag{A12}$$

where

$$\begin{aligned} I_1 &= Pr\left\{\frac{\rho_R |h_{R_n U_2}|^2 - \gamma_1}{1 + \gamma_1} \geq \gamma_2, \rho_R |h_{R_n U_2}|^2 > \gamma_1\right\} \\ &= Pr\left\{\rho_R |h_{R_n U_2}|^2 > \gamma_0, \rho_R |h_{R_n U_2}|^2 > \gamma_1\right\} \\ &= Pr\left\{\rho_R |h_{R_n U_2}|^2 > \gamma_0\right\} = 1 - F_{\rho_R |h_{R_n U_2}|^2}(\gamma_0) \\ &= \exp\left(-\frac{\sigma^2 \gamma_0}{\lambda_{R_n U_2} P_2}\right) \left(1 - \frac{\lambda_{R_n R} \sigma^2 \gamma_0}{\lambda_{R_n U_2} I_{th} + \lambda_{R_n R} \sigma^2 \gamma_0} \exp\left(-\frac{I_{th}}{\lambda_{R_n R} P_2}\right)\right), \end{aligned} \tag{A13}$$

$$\begin{aligned} I_2 &= Pr\left\{\frac{\rho_R |h_{R_n U_2}|^2 \cdot [\rho_R |h_{R_n U_1}|^2 - \gamma_1 (\rho_T |h_{TU_1}|^2 + 1)]}{\rho_R |h_{R_n U_1}|^2} \geq \gamma_2, \rho_R |h_{R_n U_1}|^2 > \gamma_1 (\rho_T |h_{TU_1}|^2 + 1)\right\} \\ &= Pr\left\{\rho_R |h_{R_n U_2}|^2 \geq \frac{\rho_R |h_{R_n U_1}|^2 \gamma_2}{\rho_R |h_{R_n U_1}|^2 - \gamma_1 (\rho_T |h_{TU_1}|^2 + 1)}, \rho_R |h_{R_n U_1}|^2 > \gamma_1 (\rho_T |h_{TU_1}|^2 + 1)\right\}. \end{aligned} \tag{A14}$$

When $\rho_R \rightarrow \infty$, we have $\frac{\rho_R |h_{R_n U_1}|^2 \gamma_2}{\rho_R |h_{R_n U_1}|^2 - \gamma_1 (\rho_T |h_{TU_1}|^2 + 1)} \rightarrow \gamma_2$, and I_2 can be rewritten as

$$\begin{aligned} I_2 &= Pr\left\{\rho_R |h_{R_n U_2}|^2 \geq \gamma_2, \rho_R |h_{R_n U_1}|^2 > \gamma_1 (\rho_T |h_{TU_1}|^2 + 1)\right\} \\ &= Pr\left\{\rho_R |h_{R_n U_2}|^2 \geq \gamma_2\right\} \cdot Pr\left\{\rho_R |h_{R_n U_1}|^2 > \gamma_1 (\rho_T |h_{TU_1}|^2 + 1)\right\} \end{aligned} \tag{A15}$$

where $Pr\{\rho_R|h_{R_nU_2}|^2 \geq \gamma_2\}$ can be obtained from (A11), and

$$Pr\{\rho_R|h_{R_nU_1}|^2 > \gamma_1(\rho_T|h_{TU_1}|^2 + 1)\} = \int_0^\infty \left[1 - F_{\rho_R|h_{R_nU_1}|^2}(\gamma_1 y)\right] \cdot f_{\rho_T|h_{TU_1}|^2+1}(y) dy \tag{A16}$$

$$= \frac{\sigma^2}{\lambda_{TU_1}P_T} \exp\left(\frac{\sigma^2}{\lambda_{TU_1}P_T}\right) \cdot (Q_9 - Q_{10})$$

where $Q_9 = \frac{1}{a_5} \exp(-a_5)$, $Q_{10} = \exp\left(-\frac{I_{th}}{(\lambda_{R_nR}P_2)}\right) \exp(a_5b_5) \left[\frac{1}{a_5} \exp(-a_5 - a_5b_5) - b_5E_1(a_5 + a_5b_5)\right]$.

In Q_9 and Q_{10} , $a_5 = \frac{\sigma^2(\gamma_1\lambda_{TU_1}P_T + \lambda_{R_nU_1}P_2)}{(\lambda_{R_nU_1}P_2\lambda_{TU_1}P_T)}$ and $b_5 = \frac{(\lambda_{R_nU_1}I_{th})}{(\lambda_{R_nR}\sigma^2\gamma_1)}$.

Substituting (A13) and (A14) into (A12), the approximate expression of $Pr\{R_{R_nU_2}^{x_2,10} < R_2\}$ can be obtained as

$$Pr\{R_{R_nU_2}^{x_2,10} \geq R_2\} = \exp\left(-\frac{\sigma^2\gamma_0}{\lambda_{R_nU_2}P_2}\right) \left(1 - \frac{\lambda_{R_nR}\sigma^2\gamma_0}{\lambda_{R_nU_2}I_{th} + \lambda_{R_nR}\sigma^2\gamma_0} \exp\left(-\frac{I_{th}}{\lambda_{R_nR}P_2}\right)\right) \tag{A17}$$

$$\times \exp\left(-\frac{\sigma^2\gamma_2}{\lambda_{R_nU_2}P_2}\right) \left(1 - \frac{\lambda_{R_nR}\sigma^2\gamma_2}{\lambda_{R_nU_2}I_{th} + \lambda_{R_nR}\sigma^2\gamma_2} \exp\left(-\frac{I_{th}}{\lambda_{R_nR}P_2}\right)\right)$$

$$\times \frac{\sigma^2}{\lambda_{TU_1}P_T} \exp\left(\frac{\sigma^2}{\lambda_{TU_1}P_T}\right) \cdot (Q_9 - Q_{10}).$$

(c) $\Theta = '01'$:

Note that the optimal relay must ensure that U_1 successfully decodes x_2 , we have

$$Pr\{R_{R_nU_2}^{x_2,01} \geq R_2\} = Pr\left\{\rho_R|h_{R_nU_2}|^2 < \gamma_2, \frac{\rho_R|h_{R_nU_1}|^2}{\rho_T|h_{TU_1}|^2+1} \geq \gamma_2\right\} \tag{A18}$$

$$= Pr\{\rho_R|h_{R_nU_2}|^2 < \gamma_2\} \cdot Pr\{\rho_R|h_{R_nU_1}|^2 \geq (\rho_T|h_{TU_1}|^2 + 1)\gamma_2\}$$

$$= \exp\left(-\frac{\sigma^2\gamma_2}{\lambda_{R_nU_2}P_2}\right) \left(1 - \frac{\lambda_{R_nR}\sigma^2\gamma_2}{\lambda_{R_nU_2}I_{th} + \lambda_{R_nR}\sigma^2\gamma_2} \exp\left(-\frac{I_{th}}{\lambda_{R_nR}P_2}\right)\right)$$

$$\times \frac{\sigma^2}{\lambda_{TU_1}P_T} \exp\left(\frac{\sigma^2}{\lambda_{TU_1}P_T}\right) \cdot (Q_{11} - Q_{12}).$$

where $Q_{11} = \frac{1}{a_6} \exp(-a_6)$, $Q_{12} = \exp\left(-\frac{I_{th}}{(\lambda_{R_nR}P_2)}\right) \exp(a_6b_6) \left[\frac{1}{a_6} \exp(-a_6 - a_6b_6) - b_6E_1(a_6 + a_6b_6)\right]$.

In Q_{11} and Q_{12} , $a_6 = \frac{\sigma^2(\gamma_2\lambda_{TU_1}P_T + \lambda_{R_nU_1}P_2)}{(\lambda_{R_nU_1}P_2 + \lambda_{TU_1}P_T)}$ and $b_6 = \frac{\lambda_{R_nU_1}I_{th}}{\lambda_{R_nR}\sigma^2\gamma_2}$.

(d) $\Theta = '00'$:

By using the same derivation method as (b), we can obtain the approximate expression of $Pr\{R_{R_nU_2}^{x_2,00} \geq R_2\}$ as

$$Pr\{R_{R_nU_2}^{x_2,00} \geq R_2\} = \exp\left(-\frac{\sigma^2\gamma_0}{\lambda_{R_nU_2}P_2}\right) \left(1 - \frac{\lambda_{R_nR}\sigma^2\gamma_0}{\lambda_{R_nU_2}I_{th} + \lambda_{R_nR}\sigma^2\gamma_0} \exp\left(-\frac{I_{th}}{\lambda_{R_nR}P_2}\right)\right) \tag{A19}$$

$$\times \exp\left(-\frac{\sigma^2(1+\gamma_1)\gamma_2}{\lambda_{R_nU_2}P_2}\right) \left(1 - \frac{\lambda_{R_nR}\sigma^2(1+\gamma_1)\gamma_2}{\lambda_{R_nU_2}I_{th} + \lambda_{R_nR}\sigma^2(1+\gamma_1)\gamma_2} \exp\left(-\frac{I_{th}}{\lambda_{R_nR}P_2}\right)\right)$$

$$\times \frac{\sigma^2}{\lambda_{TU_1}P_T} \exp\left(\frac{\sigma^2}{\lambda_{TU_1}P_T}\right) \cdot (Q_9 - Q_{10}).$$

Finally, using (A1)–(A19), the outage probability for U_2 in cooperative CR-NOMA networks can be obtained as Equation (28), which completes the proof of Theorem 1. \square

Appendix B

Proof of Theorem 2. When $\Theta = '11'$, the outage cannot happen at U_1 since U_1 has successfully acquired the desired signal at the end of the first time slot. In addition, the selected optimal relay must satisfy the QoS of U_1 ; that is, as long as the optimal relay exists, U_1 will not have an outage. Therefore, the outage event occurs at U_1 only when there is no relay in the candidate sets that meet the requirements to be the optimal relay. This also means that when all the power is allocated to the signal to be sent to U_1 , the achievable rate still does not reach the target data rate.

From the Law of Total Probability and the analyses above, the outage probability U_1 can be written as

$$P_1 = \sum_{\Theta \in \{10, 01, 00\}} P^\Theta P_{out,1}^\Theta \quad (A20)$$

where P^Θ denotes the probability for the event that a signaling sent by U_1 is Θ , which is given by (A3)–(A5), and $P_{out,1}^\Theta = \prod_{n=1}^N [1 - P_{n,1}^\Theta]$ represents the outage probability for U_1 in the case of Θ , which can be rewritten as

$$P_{n,1}^\Theta = \begin{cases} Pr\left\{R_{R_n U_1}^{x_2, \Theta} \geq R_2 \mid \beta_{n2}^\Theta = 0\right\} Pr\left\{R_{SR_n}^{x_2} \geq R_2\right\}, & \Theta = 01, \\ Pr\left\{R_{R_n U_1}^{x_1, \Theta} \geq R_1 \mid \beta_{n2}^\Theta = 0\right\} Pr\left\{R_{SR_n}^{x_2} \geq R_2, R_{SR_n}^{x_1} \geq R_1\right\}, & \Theta \in \{10, 00\}. \end{cases} \quad (A21)$$

where $Pr\left\{R_{SR_n}^{x_2} \geq R_2\right\}$ and $Pr\left\{R_{SR_n}^{x_2} \geq R_2, R_{SR_n}^{x_1} \geq R_1\right\}$ are given by (A9) and (A10), respectively, and $Pr\left\{R_{R_n U_1}^{x_1, \Theta} \geq R_1 \mid \beta_{n2}^\Theta = 0\right\}$ can be expressed as (A22)–(A24).

$$Pr\left\{R_{R_n U_1}^{x_2, 01} \geq R_2 \mid \beta_{n2}^{01} = 0\right\} = \frac{\sigma^2}{\lambda_{TU_1} P_T} \exp\left(\frac{\sigma^2}{\lambda_{TU_1} P_T}\right) (Q_{11} - Q_{12}), \quad (A22)$$

$$Pr\left\{R_{R_n U_1}^{x_1, 10} \geq R_1 \mid \beta_{n2}^{10} = 0\right\} = \frac{\sigma^2}{\lambda_{TU_1} P_T} \exp\left(\frac{\sigma^2}{\lambda_{TU_1} P_T}\right) (Q_9 - Q_{10}), \quad (A23)$$

$$Pr\left\{R_{R_n U_1}^{x_1, 00} \geq R_1 \mid \beta_{n2}^{00} = 0\right\} = \frac{\sigma^2}{\lambda_{TU_1} P_T} \exp\left(\frac{\sigma^2}{\lambda_{TU_1} P_T}\right) (Q_9 - Q_{10}). \quad (A24)$$

Finally, using (A21)–(A24), the outage probability for U_1 in cooperative CR-NOMA networks can be obtained as Equation (29), which completes the proof of Theorem 2. \square

References

1. Tavera Romero, C.A.; Ortiz, J.H.; Khalaf, O.I.; Ríos Prado, A. Business Intelligence: Business Evolution after Industry 4.0. *Sustainability* **2021**, *13*, 10026. [\[CrossRef\]](#)
2. Ding, Z.; Lei, X.; Karagiannidis, G.K.; Schober, R.; Yuan, J.; Bhargava, V.K. A Survey on Non-Orthogonal Multiple Access for 5G Networks: Research Challenges and Future Trends. *IEEE J. Sel. Areas Commun.* **2017**, *35*, 2181–2195. [\[CrossRef\]](#)
3. Anwar, A.; Seet, B.-C.; Hasan, M.A.; Li, X.J. A Survey on Application of Non-Orthogonal Multiple Access to Different Wireless Networks. *Electronics* **2019**, *8*, 1355. [\[CrossRef\]](#)
4. Islam, S.M.R.; Avazov, N.; Dobre, O.A.; Kwak, K.-S. Power-Domain Non-Orthogonal Multiple Access (NOMA) in 5G Systems: Potentials and Challenges. *IEEE Commun. Surv. Tutor.* **2017**, *19*, 721–742. [\[CrossRef\]](#)
5. Goldsmith, A.; Jafar, S.A.; Maric, I.; Srinivasa, S. Breaking Spectrum Gridlock with Cognitive Radios: An Information Theoretic Perspective. *Proc. IEEE* **2009**, *97*, 894–914. [\[CrossRef\]](#)
6. Li, G.; Mishra, D.; Hu, Y.; Huang, Y.; Jiang, H. Adaptive Relay Selection Strategies for Cooperative NOMA Networks with User and Relay Cooperation. *IEEE Trans. Veh. Technol.* **2020**, *69*, 11728–11742. [\[CrossRef\]](#)
7. Ding, Z.; Peng, M.; Poor, H.V. Cooperative Non-Orthogonal Multiple Access in 5G Systems. *IEEE Commun. Lett.* **2015**, *19*, 1462–1465. [\[CrossRef\]](#)
8. Zhang, L.; Liu, J.; Xiao, M.; Wu, G.; Liang, Y.-C.; Li, S. Performance Analysis and Optimization in Downlink NOMA Systems with Cooperative Full-Duplex Relaying. *IEEE J. Sel. Areas Commun.* **2017**, *35*, 2398–2412. [\[CrossRef\]](#)
9. Ding, Z.; Dai, H.; Poor, H.V. Relay Selection for Cooperative NOMA. *IEEE Wirel. Commun. Lett.* **2016**, *5*, 416–419. [\[CrossRef\]](#)
10. Yang, Z.; Ding, Z.; Wu, Y.; Fan, P. Novel Relay Selection Strategies for Cooperative NOMA. *IEEE Trans. Veh. Technol.* **2017**, *66*, 10114–10123. [\[CrossRef\]](#)
11. Xu, P.; Yang, Z.; Ding, Z.; Zhang, Z. Optimal Relay Selection Schemes for Cooperative NOMA. *IEEE Trans. Veh. Technol.* **2018**, *67*, 7851–7855. [\[CrossRef\]](#)
12. Yu, Z.; Zhai, C.; Liu, J.; Xu, H. Cooperative Relaying Based Non-Orthogonal Multiple Access (NOMA) With Relay Selection. *IEEE Trans. Veh. Technol.* **2018**, *67*, 11606–11618. [\[CrossRef\]](#)
13. Li, Y.; Li, Y.; Chu, X.; Ye, Y.; Zhang, H. Performance Analysis of Relay Selection in Cooperative NOMA Networks. *IEEE Commun. Lett.* **2019**, *23*, 760–763. [\[CrossRef\]](#)
14. Bletsas, A.; Khisti, A.; Reed, D.P.; Lippman, A. A Simple Cooperative Diversity Method Based on Network Path Selection. *IEEE J. Sel. Areas Commun.* **2006**, *24*, 659–672. [\[CrossRef\]](#)

15. Li, S.; Wang, F.; Gaber, J.; Zhou, Y. An Optimal Relay Number Selection Algorithm for Balancing Multiple Performance in Flying Ad Hoc Networks. *IEEE Access* **2020**, *8*, 225884–225901. [[CrossRef](#)]
16. Lima, B.K.S.; da Costa, D.B.; Yang, L.; Lima, F.R.M.; Oliveira, R.; Dias, U.S. Adaptive Power Factor Allocation for Cooperative Full-Duplex NOMA Systems with Imperfect SIC and Rate Fairness. *IEEE Trans. Veh. Technol.* **2020**, *69*, 14061–14066. [[CrossRef](#)]
17. Kim, J.-B.; Lee, I.-H. Non-Orthogonal Multiple Access in Coordinated Direct and Relay Transmission. *IEEE Commun. Lett.* **2015**, *19*, 2037–2040. [[CrossRef](#)]
18. Li, G.; Mishra, D. Cooperative NOMA Networks: User Cooperation or Relay Cooperation? In Proceedings of the ICC 2020—2020 IEEE International Conference on Communications (ICC), Dublin, Ireland, 7–11 June 2020.
19. Khalaf, O.I.; Abdulsahib, G.M. An Improved Efficient Bandwidth Allocation using TCP Connection for Switched Network. *J. Appl. Sci. Eng.* **2021**, *24*, 735–741.
20. Lv, L.; Ni, Q.; Ding, Z.; Chen, J. Application of Non-Orthogonal Multiple Access in Cooperative Spectrum-Sharing Networks Over Nakagami-m Fading Channels. *IEEE Trans. Veh. Technol.* **2017**, *66*, 5506–5511. [[CrossRef](#)]
21. Lv, L.; Chen, J.; Ni, Q. Cooperative Non-Orthogonal Multiple Access in Cognitive Radio. *IEEE Commun. Lett.* **2016**, *20*, 2059–2062. [[CrossRef](#)]
22. Singh, S.; Bansal, M. Performance Analysis of NOMA-Based AF Cooperative Overlay System with Imperfect CSI and SIC. *IEEE Access* **2021**, *9*, 40263–40273. [[CrossRef](#)]
23. Im, G.; Lee, J.H. Outage Probability for Cooperative NOMA Systems with Imperfect SIC in Cognitive Radio Networks. *IEEE Commun. Lett.* **2019**, *23*, 692–695. [[CrossRef](#)]
24. Arzykulov, S.; Nauryzbayev, G.; Tsiftsis, T.A.; Maham, B. Performance Analysis of Underlay Cognitive Radio Nonorthogonal Multiple Access Networks. *IEEE Trans. Veh. Technol.* **2019**, *68*, 9318–9322. [[CrossRef](#)]
25. Li, C.; Guo, D.; Guo, K.; Qin, Y.; Xu, R. Outage Performance of Partial Relay Selection in Underlay CR-NOMA Networks. In Proceedings of the 2019 28th Wireless and Optical Communications Conference (WOCC), Beijing, China, 9–10 May 2019.
26. Lee, S.; Duong, T.Q.; da Costa, D.B.; Ha, D.-B.; Nguyen, S.Q. Underlay Cognitive Radio Networks with Cooperative Non-Orthogonal Multiple Access. *IET Commun.* **2018**, *12*, 359–366. [[CrossRef](#)]
27. Chamisa, M.; Takawira, F.; Moualeu, J.M. Non-Orthogonal Multiple Access in Spectrum-Sharing Network under Interference Power Constraints. In Proceedings of the 2019 IEEE AFRICON, Accra, Ghana, 25–27 September 2019.
28. Nguyen, T.-T.; Vu, T.-H.; Nguyen, T.-V.; da Costa, D.B.; Ho, C.D. Underlay Cognitive NOMA-Based Coordinated Direct and Relay Transmission. *IEEE Wirel. Commun. Lett.* **2021**, *10*, 854–858. [[CrossRef](#)]
29. Sultan, K. optimal relay Selection Schemes for NOMA Based Cognitive Relay Networks in Underlay Spectrum Sharing. *IEEE Access* **2020**, *8*, 190160–190172. [[CrossRef](#)]
30. Cai, D.; Ding, Z.; Fan, P.; Yang, Z. On the Performance of NOMA with Hybrid ARQ. *IEEE Trans. Veh. Technol.* **2018**, *67*, 10033–10038. [[CrossRef](#)]
31. Choi, J. On HARQ-IR for Downlink NOMA Systems. *IEEE Trans. Commun.* **2016**, *64*, 3576–3584. [[CrossRef](#)]
32. Yu, Z.; Zhai, C.; Ni, W.; Wang, D. Non-Orthogonal Multiple Access with Cooperative Truncated ARQ and Relay Selection. *IEEE Access* **2019**, *7*, 56228–56243. [[CrossRef](#)]
33. Bariah, L.; Muhaidat, S.; Al-Dweik, A. Error Performance of NOMA-Based Cognitive Radio Networks with Partial Relay Selection and Interference Power Constraints. *IEEE Trans. Commun.* **2020**, *68*, 765–777. [[CrossRef](#)]
34. Deng, D.; Fan, L.; Lei, X.; Tan, W.; Xie, D. Joint User and Relay Selection for Cooperative NOMA Networks. *IEEE Access* **2017**, *5*, 20220–20227. [[CrossRef](#)]
35. Zwillinger, D.; Moll, V.; Gradshteyn, I.S.; Ryzhik, I.M. *Table of Integrals, Series, and Products*, 8th ed.; Academic Press: Boston, MA, USA, 2014.