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A Modified Conjugate Gradient Approach for Reliability-Based Design Optimization

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ABSTRACT To improve the efficiency of structural reliability-based design optimization (RBDO) based on the performance measure approach (PMA), a modified conjugate gradient approach (MCGA) is proposed for RBDO with nonlinear performance function. In PMA, the advanced mean value (AMV) approach is widely used in engineering because its simplicity and efficiency. However, the AMV method shows the inefficient and unstable results for structural performance function with high nonlinearity in RBDO. To overcome this shortcoming, the proposed MCGA method improves the efficiency of solution by modifying the relevant parameters of conjugate gradient approach (CGA) and the direction of conjugate gradient algorithm for searching the optimal design point. Finally, three numerical examples with highly nonlinear performance function and an optimization design example of speed reducer are presented. Compared with different methods, the results show that the MCGA method exhibited the better efficiency and robustness in structural reliability and RBDO analyses.

INDEX TERMS Reliability-based design optimization, modified conjugate gradient approach, advanced mean value, performance measure approach, reliability analysis.

I. INTRODUCTION

In the structural optimization design, the traditional optimization methods are often used to solve the deterministic optimization problem, which means that the design variables are deterministic variables. However, there are certain uncertainties in the dimensional parameters, material properties and external loads of the structure owing to machining errors, internal dispersion of materials and accidental factors, etc [1], [2]. To solve this problem, the reliability-based design optimization (RBDO) has been proposed. Generally, the RBDO methods can be divided into three categories: decoupling method, single loop method and double loop method. Decoupling method can be used to solve the optimization design and the reliability analysis as well. By using this method, new design parameters are obtained through the optimization loop, and then the reliability analysis method is used to evaluate the feasibility of the design parameters for the probability constraint. In the step of reliability analysis, the most probable point (MPP) is obtained, which is used to get optimal design parameters in subsequent optimization loops [3]. Generally, relatively high solution efficiency can be obtained by the decoupling method. Unlike decoupling method, single-loop method solves the RBDO problem by replacing the reliability analysis loop with the Karush-Kuhn-Tucker optimal condition. However, the relevant research shows that single-loop method encounters the numerical instability and non-convergence for highly nonlinear problems [4]. For double loop method, an external optimization loop and an internal reliability constraint loop are usually used to solve the RBDO problem. Compared with single-loop method, the double loop method is relatively simple.

The RBDO problem is actually optimization design problem based on probability constraints. In order to ensure the reliability of the structure, it is very important to obtain the efficient and accurate solutions of the probability constraints. In RBDO, double loop method based on the reliability index approach (RIA) and performance measure approach (PMA) is widely used [5]–[8]. The RIA usually transforms the probability constraint problems into reliability index constraint problems, and then establishes a constraint relationship with the target reliability index. In general, the reliability index can be solved by first order reliability method (FORM)

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and second order reliability method (SORM) [9]–[11]. However, second order Taylor series expansion is needed to approximate the performance function at MPP in SORM, which is difficult to use in engineering applications because of the expensive computing for complicated engineering problems. On the other hand, FORM just needs first order Taylor series expansion at MPP. Therefore, FORM has high efficiency. In FORM, the HL-RF method is often used in engineering practice because of its simple and efficient characteristics. However, iterative oscillation and low efficiency could be caused for a performance function with a high nonlinearity [12]–[14]. In RBDO, the PMA of the double loop method transforms the probability constraint problem into an optimization problem with the constraint function of target reliability index, then the minimum performance target point (MPTP) can be obtained through continuously searching in PMA. Compared with RIA, PMA has higher efficiency and robustness [15]–[17].

In the PMA, the advanced mean value (AMV) method has been often used because of its simple and efficient characteristics [18]. However, for nonlinear performance functions, its solutions maybe not converge owing to the phenomena of chaotic and periodic cycles. In order to ensure the convergent stability and efficiency of the AMV, conjugate mean value (CMV) and hybrid mean value (HMV) methods have been proposed [19]–[21]. However, unstable solutions still occur for some highly nonlinear concave functions. In response to the highly nonlinear problem, Yang and Yi [22] successfully applied the chaotic control method to solve the nonlinear performance function problems, and significantly improved the convergence compared with the HMV method, but the efficiency is low. Furthermore, to solve highly nonlinear convex functions, based on the work of Yang, Meng *et al.* [23] proposed a modified chaotic control (MCC) method, which improved the chaotic control method through modifying the iterative step, and then an efficient hybrid chaos control (HCC) method is proposed combining AMV and MCC. There are other advanced methods for RBDO are presented in recent years, such as enhanced chaos control (ECC) method [24] and isogeometric analysis method [25]. In addition, Ezzati *et al.* [26] proposed a conjugate gradient approach (CGA), which can quickly achieve the convergence of the solution by using the CGA optimization method, which has very high efficiency. However, in CGA, the efficiency of solution for some highly nonlinear performance functions still needs to be improved.

Based on CGA, this paper proposes a more efficient RBDO method, which modifies the relevant iterative parameters of the conjugate gradient method and the direction of conjugate gradient algorithm for searching the optimal design point. The method is tested by three RBDO numerical examples with highly nonlinear probability constraints and RBDO of a speed reducer. Compared with the other methods, the results show that the proposed method has good robustness and high efficiency.

II. RELIABILITY-BASED DESIGN OPTIMIZATION MODEL AND ANALYSIS METHODS

In this section, we introduce the mathematical model of RBDO and reliability analysis methods. To begin with, we introduce the mathematical model of RBDO, RIA, and PMA in subsection II-A. Then, the common reliability analysis methods of RBDO are showed in subsection II-B. Finally, a modified conjugate gradient approach of RBDO is proposed to improve the efficiency of RBDO in subsection II-C.

A. RELIABILITY-BASED DESIGN OPTIMIZATION MODEL

In RBDO, its mathematical model is generally defined as follows [27], [28]

$$\begin{aligned} &\text{find } \boldsymbol{d}, \boldsymbol{\mu}_X \\ &\min f(\boldsymbol{d}, \boldsymbol{X}) \\ &\text{s.t. } P[g_i(\boldsymbol{d}, \boldsymbol{X}) \le 0] \le \Phi\left(-\beta_t^i\right) \quad i = 1, 2, \cdots, n \\ &\quad \boldsymbol{d}^L \le \boldsymbol{d} \le \boldsymbol{d}^U, \quad \boldsymbol{\mu}_X^L \le \boldsymbol{\mu}_X \le \boldsymbol{\mu}_X^U \end{aligned} \tag{1}$$

where $f(\cdot)$ is the objective function, β_t^i represents the ith target reliability index for constraint performance function $g_i(\cdot)$, and $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution, $\boldsymbol{d}$ stands for the vector of design variables, $\boldsymbol{X}$ is random variables, μ_X is the mean of $\boldsymbol{X}$, $P(g_i(\boldsymbol{d},\boldsymbol{X}) \le 0)$ represents the failure probability of the ith constraint function, $\boldsymbol{\mu}_X^L$ and $\boldsymbol{\mu}_X^U$ represent the lower and upper value of μ_X respectively, $\boldsymbol{d}^L$ and $\boldsymbol{d}^U$ stand for the lower and upper value of $\boldsymbol{d}$.

In general, multi-dimensional integration can be used to solve the constraint function of probability, which can be computed by

$$P[g(\boldsymbol{d}, \boldsymbol{X}) \le 0] = F_g(0) = \int_{g(\boldsymbol{d},\boldsymbol{X}) \le 0} \cdots \int f_X(\boldsymbol{x}) \mathrm{d}\boldsymbol{x} \le \Phi(-\beta_t) \tag{2}$$

where $f_X(\boldsymbol{x})$ is the joint probability density function of $\boldsymbol{X}$, $F_g(\boldsymbol{X})$ represents cumulative distribution function of $g(\cdot)$, $g(\boldsymbol{d},\boldsymbol{X}) \le 0$ stands for failure domain.

However, (2) is difficult to compute in practical engineering. Thus, some efficient approximation methods have been developed, such as FORM and SORM. FORM is often used in engineering practice for its simplicity and efficiency [29]–[31]. In FORM method, it is generally necessary to transform the original space ($\boldsymbol{X}$-space) into a standard normal space ($\boldsymbol{U}$-space) as $\boldsymbol{U} = \Phi^{-1}[F_X(\boldsymbol{X})]$. In addition, the probability constraint problem of (2) can be transformed into the following using the inverse transformation of the cumulative distribution function [20], [21]

$$\beta = \left(-\Phi^{-1}\left(F_g(0)\right)\right) \le \beta_t \tag{3}$$

$$g_p(\boldsymbol{d}, \boldsymbol{X}) = F_g^{-1}(\Phi(-\beta_t)) \ge 0 \tag{4}$$

where β represents reliability index of the performance function, g_p is probabilistic performance measure. In RBDO, the probability constraint in (1) can be replaced by (3) and (4) using the constraint of reliability index and the probabilistic performance measure respectively. Therefore, they are called reliability index approach (RIA) and performance measurement approach (PMA) respectively.

B. EXISTING RELIABILITY ANALYSIS TOOLS IN RBDO

Aiming at the problem that the HMV method does not converge for solving highly nonlinear performance functions, Yang and Yi [22] successfully applied chaotic dynamics theory in RBDO, which partially solved the problem of non-convergence of nonlinear performance functions. However, the speed of convergence needs to be improved. Due to the chaos control method is not efficient, Meng *et al.* [23] proposed a modified chaos control (MCC) method, which improved the efficiency of the solution by correcting the relevant parameters of the chaos control method, and the principle of the MCC can be depicted as

$$\begin{cases} \tilde{\boldsymbol{n}}\left(\boldsymbol{\mu}^{k+1}\right) = \boldsymbol{\mu}^{k} + \lambda \boldsymbol{C}\left(f\left(\boldsymbol{\mu}^{k}\right) - \boldsymbol{\mu}^{k}\right) 0<\lambda<1 \\ \boldsymbol{\mu}^{k+1} = \beta_t \dfrac{\tilde{\boldsymbol{n}}\left(\boldsymbol{\mu}^{k+1}\right)}{\left\|\tilde{\boldsymbol{n}}\left(\boldsymbol{\mu}^{k+1}\right)\right\|} \end{cases} \tag{5}$$

where

$$f\left(\boldsymbol{u}^{k}\right) = -\beta_t \frac{\nabla g\left(\boldsymbol{d}, \boldsymbol{u}_k\right)}{\left\|\nabla g\left(\boldsymbol{d}, \boldsymbol{u}_k\right)\right\|} \tag{6}$$

and $\tilde{\boldsymbol{n}}\left(\boldsymbol{\mu}^{k+1}\right)$ is the modified descent direction based on the chaos control on performance measure functions. $\boldsymbol{C}$ represents the $n \times n$ dimensional involutory matrix, in general, $\boldsymbol{C}$ is the unit matrix $\boldsymbol{I}$, factor λ is generally recommended to be 0.1 or 0.5.

Although MCC method can improve the efficiency of solving the highly nonlinear probability constraint function, the selection of its parameter λ has a great influence on the accuracy of the solution. Since MCC method has a low efficiency when a small value of λ is selected. On the contrary, MCC method has a high efficiency when a large value of λ is selected. Therefore, the method needs to select the appropriate λ in the process of practical application, which reduces the efficiency of the method. Furthermore, Ezzati *et al.* [26] successfully introduced the conjugate gradient analysis (CGA) optimization algorithm in RBDO. Compared with the MCC method, CGA is simple and has higher efficiency for the nonlinear probability constraint problem. The iterative formula of the CGA is formulated as [32]

$$\begin{cases} \boldsymbol{\mu}_{k+1} = \beta_t \boldsymbol{n}\left(\boldsymbol{\mu}_k\right) \\ n\left(\boldsymbol{\mu}_k\right) = \dfrac{\boldsymbol{w}_k}{\left\|\boldsymbol{w}_k\right\|} \end{cases} \tag{7}$$

where $\boldsymbol{n}(\mu_k)$ stands for the conjugate search direction, $\boldsymbol{w}_k$ represents conjugate gradient vector written as

$$\boldsymbol{w}_k = -\nabla g\left(\boldsymbol{d}, \boldsymbol{\mu}_k\right) + \theta_k \boldsymbol{w}_{k-1} \tag{8}$$

in which, θ_k stands for conjugate scalar factor computed by

$$\theta_k = \frac{\left\|\nabla g\left(\boldsymbol{d}, \boldsymbol{\mu}_k\right)\right\|^2}{\left\|\nabla g\left(\boldsymbol{d}, \boldsymbol{\mu}_{k-1}\right)\right\|^2} \tag{9}$$

C. MODIFIED CONJUGATE GRADIENT APPROACH (MCGA)

In CGA method, a new design point μ is determined by the vector $\boldsymbol{w}_k$ and the conjugate factor θ_k. During the iteration of the CGA, θ_k can be determined by the previous conjugate of the performance function, and $\boldsymbol{w}_k$ is mainly used to determine the direction of the new design point. Therefore, $\boldsymbol{w}_k$ has an important influence on the efficiency and accuracy in CGA. For the above reasons, this paper improves the efficiency of the CGA method by modifying the vector $\boldsymbol{w}_k$, which considers the influence of vector $\boldsymbol{w}_{k-1}$ on $\boldsymbol{w}_k$. After the modification of $\boldsymbol{w}_k$, the CGA can significantly improve its convergence speed. In addition, researches show that θ_k is not beneficial to improve computation efficiency in CGA. Therefore, θ_k is changed into (14) in MCGA, and the iterative approach of the MCGA is formulated by

$$\begin{cases} \boldsymbol{\mu}_{k+1} = \beta_t \boldsymbol{n}\left(\boldsymbol{\mu}_k\right) \\ n\left(\boldsymbol{\mu}_k\right) = \dfrac{\boldsymbol{w}_k^{\mathrm{MCGA}}}{\left\|\boldsymbol{w}_k^{\mathrm{MCGA}}\right\|} \end{cases} \tag{10}$$

where $\boldsymbol{w}_k^{\mathrm{MCGA}}$ is modified conjugate gradient vector, and it can be given by

$$\boldsymbol{w}_k^{\mathrm{MCGA}} = \alpha \boldsymbol{w}_k \tag{11}$$

where α stands for conjugate gradient vector factor, which considers the influence of vector $\boldsymbol{w}_{k-1}$ on $\boldsymbol{w}_k$. $\boldsymbol{w}_k$ and α are calculated by

$$\boldsymbol{w}_k = -\nabla g\left(\boldsymbol{d}, \boldsymbol{\mu}_k\right) + \theta_k \boldsymbol{w}_{k-1} \tag{12}$$

and

$$\alpha = \sqrt{\frac{\left\|\boldsymbol{w}_{k-1}\right\|}{\left\|\boldsymbol{w}_k\right\|}} \tag{13}$$

respectively, θ_k can be obtained by

$$\theta_k = \frac{\left\|\nabla g\left(\boldsymbol{d}, \boldsymbol{\mu}_k\right)\right\|}{\left\|\nabla g\left(\boldsymbol{d}, \boldsymbol{\mu}_{k-1}\right)\right\|} \tag{14}$$

In the next section, we would demonstrate the efficiency and robustness of the MCGA method compared with the AMV, CC, HCC, and CGA methods. The flowchart of MCGA method is shown as Figure 1.

III. EXAMPLES ANALYSES

In this section, the proposed MCGA is compared with different methods including AMV, CC, MCC, and CGA method through three mathematical examples in subsection III-A. In addition, a speed reducer design example is showed in subsection III-B.

A. NONLINEAR PERFORMANCE FUNCTION EXAMPLES

Three examples of probability constraints are solved through the MCGA proposed in this paper. Where, example 1 and example 2 are highly nonlinear performance functions. And example 3 is a large reliability index problem, which also increases the difficulty to solve. These three mathematical

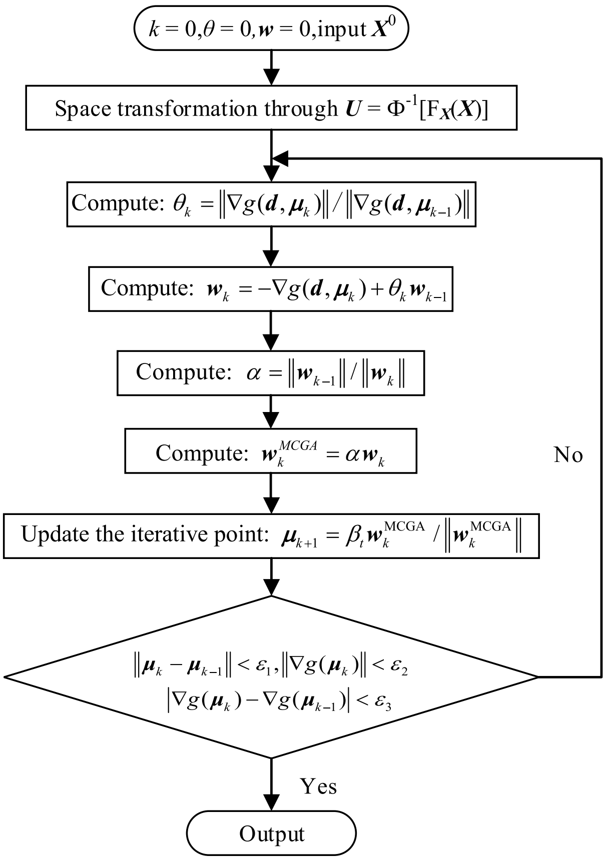

FIGURE 1. Flowchart of the MCGA method.

examples are often analyzed in the literature, so we still use them and given as follows [23]:

Example 1:

$$\begin{cases} g_1(\boldsymbol{x}) = x_1^3 + x_2^3 - 18 \\ x_1 \sim N\left(10, 5^2\right), x_2 \sim N\left(9.9, 5^2\right), \quad \beta_t = 3.0 \end{cases}$$

Example 2:

$$\begin{cases} g_2(\boldsymbol{x}) = x_1^4 + 2x_2^4 - 20 \\ x_1 \sim N\left(10, 5^2\right), x_2 \sim N\left(12, 5^2\right), \quad \beta_t = 2.5 \end{cases}$$

Example 3:

$$\begin{cases} g_3(x) = 0.3x_1^2x_2 - x_2 + 0.8x_1 + 1 \\ x_1 \sim N(1.2, 0.42^2), x_2 \sim N(1.0, .042^2), \quad \beta_t = 6.0 \end{cases}$$

TABLE 1. Results of the different methods.

	AMV	CC	MCC	CGA	MCGA
$g_1(x)$	-	-31.0613(113)	-31.0665(25)	-31.0664(45)	31.0665(15)
$g_2(x)$	-	50.3520(98)	50.3098(12)	50.3098(23)	50.3098(10)
$g_3(x)$	-	-2.2293(109)	-2.2293(29)	-2.2293(8)	-2.2293(9)

The computing results including the AMV, CC, MCC and CGA method as well as MCGA are all given in Table 1. And the number of iterations of the different methods, which can be used as the evaluation criterion of efficiency, is given in the bracket in Table 1.

It can be seen that the AMV method does not converge for all the examples. This is because that the performance function has high nonlinearity which leads to chaos in the iterative process. In contrast to the AMV method, all of the results of the CC, MCC, CGA, and MCGA methods converge accurately.

For example 1, as shown in Figure 2, although the CC method can converge, its convergence speed is extremely slow, because its iterative step size decreases continuously as it approaches MPTP. Unlike the CC method, the MCC, CGA, and MCGA methods converge very quickly because the step size is larger when μ is far away from the MPTP, but the step size is small when μ approaches MPTP. Therefore, in MCC, CGA, and MCGA, the iterations mainly concentrate near the MPTP. However, as shown in Figures 2(2) and (3), the speed is slow when MCC and CGA converge in areas close to the MPTP, which limits the rate of convergence. Unlike them, MCGA's convergence rate in the areas close to the MPTP is significantly better than MCC and CGA, as shown in Figures 3(2), (3) and (4).

For Example 2 in Table 1, the MCGA method is significantly more efficient than the CC and CGA methods. The proposed MCGA method converges about 10 times faster than the CC method and about 2 times faster than the CGA method. Similarly, for Example 3 in Table 1, the MCGA method is 12 times faster than the CC method and 4 times faster than the MCC method.

As shown in three examples in Table 1, compared with the MCGA method, the efficiency of the CC method is lower for all three examples. The MCC method is inefficient for examples 1 and 3, and the efficiency of the CGA method is relatively low for examples 1 and 2, and the MCGA method has fewer the number of iteration times for all examples. Therefore, the proposed MCGA method has higher efficiency and robustness.

B. A SPEED REDUCER DESIGN EXAMPLE BASE

As an important part of mining machinery, cranes and transportation machinery, the RBDO of speed reducer is very important, because it not only ensures its reliability, but also reduces its production cost. A schematic diagram of a reducer is shown in Figure 4. There are 7 random variables and 11 probability constraint functions in the model of reliability-based optimization design of the speed reducer. Generally, the design goal is that the speed reducer is the lightest in weight and it also meets the requirement of probability constraint. Besides, the constraint functions of the speed reducer correspond to displacement constraints, stress constraints, and other constraints, respectively. In the RBDO model of the speed reducer, the random variables X_1, X_2 and X_3 are the gear width, the gear module and the number of pinion teeth, X_4 and X_5 are the bearing distance, and X_6 and X_7 are the diameter of each shaft.

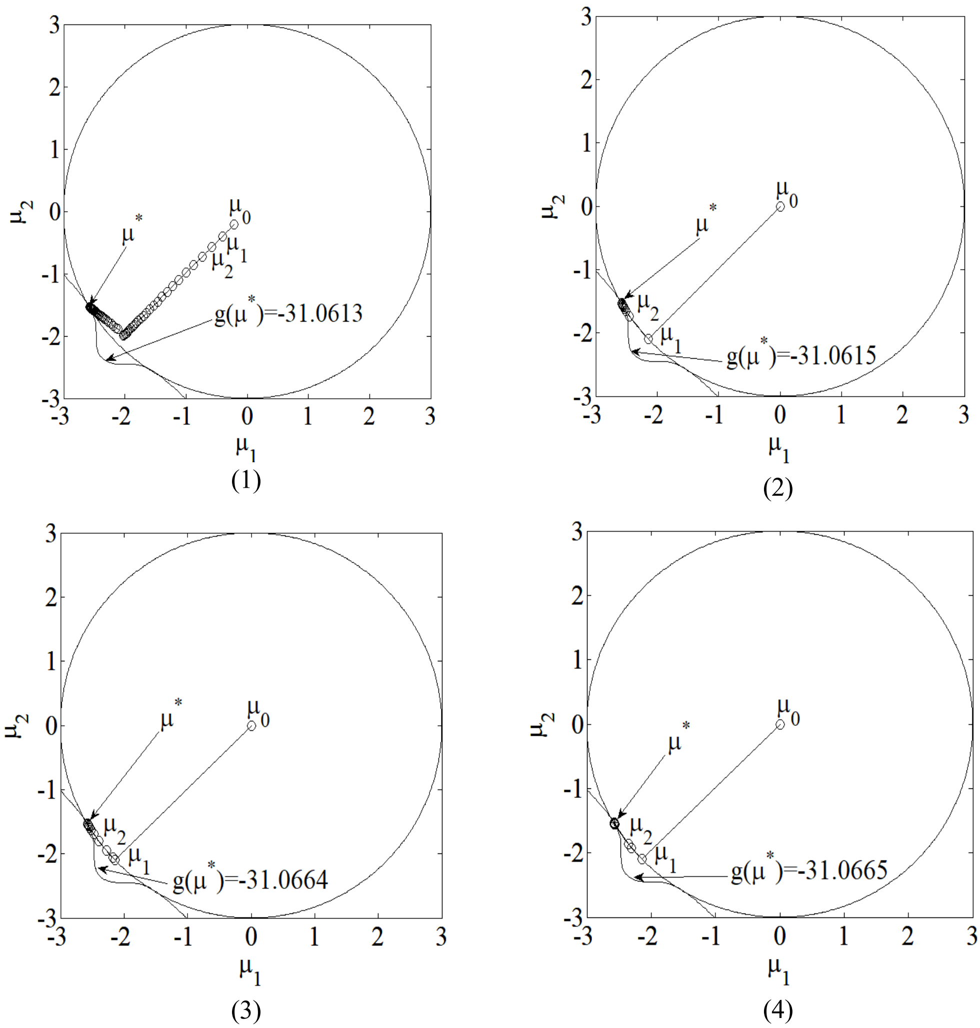

FIGURE 2. Iterative results of MPTP for example 1: (1) CC method; (2) CGA method; (3) MCC method; (4) MCGA method.

TABLE 2. Reliability-based optimization design results of the speed reducer.

	$f(\boldsymbol{d})$	Variables	Number of iterations	
			Objective function	Constraint function
CC	3038.6074	(3.5765,0.7000,17.0000,7.3000,7.7541,3.3652,5.3017)	10	1540
HCC	3038.6128	(3.5765,0.7000,17.0000,7.3000,7.7541,3.3652,5.3017)	10	880
CGA	3038.6128	(3.5765,0.7000,17.0000,7.3000,7.7541,3.3652,5.3017)	10	756
MCC	3038.6128	(3.5765,0.7000,17.0000,7.3000,7.7541,3.3652,5.3017)	10	1140
MCGA	3038.6127	(3.5765,0.7000,17.0000,7.3000,7.7541,3.3652,5.3017)	10	728

Therefore, the mathematical model of RBDO for the speed reducer can be given as follows [33]:

$$\text{find } \boldsymbol{d} = [d_1, d_2, d_3, d_4, d_5, d_6, d_7]$$

$$\begin{aligned}\min f(\boldsymbol{d}) &= 0.7854 d_1 d_2^2 (3.3333 d_3^2 + 14.9334 d_3 - 43.0934) \\ &\quad - 1.508 d_1 (d_6^2 + d_7^2) + 7.477 (d_6^3 + d_7^3) \\ &\quad + 0.7854 (d_4 d_6^2 + d_5 d_7^2)\end{aligned}$$

$$\text{s.t. } P\left[g_i(\boldsymbol{d}, \boldsymbol{X}) \le 0\right] \le \Phi\left(-\beta_t^i\right) \quad i = 1, 2, \cdots, 11$$

$$g_1(\boldsymbol{X}) = \frac{27}{X_1 X_2^2 X_3} - 1, \quad g_2(\boldsymbol{X}) = \frac{397.5}{X_1 X_2^2 X_3^2} - 1,$$

$$g_3(\boldsymbol{X}) = \frac{1.93 X_3^4}{X_2 X_3 X_6^4} - 1, \quad g_4(\boldsymbol{X}) = \frac{1.93 X_5^4}{X_2 X_3 X_7^4} - 1,$$

$$g_5(\boldsymbol{X}) = \frac{\sqrt{\left(745 X_4 / (X_2 X_3)\right)^2 + 16.9 \times 10^6}}{0.1 X_6^3} - 1100,$$

$$g_6(\boldsymbol{X}) = \frac{\sqrt{\left(745 X_5 / (X_2 X_3)\right)^2 + 157.5 \times 10^6}}{0.1 X_7^3} - 850,$$

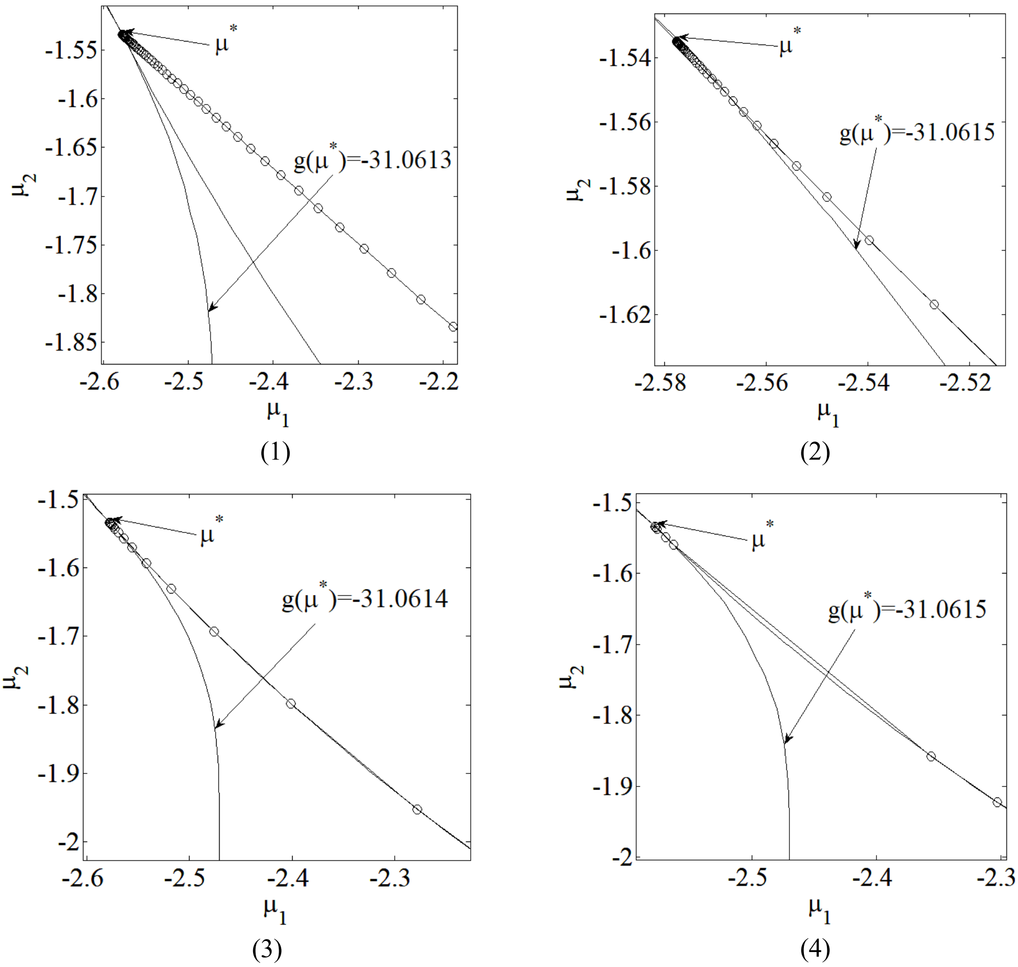

FIGURE 3. **Iterative results near MPTP for example 1: (1) CC method; (2) CGA method; (3) MCC method; (4) MCGA method.**

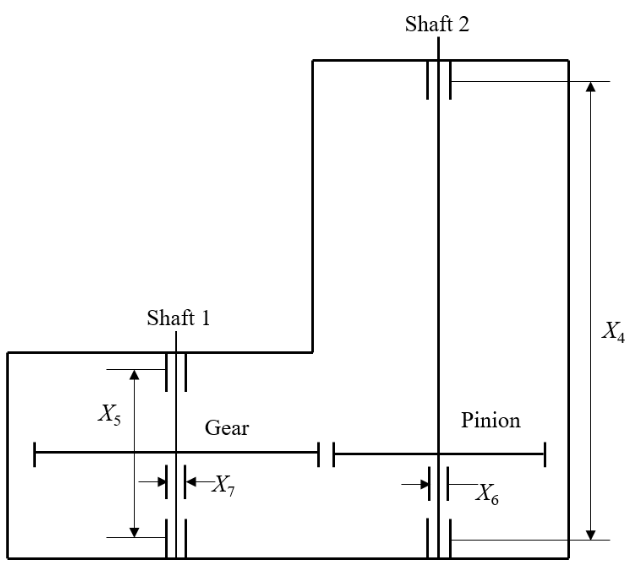

FIGURE 4. **A speed reducer diagram.**

$$g_7(\boldsymbol{X}) = X_2X_3 - 40,\ g_8(\boldsymbol{X}) = 5 - \frac{X_1}{X_2},$$

$$g_9(\boldsymbol{X}) = \frac{X_1}{X_2} - 12,$$

$$g_{10}(\boldsymbol{X}) = \frac{(1.5X_6 + 1.9)}{X_4} - 1,$$

$$g_{11}(\boldsymbol{X}) = \frac{(1.1X_7 + 1.9)}{X_5} - 1,$$

$$2.6 \le X_1 \le 3.6, \quad 0.7 \le X_2 \le 0.8,\ 17 \le X_3 \le 28,$$

$$7.3 \le X_4 \le 8.3, \quad 7.3 \le X_5 \le 8.3,\ 2.9 \le X_6 \le 3.9,$$

$$5.0 \le X_7 \le 5.5,$$

$$\boldsymbol{d}^0 = [3.5,0.7,17.0,7.3,7.72,2.35,5.29]^T,$$

$$\beta_t^1 = \beta_t^2 = \cdots = \beta_t^{11} = 3.0,$$

$$X_j \sim N\left(d_j, 0.005^2\right), \quad j = 1, 2, \cdots, 7$$

where $\boldsymbol{d}_0$ is initial value of the design variable, d_j stands for the random design variable, $j = 1, 2, \cdots, 7$.

The optimization results of the speed reducer is shown in Table 2, the five methods of CC, HCC, CGA, MCC and MCGA based PMA are adopted. As shown in Table 2, the number of iterations of the objective function for all methods is 10, and all methods give almost the identical optimal design points. In addition, although it has an accurate solution, the CC method has the slowest convergence speed. Apparently, compared with the CC method, the MCC is improved. Furthermore, HCC and CGA method is obviously more efficient than CC and MCC method, because the number of iterations of the constraint function is 880 and 756 in HCC and CGA respectively. And MCGA is more efficient than CC, MCC HCC and CGA. Where, MCGA converges about twice as faster than CC and the efficiency of MCGA is significantly better than MCC. Besides, the number of iterations of MCGA is less than HCC and CGA. In summary, the MCGA method has a higher efficiency in RBDO.

IV. CONCLUSION

In the process of structural RBDO, efficient solution of structural performance function with the highly nonlinear has always been a problem needed to be solved. Therefore, how to achieve efficient and accurate solution of nonlinear problems has always been an important research topic. In this paper, a RBDO reliability method based modified conjugate gradient (MCGA) is proposed. The proposed method is validated through comparative analyses of three numerical examples and a speed reducer engineering example. The results show that this method not only can significantly improve the efficiency of reliability optimization design. Therefore, the method has the advantages of high efficiency and good solution accuracy, and can be applied to practical large-scale engineering reliability optimization design.

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