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REGULAR PAPER

Influence of Feed Velocity on Nonlinear Dynamics of Turning Process

An Wang[1] · Wuyin Jin[1,2]

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Abstract

A more comprehensive orthogonal turning model is developed in order to further study the influence of feed velocity on frictional chatter. Nonlinear dynamic behavior of the cutting tool in two directions is presented by using bifurcation diagram, phase portrait, and Poincaré section. It can be found that the cutting tool has a variety of dynamic behaviors at different feed velocity and cutting velocity, such as periodic motion, quasi-periodic motion, and chaotic motion. Furthermore, the vibration displacement of the cutting tool is affected by the feed velocity, especially for relatively high feed velocity which will result in the cutting tool vibration displacement increase in the cutting direction but a decrease in the feed direction. In addition, it is clear that the stick–slip phenomenon only appears in the cutting direction in our work.

Keywords Turning process · Frictional chatter · Feed velocity · Bifurcation · Chaos

1 Introduction

Chatter vibration has been studied for a long time because of its serious impact on the productivity and machining accuracy. Self-excited vibrations will directly affect cutting process, they can reduce efficiency, affect the surface finish, dramatically affect tool life, and reduce production efficiency.

As we all know, regenerative chatter, frictional chatter, mode coupling chatter, and thermo-mechanical chatter are mainly caused by self-excitation vibration [1]. The friction and regenerative factors are the main cause of chatter. Frictional chatter is mainly induced by the nonlinear friction force that exists between the cutting tool, the workpiece, and the chip [2–4]. Regenerative chatter is mainly due to the mutative cutting force that determined by the position of the cutting tool at the current time and the last time [5–8]. The presence of mode coupling chatter is due to the relatively coupled motion of the cutting system in multiple directions [9]. Thermo-mechanical chatter is caused by the temperature change and plastic deformation of the cutting system in the cutting process [10]. As early as 1800s, Taylor studied chatter and realized that chatter would affect the cutting process to limit production efficiency [11], then, Arnold first applied negative damping effect to explain the occurrence of chatter in the cutting process [12], moreover, Grabec established the two degrees of freedom nonlinear dynamic model of cutting process who investigated the frictional chatter where the cutting force is determined by the relative velocity between the cutting tool, workpiece, and the chip [13], and many of the nonlinear dynamic models of the cutting process have been proposed. The typical factors that can influence the nonlinear dynamic model are the cutting tool and the workpiece separated from each other [14], the nonlinear friction coefficient [15], and the workpiece and tool geometry [15, 16]. Especially, the nonlinear dynamic model considers loss of contact between the tool and workpiece was established by Wiercigroch [10]. In the past, many researchers have analyzed the stability and dynamic response characteristics of the cutting process [17–25].

In recent years, Rusinek et al. established a frictional chatter model in which the cutting force of the cutting system consists of the normal force and the frictional force between the tool, workpiece, and the chip. In their work, the influence of many different parameters on the nonlinear dynamics of the cutting system was studied. Such as, effect of the specific cutting force coefficient, cutting velocity, and

✉ Wuyin Jin
wuyinjin@hotmail.com

An Wang
wangan1981@126.com

[1] School of Mechanical and Electronical Engineering, Lanzhou University of Technology, Lanzhou, China

[2] Key Laboratory of Digital Manufacturing Technology and Application, Ministry of Education of China, Lanzhou 730050, China

the ratio of the stiffness has been studied through bifurcation diagrams [3, 18]. However, a very important factor influencing the turning process, feed velocity is not involved in their model. In order to further investigate the influence of feed velocity on the dynamic characteristics of cutting system, a more novel and comprehensive two degrees of freedom orthogonal turning model considering the influence of feed velocity is established in this work. Meanwhile, aiming at improving the stability of cutting process, the dynamic responses of the cutting tool under the influence of feed velocity for different cutting velocity are investigated.

2 Modeling for a Turning System

In the orthogonal turning system the normal force under consideration, the normal force applied to both surfaces of the cutting tool is presented in Fig. 1., the cutting tool can be described as a two degrees of freedom system vibrating in the X and Y directions, meanwhile the workpiece is treated as a rigid body that does not vibrate. Hence, vibration response of the cutting tool motion in the X and Y directions can be written as follows

$$\begin{aligned} m\ddot{x} + c_x\dot{x} + k_x x &= F_x + N_x \\ m\ddot{y} + c_y\dot{y} + k_y y &= F_y + N_y \end{aligned} \tag{1}$$

where, m is the tool equivalent mass, c_x and c_y,are equivalent damper coefficients, k_x and k_y are equivalent spring coefficients in the X and Y directions, respectively. x and y stands for the cutting tool vibration displacement in the X and Y directions, respectively. A cutting force consists of two parts, the normal force applied to the cutting tool and the frictional forces exists in the workpiece, tool, and the chip.

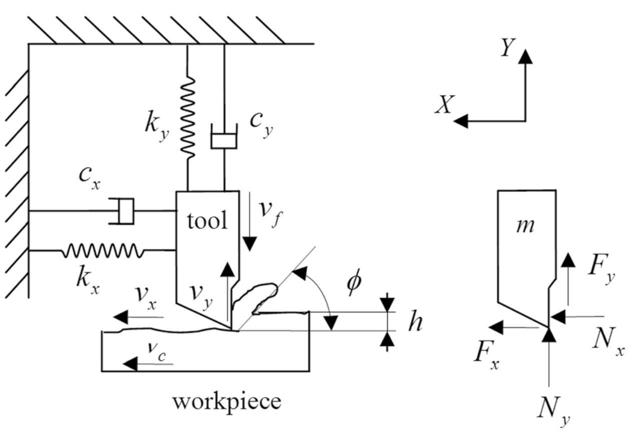

Fig. 1 The two degrees of freedom model of orthogonal turning process, where m is the equivalent mass of the cutting tool,k_x and k_y,c_x and c_y are equivalent spring coefficients and equivalent damper coefficients in the X and Y directions, respectively,h is the instantaneous chip thickness,φ is the shear angle of the workpiece material,v_c is the nominal cutting velocity,v_f is feed velocity of the cutting tool,v_x is the relative velocity of the cutting tool and the workpiece, and v_y is relative velocity between the cutting tool and the chip,N_x,N_y and F_x,F_y are the normal forces and the dry friction forces applied to the cutting tool surfaces, respectively

Normally, The normal forces can be presented as [3, 17, 18]

$$\begin{aligned} N_x &= Qh(c(v_x - 1)^2 + 1)H(h)H(v_x) \\ N_y &= KhH(h) \end{aligned} \tag{2}$$

where Q means the specific cutting force modulus, h means the instantaneous chip thickness, c is a constant associated with the cutting force,K means the contact stiffness, v_x means the relative velocity between the cutting tool and the workpiece, and H(.) represents the Heaviside function.

In a real cutting process, the frictional forces always exist between the workpiece, cutting tool, and the chip. It can be expressed as

$$\begin{aligned} F_x &= N_y\mu_x(\mathrm{sgn}(v_x) - a_x v_x + \beta_x v_x^3) \\ F_y &= N_x\mu_y(\mathrm{sgn}(v_y) - a_y v_y + \beta_y v_y^3) \end{aligned} \tag{3}$$

where μ_x and μ_y are the static coefficient of friction between the workpiece and the cutting tool, and the chip and the cutting tool, respectively. a_x,β_x,a_y,and β_y are constants that determine the characteristics of the dry friction force,v_y is the relative velocity between the chip and the cutting tool, and sgn(.) represents the sign function.

More specifically, the instantaneous chip thickness h and the workpiece revolution period τ can be expressed in the following form

$$h = v_f\tau - y,\ \ \tau = 2\pi/\Omega \tag{4}$$

The relative velocity between the workpiece and the cutting tool v_x, between the chip and the cutting tool v_y can be written as

$$v_x = v_c - \dot{x},\ \ v_y = v_x \tan\varphi + v_f - \dot{y} \tag{5}$$

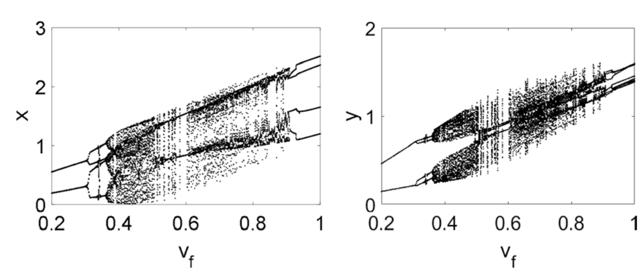

Fig. 2 Bifurcation diagram of the cutting tool vibration displacement x and y versus feed velocity v_f from 0.2 to 1 when the spindle velocity $\Omega = 2$

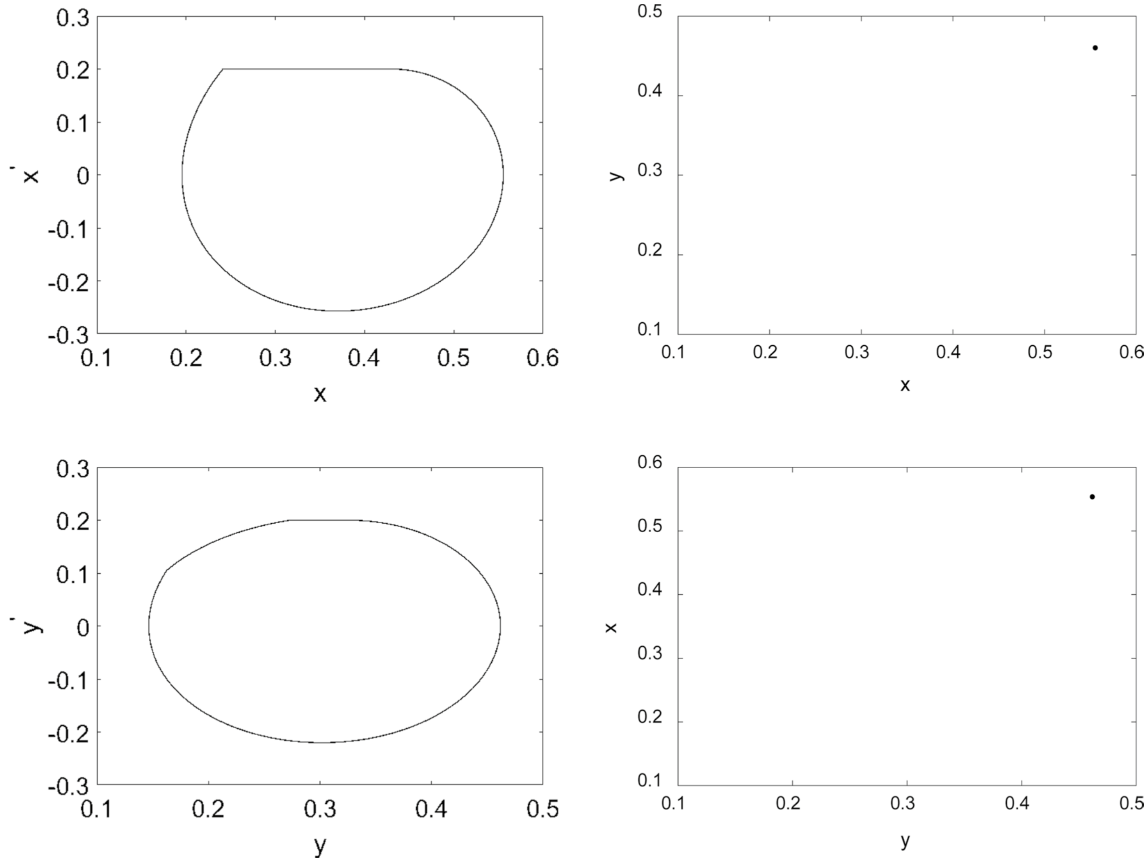

Fig. 3 Phase portrait and Poincaré section of the vibration displacement of the cutting tool in the X and Y directions when the spindle velocity $\Omega = 2$, $v_f = 0.2$

where v_c means the nominal cutting velocity, and $v_c = \Omega R$,v_f is the velocity of the feed,φ means the shear angle of the workpiece material, here Ω is the angular velocity of the workpiece,R is the radius of the workpiece.

For clarity, the cutting tool vibration Eq. (6) and the dimensionless vibration Eq. (7) are put in the appendix.

3 Details of the Dynamic Response of the Cutting Tool

Firstly, the complicated dynamic response of the cutting tool will be revealed with the help of the numerical simulation method. Feed velocity v_f is chosen as the bifurcation parameter that the values choose from 0.2 to 1. It is convenient to use the Poincaré section to locate the periodic and chaotic solutions, therefore, the bifurcation diagrams correspond to the maximum value of the vibration displacement of the cutting tool in X and Y directions, respectively. There are many parameters in the above model, but this research mainly focuses on the impact of feed velocity on the dynamic response of the cutting tool for the three different dimensionless spindle velocity Ω.

In the following simulation, the bifurcation diagram, phase portrait, and the Poincaré section are employed to study the cutting tool dynamic response in both directions. For the Poincaré section, we collected all of the points of intersection of the trajectory with the surface of section $\dot{x}=0$

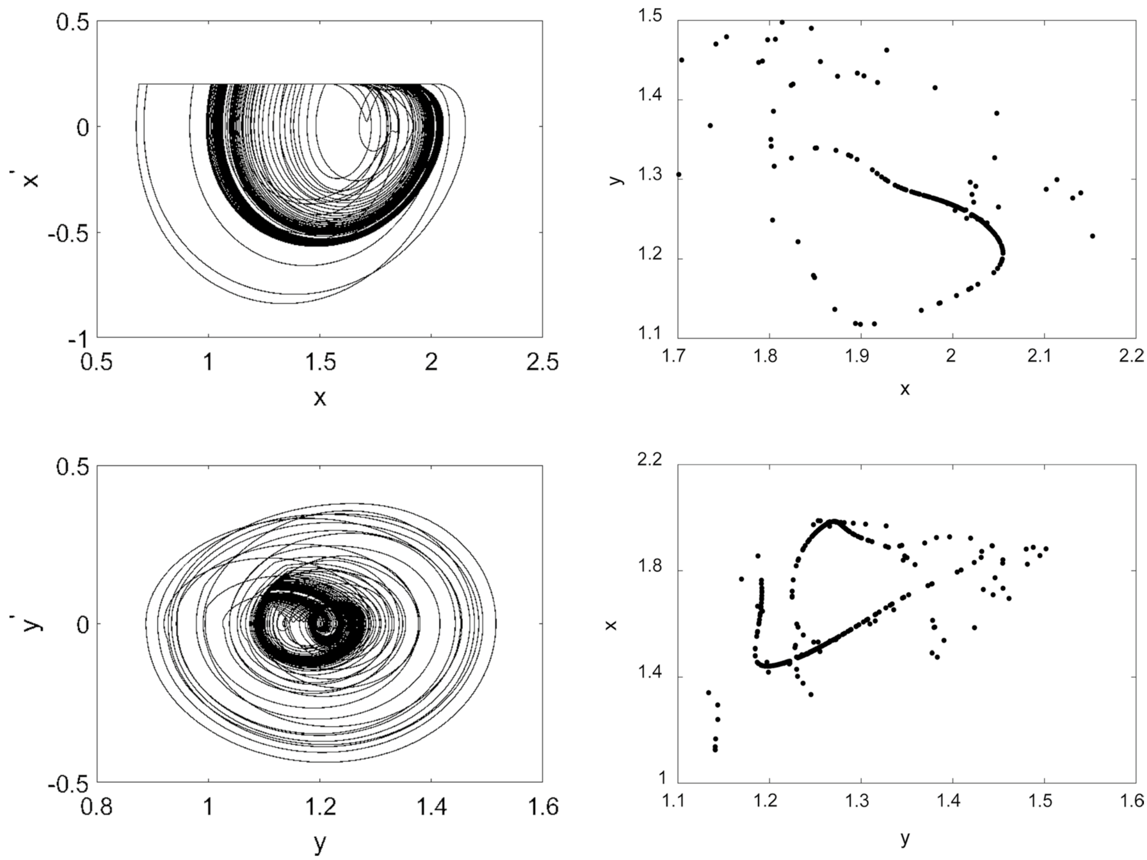

Fig. 4 Phase portrait and Poincaré section of the vibration displacement of the cutting tool in the X and Y directions when the spindle velocity $\Omega = 2$, $v_f = 0.8$

when $\ddot{x} < 0$ in three dimensions $\{x, y, \dot{x}\}$, and $\dot{y} = 0$ when $\ddot{y} < 0$ in three dimensions $\{y, x, \dot{y}\}$. Assuming the following initial conditions and parameters:

$x = \dot{x} = 0$, $y = \dot{y} = 0$, $\xi_x = \xi_y = 0.01$, $\alpha_x = \alpha_y = 0.3$,$\beta_x = \beta_y = 0.1$,$\mu_x = \mu_y = 0.5$,$q = 0.9$,$\tan\varphi = 0.45$,$k = 0.5$,$c = 0.3$,$\alpha = 1$,$R = 0.1$,Ω=2,4,6, respectively.

Obviously, for the case of spindle velocity $\Omega = 2$, the dynamic responses of the vibration displacement of the cutting tool in two directions are very complicated. In the midst of chaos there are some windows of other period and some windows have an abrupt transition from chaos to order. All of this phenomenon can be observed in the Fig. 2. In addition, as shown in Fig. 3 and Fig. 4, there is period 1 motion when $v_f = 0.2$, and chaotic motion can be observed at around $v_f = 0.8$ in both directions. Similarly, a single point and many irregular points appearing in the Poincaré section can further determine that the type of tool dynamic response is periodic-1 motion and chaotic motion. Especially a period 2 motion appeared in the X direction, meanwhile a period 4 motion appeared in the Y direction at about $v_f = 1$ as shown in Fig. 5. In addition, the periodic motion can be confirmed by two points and four points on the Poincaré section. Moreover, there are very obvious stick–slip motion in the X direction but hardly appears in the Y direction as shown in the Fig. 3, 4 and 5.

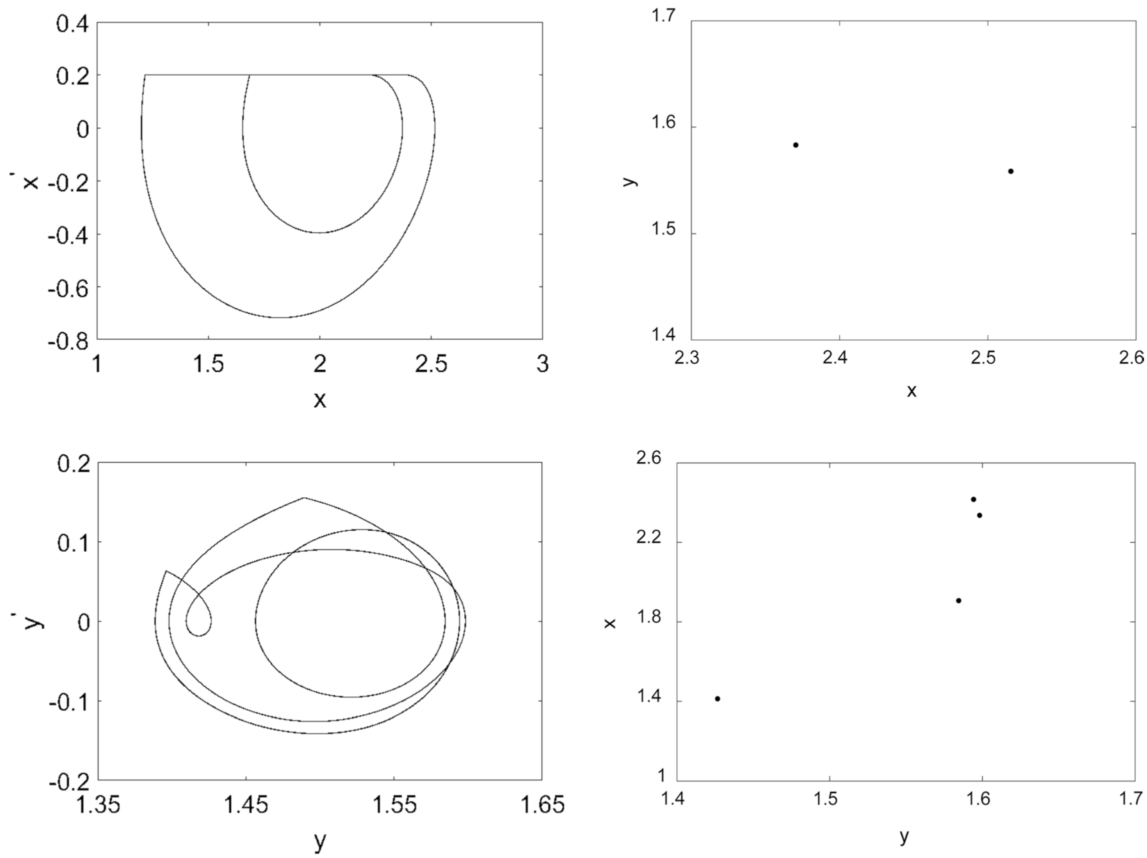

Fig. 5 Phase portrait and Poincaré section of the vibration displacement of the cutting tool in the X and Y directions when the spindle velocity $\Omega = 2$, $v_f = 1$

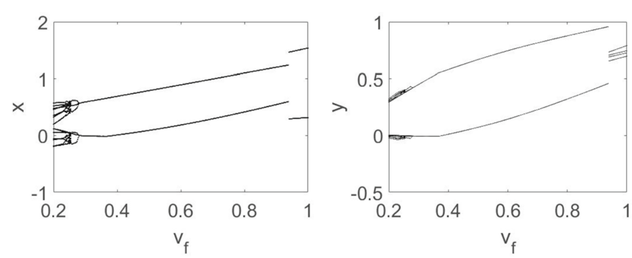

Fig. 6 Bifurcation diagram of the cutting tool vibration displacement x and y versus feed velocity v_f from 0.2 to 1 when the spindle velocity $\Omega = 4$

Furthermore, for the case of spindle velocity $\Omega = 4$, the more interesting phenomenon is there is almost no chaotic motion when feed velocity continues to increase from 0.2 to 1 as shown in Fig. 6. It can be seen from Figs. 7 and 8 that with the increase of feed velocity, the motion of the cutting tool from period 7 to period 3 and then turn into period 1 in both directions, while feed velocity beyond 0.94, the motion of the cutting tool is still period 1 but period 2 motion appears in the Y direction. Also, the Poincaré section

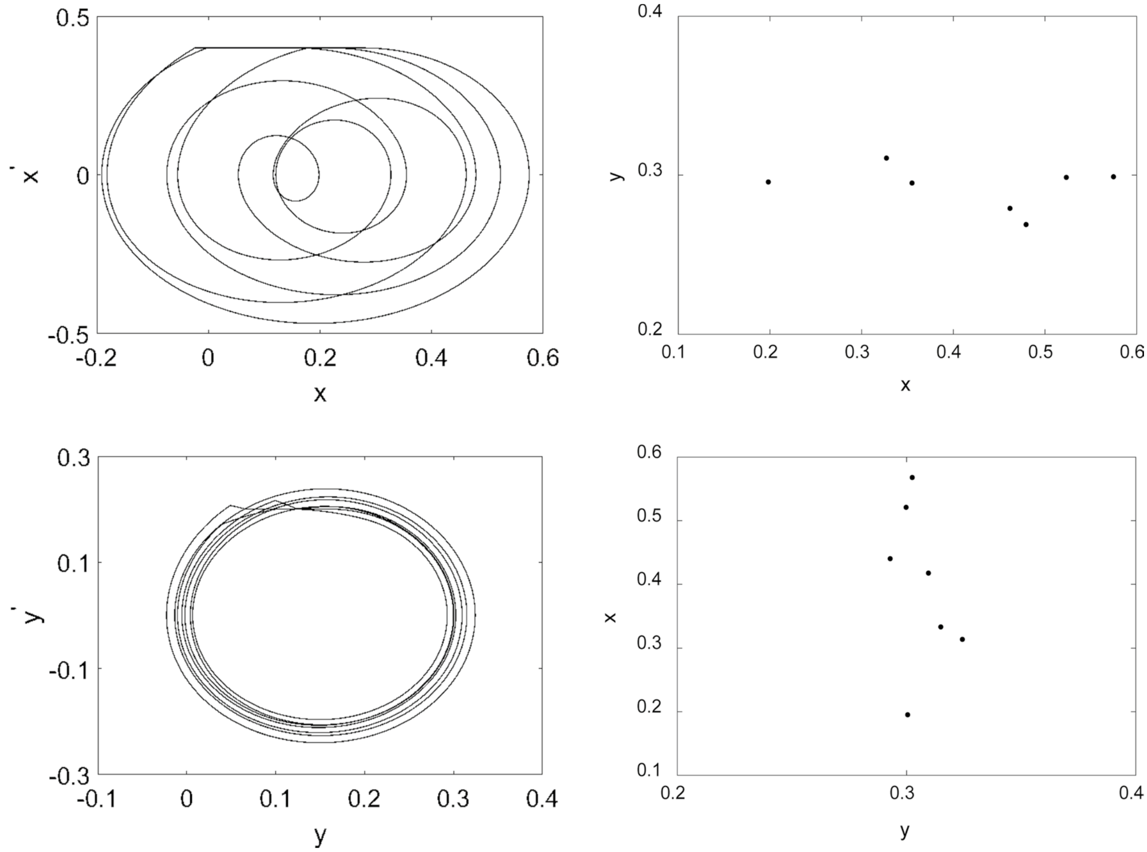

Fig. 7 Phase portrait and Poincaré section of the vibration displacement of the cutting tool in the X and Y directions when the spindle velocity $\Omega = 4$, $v_f = 0.2$

consists of the seven points and three points in Fig. 7 and Fig. 8, meaning the periodic-7 motion and periodic-3 motion of the cutting tool. Moreover, the cutting tool vibration displacement is increased with feed velocity more than 0.94 in the X direction, on the contrary, the cutting tool vibration displacement decreases in the Y direction. Similarly, stick–slip phenomenon only appears in the X direction.

While for the case of $\Omega = 6$, bifurcation diagram shown in Fig. 9, it is readily distinguished that there exist complicated dynamics motions with feed velocity changed. The cutting tool motions experience quasi-periodic and periodic motions in both directions. It can be seen from Fig. 10, the response of the cutting tool is quasi-periodic motion in both directions. The Poincaré section in Fig. 10 means that the motion of the tool is quasi-periodic because it consists of a large number of points falling on a closed curve. Furthermore, with the feed rate over 0.83 as shown in Fig. 11, period 1 and period 2 motions appear in the X and Y directions, respectively. The Poincaré section in Fig. 11 consists of one point and two point, confirming the periodic-1 and periodic-2 motion of the cutting tool. Similar behavior is presented in Fig. 9, vibration displacement of the cutting tool increases in the X direction but decreases in the Y direction when feed velocity is greater than 0.83. Similar to the above two cases, stick–slip phenomenon still appears in the X direction.

The effect of feed velocity on the vibration displacement of the cutting tool and the system dynamics under three different cutting velocity is shown in Fig. 2, Fig. 6, and Fig. 9. It is turned out that the relatively high cutting velocity

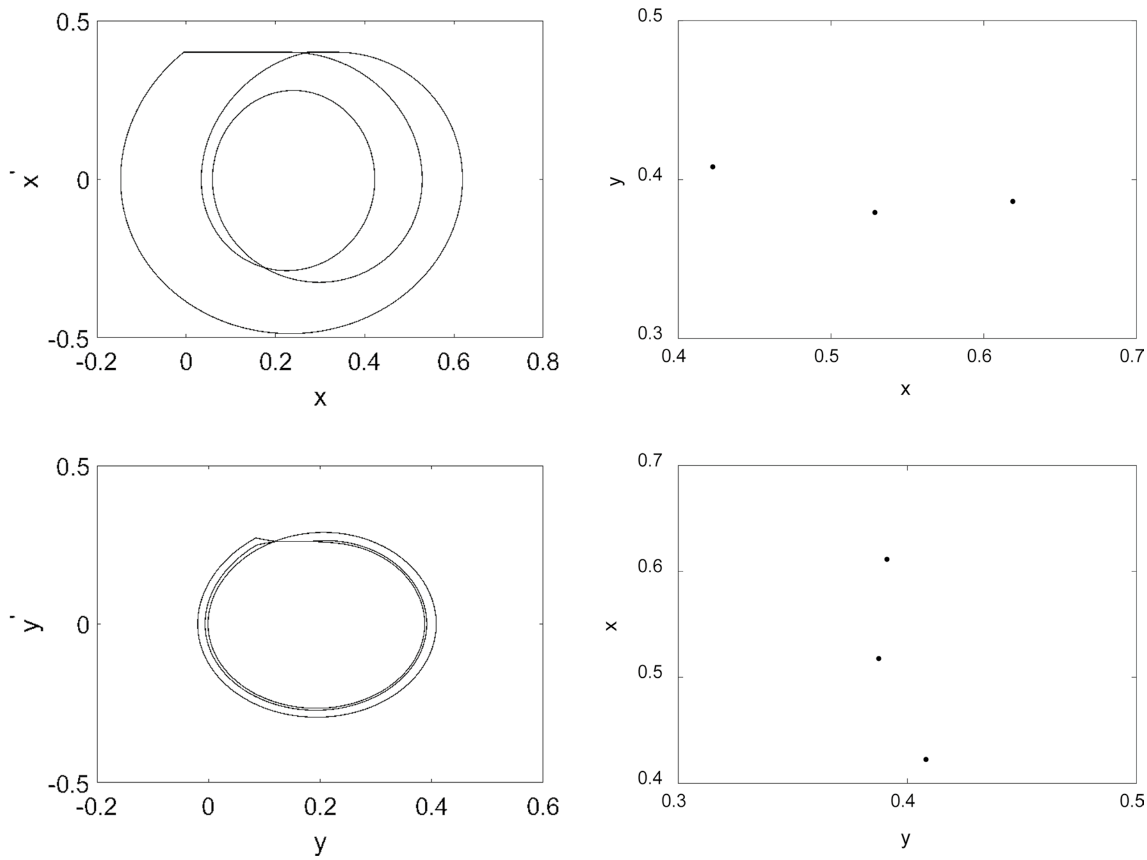

Fig. 8 Phase portrait and Poincaré section of the vibration displacement of the cutting tool in the X and Y directions when the spindle velocity $\Omega = 4$, $v_f = 0.26$

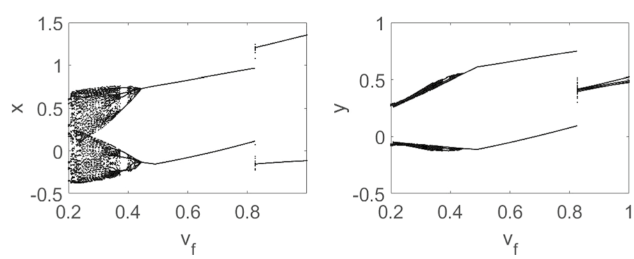

Fig. 9 Bifurcation diagram of the cutting tool vibration displacement x and y versus feed velocity v_f from 0.2 to 1 when the spindle velocity $\Omega = 6$

results in smaller vibration displacement while feed velocity seriously affects the dynamic response of the cutting tool. Besides, for relatively high feed velocity, the cutting tool vibration displacement increases in the X direction while the vibration displacement of the cutting tool decreases in the Y direction. Therefore, it is very important to choose a reasonable cutting velocity and feed velocity for stable cutting.

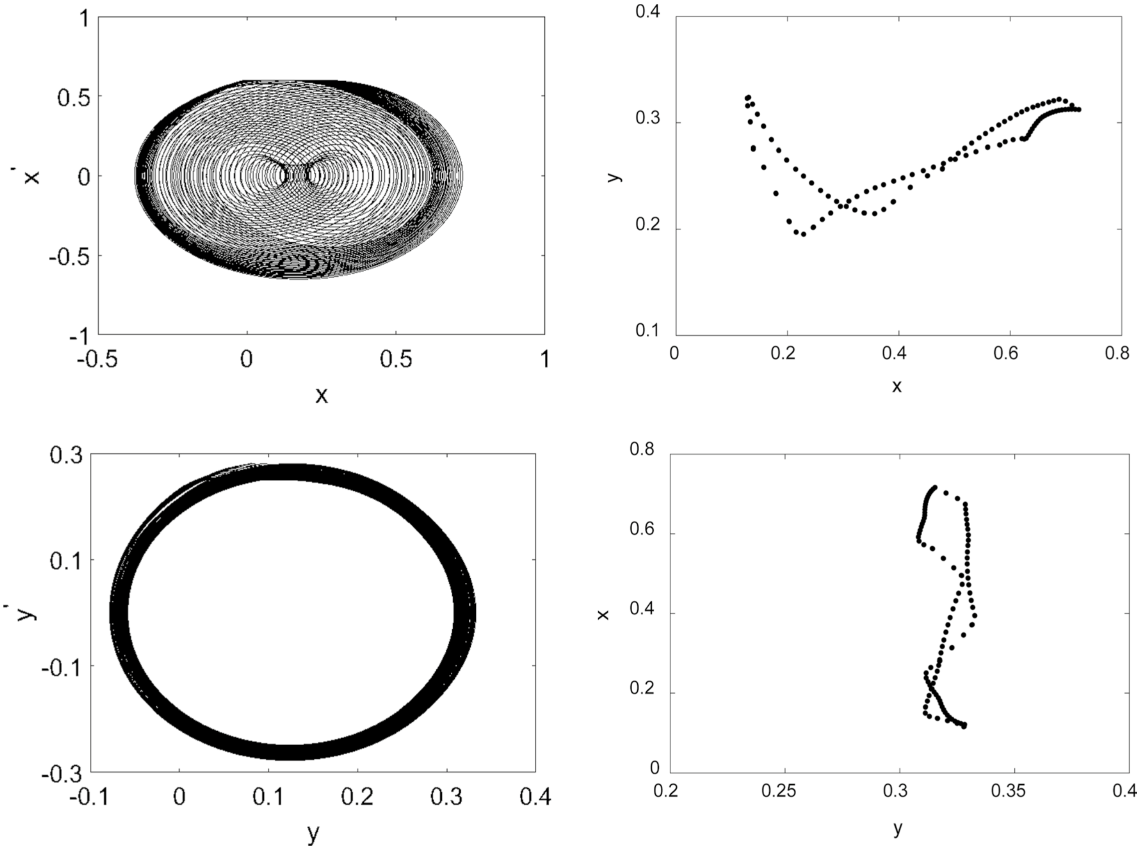

Fig. 10 Phase portrait and Poincaré section of the vibration displacement of the cutting tool in the X and Y directions when the spindle velocity $\Omega = 6$, $v_f = 0.25$

4 Conclusions

In this paper, a two degrees of freedom orthogonal turning model is proposed based on friction model proposed by other scholars. The novelty of our model is to consider the effect of cutting velocity and feed velocity on dynamic frictional force in turning process. The nonlinear dynamic behavior of the cutting system is analyzed by using the feed rate as the bifurcation parameter. It can be found that the response of cutting tool has periodic motion, quasi-periodic motion, and chaotic motion with the different feed velocity. In addition, the stick–slip phenomenon is only found in the cutting direction. Based on the model in this study, both the cutting velocity and feed velocity have a significant effect on the vibration displacement and dynamic response of the cutting tool, especially for relatively high feed velocity which will result in the cutting tool vibration displacement increase in the cutting direction but decrease in the feed direction. For the cutting process, the phenomenon of increased vibration displacement of the cutting tool is a harmful behavior. Therefore, it is important to choose reasonable cutting parameters during cutting process. Furthermore, related experiments and analytical method are a very useful research methods to further validate theoretical research. Similarly, based on current research work, relevant research on effective control of chatter can be conducted in the future.

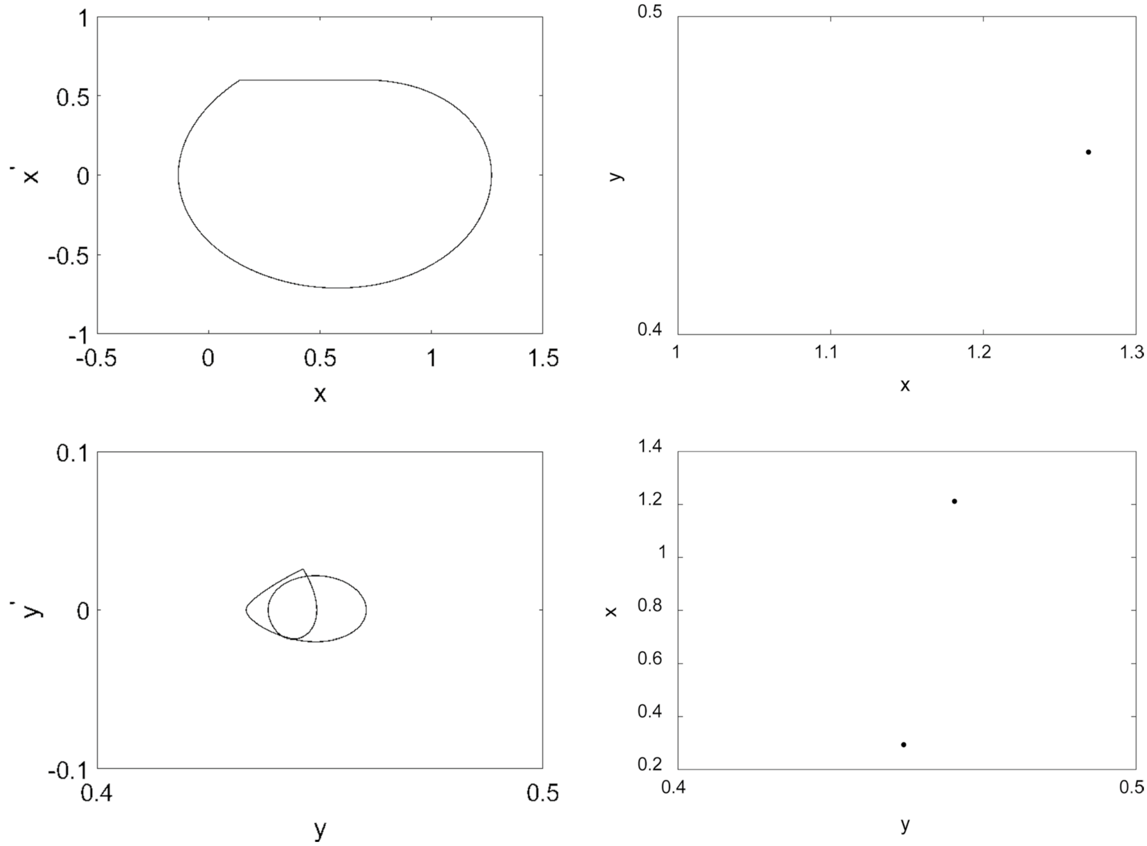

Fig. 11 Phase portrait and Poincaré section of the vibration displacement of the cutting tool in the X and Y directions when the spindle velocity $\Omega = 6$, $v_f = 0.9$

Appendix

Substituting Eqs. 2,3,4 and 5 into Eq. (1), we obtain the vibration response of the cutting tool motion in both directions

In order to reduce the number of parameters, Eq. (6) is simplified as the following dimensionless equation.

$$
\begin{aligned}
&m\ddot{x} + c_x\dot{x} + k_x x = K(2\pi v_f/\Omega - y)H(2\pi v_f/\Omega - y)\mu_x(\mathrm{sgn}(\Omega R - \dot{x}) \\
&\quad - a_x(\Omega R - \dot{x}) + \beta_x(\Omega R - \dot{x})^3) + Q(2\pi v_f/\Omega - y)(c(\Omega R - \dot{x} - 1)^2 + 1) \\
&\quad H(2\pi v_f/\Omega - y)H(\Omega R - \dot{x}) \\
&m\ddot{y} + c_y\dot{y} + k_y y = Q(2\pi v_f/\Omega - y)(c(\Omega R - \dot{x} - 1)^2 + 1) \\
&\quad H(2\pi v_f/\Omega - y)H(\Omega R - \dot{x})\mu_y(\mathrm{sgn}((\Omega R - \dot{x}) \\
&\quad \tan\varphi + v_f - \dot{y}) - a_y((\Omega R - \dot{x})\tan\varphi + v_f - \dot{y}) + \beta_y((\Omega R - \dot{x})\tan\varphi + v_f - \dot{y})^3) \\
&\quad + K(2\pi v_f/\Omega - y)H(2\pi v_f/\Omega - y)
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
&\ddot{x}+2\xi_x\dot{x}+x=k(2\pi v_f/\Omega-y)H(2\pi v_f/\Omega-y)\mu_x(\mathrm{sgn}(\Omega R-\dot{x})-a_x(\Omega R-\dot{x})+\beta_x(\Omega R-\dot{x})^3)\\
&\quad+q(2\pi v_f/\Omega-y)(c(\Omega R-\dot{x}-1)^2+1)H(2\pi v_f/\Omega-y)H(\Omega R-\dot{x})\\
&\ddot{y}+2\xi_y\sqrt{\alpha}\dot{y}+\alpha y=q(2\pi v_f/\Omega-y)(c(\Omega R-\dot{x}-1)^2+1)\\
&\quad H(2\pi v_f/\Omega-y)H(\Omega R-\dot{x})\mu_y(\mathrm{sgn}((\Omega R-\dot{x})\tan\varphi+v_f-\dot{y})-a_y((\Omega R-\dot{x})\tan\varphi+v_f-\dot{y})\\
&\quad+\beta_y((\Omega R-\dot{x})\tan\varphi+v_f-\dot{y})^3)+k(2\pi v_f/\Omega-y)H(2\pi v_f/\Omega-y)
\end{aligned}
\tag{7}
$$

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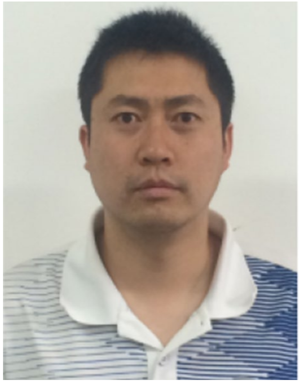
References

1. Quintana, G., & Ciurana, J. (2011). Chatter in machining processes: A review. *International Journal of Machine Tools and Manufacture, 51*(5), 363.
2. Wiercigroch, M., & Krivtsov, A. M. (2001). Frictional chatter in orthogonal metal cutting. *Philosophical Transactions of the Royal Society of London Series A: Mathematical, Physical and Engineering Sciences, 359*(1781), 713–738.
3. Rusinek, R., Wiercigroch, M., & Wahi, P. (2015). Orthogonal cutting process modelling considering tool-workpiece frictional effect. *Procedia CIRP, 31*, 429–434.
4. Kecik, K., Rusinek, R., & Warminski, J. (2013). Modeling of high-speed milling process with frictional effect. *Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multibody Dynamics, 227*(1), 3–11.
5. Faassen, R. P. H., Van de Wouw, N., Oosterling, J. A. J., & Nijmeijer, H. (2003). Prediction of regenerative chatter by modelling and analysis of high-speed milling. *International Journal of Machine Tools and Manufacture, 43*(14), 1437–1446.
6. Molnár, T. G., Insperger, T., & Stépán, G. (2016). State-dependent distributed-delay model of orthogonal cutting. *Nonlinear Dynamics, 84*(3), 1147–1156.
7. Nayfeh, A. H., & Nayfeh, N. A. (2011). Analysis of the cutting tool on a lathe. *Nonlinear Dynamics, 63*(3), 395–416.
8. Kalmár-Nagy, T., Stépán, G., & Moon, F. C. (2015). Subcritical hopf bifurcation in the delay equation model for machine tool vibrations. *Nonlinear Dynamics, 26*, 121–142.
9. Tlusty, .J, Polacek, M. The stability of the machine tool against self-excited vibration in machining. Mach Sci Technol.1963.
10. Wiercigroch, M., & Budak, E. (2001). Sources of nonlinearities, chatter generation and suppression in metal cutting. *Philosophical Transactions of the Royal Society of London Series A: Mathematical, Physical and Engineering Sciences, 359*(1781), 663–693.
11. Taylor, F. W. (1906). On the art of cutting metals. *American Society of Mechanical Engineers., 63*(1619), 25942–25944.
12. Arnold, R. (1946). Cutting tools research: report of subcommittee on carbide tools: the mechanism of tool vibration in the cutting of steel. *Proceedings of the Institution of Mechanical Engineers, 154*(1), 261–284.
13. Grabec, I. (1986). Chaos generated by the cutting process. *Physics Letters A, 117*, 384–386.
14. Kotaiah, K. R., & Srinivas, J. (2010). Dynamic analysis of a turning tool with a discrete model of the workpiece. *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture, 224*(2), 207–215.
15. Nosyreva, E. P., & Molinari, A. (1998). Analysis of nonlinear vibrations in metal cutting. *International Journal of Mechanical Sciences, 40*, 735–748.
16. Vogler, M. P., Devor, R. E., & Kapoor, S. G. (2002). Nonlinear influence of effective lead angle in turning process stability. *Journal of Manufacturing Science and Engineering Transaction of the ASME, 124*, 473–475.
17. Rusinek, R., Wiercigroch, M., & Wahi, P. (2014). Influence of tool flank forces on complex dynamics of cutting process. *International Journal of Bifurcation and Chaos, 24*(09), 1450115.
18. Rusinek, R., Wiercigroch, M., & Wahi, P. (2014). Modelling of frictional chatter in metal cutting. *International Journal of Mechanical Sciences, 89*, 167–176.
19. Wang, A., Jin, W. Y., Wang, G. P., & Li, X. Y. (2016). Analysis on dynamics of a cutting tool with the thermal distortion in turning process. *Nonlinear Dynamics, 2*, 1183–1191.
20. Weremczuk, A., & Rusinek, R. (2017). Influence of frictional mechanism on chatter vibrations in the cutting process–analytical approach. *The International Journal of Advanced Manufacturing Technology, 89*(9–12), 2837–2844.
21. Yan, Y., Xu, J., & Wiercigroch, M. (2016). Regenerative chatter in self-interrupted plunge grinding. *Meccanica, 51*, 3185–3202.
22. Yan, Y., Xu, J., & Wiercigroch, M. (2017). Influence of workpiece imbalance on regenerative and frictional grinding chatters. *Procedia IUTAM, 22*, 146–153.
23. Yi, S., Nelson, P. W., & Ulsoy, A. G. (2007). Delay differential equations via the matrix lambert w function and bifurcation analysis: application to machine tool chatter. *Mathematical Biosciences and Engineering, 4*(2), 355–368.
24. Wang, A., Jin, W. Y., Chen, W. C., Feng, R. C., & Xu, C. W. (2018). Bifurcation and chaotic vibration of frictional chatter in turning process. *Advances in Mechanical Engineering, 10*(4), 168781401877126.
25. Grossi, N., Montevecchi, F., Sallese, L., Scippa, A., & Campatelli, G. (2018). Correction to: Chatter stability prediction for high-speedmilling through a novel experimental-analytical approach. *The International Journal of Advanced Manufacturing Technology, 96*(9), 4541–4544.



An Wang received his M.E. in Mechanical Manufacturing and Automation from Lanzhou University of Technology, Lanzhou, China, in 2008. He received his Ph.D. from Lanzhou University of Technology in 2020. His research interests include nonlinear vibrations and nonlinear dynamics and control.

Wuyin Jin is a Professor at the School of Mechanical and Electronical Engineering, Lanzhou University of Technology. He received his Ph.D. from Xi'an Jiaotong University in 2004. His major research includes neuroscience and dynamic system analysis, machine vision and image processing, embedded systems and signal processing, and nonlinear dynamics theory and method.