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Pinning Synchronization of Independent Chaotic Systems on Complex Networks with Double Delays

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Abstract

In this paper, aiming at the problem of different signals acting on the same node on a complex network with double time delay, two independent chaotic systems are established, a complex network model with both node delay and coupling delay is constructed. And the synchronization error condition of complex networks with double time delay is analyzed, by applying appropriate pinning controller on the complex network nodes with double time delay, some network nodes are controlled to investigate the problem of hybrid synchronization. Selecting the appropriate Lyapunov function, basing on the Lasalle invariance principle and the linear matrix inequality characteristic, the sufficient conditions for hybrid synchronization of node delay and coupling delay complex networks are given. Finally, the numerical simulation is presented, and the results indicate it is feasible and verified to realize the hybrid synchronization of two independent chaotic systems on the complex network with node delay and coupling delay.

Keywords Double delay · Pinning controller · Hybrid synchronization · Lyapunov function · Lasalle invariance principle

1 Introduction

Synchronization is a common phenomenon in nature. At present, the control and synchronization of complex networks has become a hot topic to researchers. The synchronization of complex network has been applied into many subjects, such as sociology, transportation, biology, computer science and even economics etc. [1–7]. The time delay is inevitable in information processing, computer network and secure communication system and so on [8–12]. Therefore, it is very important to study the synchronization of complex networks with time-delay systems [13, 14]. In the condition of complex network with node time delay, Ahmed et al. studied the complex network synchronization of a class of directional network topology [15]. In Ref. [16], the system coupling delay was considered, and the finite-time synchronization of multi-weighted complex networks was studied. Xu et al. [17]

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studied the synchronization of complex dynamic networks (CDNs) with and without node time delay. However, the complex network generally contains both node delay and coupling delay, and the synchronization effect of two kinds of delay on the complex network can make great influence. Therefore, it is significant to study the synchronization of complex networks with both node delay and coupling delay.

In addition, researchers often study synchronization by adding one or more of the same chaotic systems on complex networks, and had made a lot of significant achievements [18, 19]. For example: Oooka et al. studied local synchronization and global synchronization of time-delay chaotic systems on complex networks [20]. In [21], the synchronization of chaotic signals on uncertain time-varying networks was discussed. Li et al. studied the synchronization of two stochastic memristive chaotic neural networks with time delay [22]. The above achievements have greatly promoted the synchronization of chaotic systems on complex networks. However, the synchronization of the independent chaotic system on the time-delay complex network becomes the problem that has to be solved in the research process, and there are very few achievements in this area [23].

In recent years, researchers have proposed the pinning method to control the complex networks of some nodes. Chen et al. showed that adding a controller to a node in the network system can achieve the overall synchronization of the network system [24]. Han et al. studied the time-varying coupled complex network with double time delay, and realized the external synchronization of two complex systems using the method of pinning synchronization [25]. Ren et al. used the control method of the pinning to realize the synchronization of fractional general complex dynamic networks with time-delay [26]. All the above results have used pinning method to achieve the synchronization of single complex network. However, the actual complex network is often composed of two or more independent network systems. In order to solve the synchronization problem of the independent time-delay chaotic system on the complex network, this paper designs an adaptive pinning controller as a technical means.

Based on the above discussion, hybrid synchronization of two independent chaotic systems with node delay and coupled delay are studied in this paper. Aiming at different signals acting on the same node of complex network, two independent chaotic systems are established, the adaptive pinning synchronization controller is designed, the sufficient conditions for synchronization control are given based on the special control strategy. The validity and feasibility of this method are verified by theoretical analysis and numerical simulation.

This paper is organized as follows. In Sect. 2 the complex networks model will be described in detail and lots of assumptions and lemmas are given. In Sect. 3 the synchronization error is analyzed and the appropriate controller is designed to control the complex networks. The simulation result is presented in Sect. 4. Finally, some conclusions for this paper are given in Sect. 5.

2 System Model and Assumptions

Considering two different independent chaotic systems, the systems of nonlinear dynamics mathematical model are as follows:

$$\dot{X}_i(t) = f(x_i(t)) \tag{1}$$

$$\dot{Y}_i(t) = g(y_i(t)) \tag{2}$$

Type in $x, y \subset R$; f and g are non-linear smooth functions. Describe the network dynamics systems:

$$\dot{X}_i(t) = f(x_i(t)) + f(x_i(t-\tau_1)) + \sigma \sum_{j=1}^{N} a_{ij}\Phi(t, x_j(t)) + \sigma \sum_{j=1}^{N} b_{ij}\Phi(t, x_j(t-\tau_2)) \tag{3}$$

$$\dot{Y}_i(t) = g(y_i(t)) + g(y_i(t-\tau_1)) + \sigma \sum_{j=1}^{N} a_{ij}\Phi(t, y_j(t)) + \sigma \sum_{j=1}^{N} b_{ij}\Phi(t, y_j(t-\tau_2)) \tag{4}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), x_{i3}(t), \ldots, x_{iN}(t))$ $y_i(t) = (y_{i1}(t), y_{i2}(t), y_{i3}(t), \ldots, y_{iN}(t))$ represents the state of the nth node. f and g represent nonlinear smooth functions respectively, σ is the coupling strength, $\Phi : R^n \to R^n$ is the inter-node coupling function. The definitions $A = a_{ij}$ and $B = b_{ij}$ are non-delay coupling and time-delayed coupling matrices, respectively, If node i and node j exist without time-delay coupling $(i \neq j)$, then $a_{ij} > 0$, otherwise $a_{ij} = 0$; If node i and node j have time-delay coupling edges $(i \neq j)$, then $a_{ij} > 0$, otherwise $a_{ij} = 0$. τ_1 and τ_2 are time delays. And assumptions:

$$a_{ii} = -\sum_{\substack{j=1 \\ i \neq j}}^{N} a_{ij} \tag{5}$$

$$b_{ii} = -\sum_{\substack{j=1 \\ i \neq j}}^{N} b_{ij} \tag{6}$$

meet the dissipation conditions. It is considered that the coupling strength between the two nodes is equal.

The equation of state for a hybrid chaotic system is $M = H(M)$, and here we consider the linear superposition of the two hybrid systems [23], namely:

$$M_{i(t)} = H(M) = X_{i(t)} + Y_{i(t)} \tag{7}$$

where $M_{i(t)} = (M_{i1(t)}, M_{i2(t)}, M_{i3(t)}, \ldots, M_{iN(t)})$ represents a vector of hybrid states.

$$\dot{X}_i(t) = f(x_i(t)) + f(x_i(t-\tau_1)) + \sigma \sum_{j=1}^{N} a_{ij}\Phi(t, x_j(t)) + \sigma \sum_{j=1}^{N} b_{ij}\Phi(t, x_j(t-\tau_2)) \tag{8}$$

$$\dot{Y}_i(t) = g(y_i(t)) + g(y_i(t-\tau_1)) + \sigma \sum_{j=1}^{N} a_{ij}\Phi(t, y_j(t)) + \sigma \sum_{j=1}^{N} b_{ij}\Phi(t, y_j(t-\tau_2)) \tag{9}$$

Then the superposition of $X_{i(t)}$ and $Y_{i(t)}$ constitutes a new chaotic system.

In this paper, t is a time variable to describe the synchronization process of the new system. $S(t)$ is a nonlinear smooth or chaotic orbit of a hybrid system.

$H_1(Z), H_2(Z), H_3(Z), \dots, H_N(Z)$ and $H(S)$ represent the hybrid state of the system. The goal here is to design the proper controller to synchronize network $H(M)$ over time over time to $S(t)$, namely:

$$\lim_{t\to\infty} \left|M_i(t) - S(t)\right| = 0, \quad i = 0, 1, 2, 3, \dots, N \tag{10}$$

The following assumptions and lemmas are given in this paper:

Assumption 1 [27] Let X, Y be an any n-dimensional vector, $A \subset R^{n\times n}$, Then

$$2X^TAY \le X^TAA^TX + Y^TY$$

Assumption 2 There is non-negative constant α, make

$$\left\|\Phi(x_j(t)) - \Phi(s)\right\| \le \alpha\left\|e_j(t)\right\|$$
$$\left\|\Phi(y_j(t)) - \Phi(s)\right\| \le \alpha\left\|e_j(t)\right\|$$

where $e_i(t)$ is the error vector.

Lemma 1 [28] *The diagonal element h_{ii} of the matrix H is replaced by $(\rho_{\min}/a)h_{ii}$ to obtain the correction matrix $\tilde{H}$ of H, write $\hat{H} = \frac{\tilde{H}+\tilde{H}^T}{2}$. $\rho_{\min}$ is the minimum characteristic root of $\hat{A} = \frac{A+A^T}{2}$, $a = \|A\|$.*

Lemma 2 *Let the characteristic sequence of matrix A are $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n$, the characteristic sequence of matrix B are $\mu_1, \mu_2, \mu_3, \dots, \mu_n$, then the $n \cdot m$ characteristic sequence of $A \otimes B$ are $\lambda_i\mu_j (i = 1, 2 \dots, n; j = 1, 2 \dots, m)$.*

Lemma 3 [29] *Linear inequality*

$$M=\begin{pmatrix} A & B \\ B^T & C \end{pmatrix} < 0$$

If it's equivalent to one of the following conditions:

1 $A < 0$, 且 $C - B^TA^{-1}B < 0$;
2 $C < 0$, 且 $A - BC^{-1}B^T < 0$;

The following lemma is present, Matrix $\bar{A} = \begin{pmatrix} A_1 & A_3 \\ A_3^T & A_2 \end{pmatrix}$, $\bar{B} = \begin{pmatrix} B_1 & 0 \\ 0 & 0 \end{pmatrix}$, where $B_1 = diag(b_1, b_2, \dots, b_r)$ is a positive definite diagonal matrix. $A_1^T = A_1, A_2^T = A_2$, $A_2 < 0$ is equivalent to $\bar{A} - \bar{B} < 0$ when $b_i (1 \le i \le r)$ is big enough.

3 Simultaneous Analysis and Controller Design

The synchronization of complex network models is closely related to the type of controller. The same network model using different controllers, the synchronization effect will show a great difference. As for complex network model with double time-delay, this paper uses the adaptive pinning synchronization method to control the complex network and studies its process of synchronization.

The proper controller $u_i(t)$ is designed to control two chaotic systems. Describe the chaotic nonlinear dynamics system as

$$\begin{cases} \dot{X}_i(t) = f(x_i(t)) + f(x_i(t-\tau_1)) + \sigma \sum_{j=1}^{N} a_{ij}\Phi(t, x_j(t)) + \sigma \sum_{j=1}^{N} b_{ij}\Phi(t, x_j(t-\tau_2)) + \sigma u_i(t) & 0 < i \le l \\ \dot{X}_i(t) = f(x_i(t)) + f(x_i(t-\tau_1)) + \sigma \sum_{j=1}^{N} a_{ij}\Phi(t, x_j(t)) + \sigma \sum_{j=1}^{N} b_{ij}\Phi(t, x_j(t-\tau_2)) & l+1 < i < n \end{cases} \tag{11}$$

$$\begin{cases} \dot{Y}_i(t) = g(y_i(t)) + g(y_i(t-\tau_1)) + \sigma \sum_{j=1}^{N} a_{ij}\Phi(t, y_j(t)) + \sigma \sum_{j=1}^{N} b_{ij}\Phi(t, y_j(t-\tau_2)) + \sigma u_i(t) & 0 < i \le l \\ \dot{Y}_i(t) = g(y_i(t)) + g(y_i(t-\tau_1)) + \sigma \sum_{j=1}^{N} a_{ij}\Phi(t, x_j(t)) + \sigma \sum_{j=1}^{N} b_{ij}\Phi(t, y_j(t-\tau_2)) & l+1 < i < n \end{cases} \tag{12}$$

The system error equation is described as:

$$e_i(t) = M_i(t) - S(t) = (e_{i1}, e_{i2}, e_{i3}, \dots, e_{in})^T \tag{13}$$

Dynamic system error:

$$\begin{aligned} \dot{e}_i^{(1)} &= X_i(t) - S(t) \\ &= f(x_i(t)) + f(x_i(t-\tau_1)) - 2m(s) + \sigma \sum_{j=1}^{N} a_{ij}(\Phi(t, x_j(t)) - \Phi(t, S(t))) \\ &\quad + \sigma \sum_{j=1}^{N} b_{ij}(\Phi(t, x_j(t-\tau_2)) - \Phi(t, S(t))) + \sigma u_i(t) \end{aligned} \tag{14}$$

$$\begin{aligned} \dot{e}_i^{(2)} &= Y_i(t) - S(t) \\ &= g(y_i(t)) + g(y_i(t-\tau_1)) - 2m(s) + \sigma \sum_{j=1}^{N} a_{ij}(\Phi(t, y_j(t)) - \Phi(t, S(t))) \\ &\quad + \sigma \sum_{j=1}^{N} b_{ij}(\Phi(t, x_j(t-\tau_2)) - \Phi(t, S(t))) + \sigma u_i(t) \end{aligned} \tag{15}$$

Here we define the error vector:

$$\begin{aligned} \dot{e}_i &= \dot{e}_i^{(1)} + \dot{e}_i^{(2)} \\ &= M(Z_i(t)) - M(S(t)) \end{aligned} \tag{16}$$

The error is linearly transformed to:

$$
\begin{aligned}
\dot{e}_i &= Pe_i(t) + Pe_i(t-\tau_1) + Qe_i(t) + Qe_i(t-\tau_1) \\
&\quad + \sigma \sum_{j=1}^{N} a_{ij}(\Phi(t, x_j(t)) - \Phi(t, S(t))) + \sigma \sum_{j=1}^{N} b_{ij}(\Phi(t, x_j(t-\tau_2)) - \Phi(t, S(t))) \\
&\quad + \sigma \sum_{j=1}^{N} a_{ij}(\Phi(t, y_j(t)) - \Phi(t, S(t))) + \sigma \sum_{j=1}^{N} b_{ij}(\Phi(t, y_j(t-\tau_2)) - \Phi(t, S(t)))
\end{aligned}
\tag{17}
$$

where P, Q is the Jacobian matrix of $f(x_i(t)),\ g(y_i(t))$ at $x_i(t),\ y_i(t)$, respectively. From the above equation, we know that P, Q must be bounded matrix, then there must be the following inequality:

$$
\begin{aligned}
\|P\|_2 &\le \delta \\
\|Q\|_2 &\le \delta
\end{aligned}
$$

Theorem *If* f, g *satisfies the condition in Assumptions* 1 *and* 2, *When* $\alpha I_N + 2\alpha\hat{A} + \alpha\hat{B}^2 - \bar{\mathrm{D}} < 0$, *the complex network to achieve hybrid synchronization, and the gradual and stable synchronization. Where D=diag*$\{\underbrace{d_1, d_2, d_3, \dots, d_l}_{l}, \underbrace{0, 0, \dots, 0}_{n-l}\}$, I_N *is the N-order identity matrix. Make controller*:

$$
u_i(t) = \begin{cases} -K_i e_i(t) & 0 < i \le l \\ 0 & l+1 < i < n \end{cases} \tag{18}
$$

$K_i = diag(k_{i1}, k_{i2}, \dots, k_{il}, \underbrace{0, \dots, 0}_{n-l})$, *The adaptive update rates are as follows*:

$$
\begin{cases}
\dot{k}_{i1} = \partial_i e_{i1}^T e_{i1} \\
\dot{k}_{i2} = \partial_i e_{i2}^T e_{i2} \\
\dot{k}_{i3} = \partial_i e_{i3}^T e_{i3} \\
\dots \\
\dot{k}_{in} = \partial_i e_{in}^T e_{in}
\end{cases}
\tag{19}
$$

Construct the *Lyapunov* function $V(t)$ as follows:

$$
V(t) = \frac{1}{2} \sum_{i=1}^{N} e_t^T(t) e_i(t) + \delta \sum_{i=1}^{N} \int_{t-\tau_1}^{t} e_i^T(s) e_i(s) ds + \alpha\sigma \sum_{i=1}^{N} \int_{t-\tau_2}^{t} e_i^T(s) e_i(s) ds + \frac{\sigma}{2} \sum_{i=1}^{N} \frac{1}{\partial_i} D_i^T D_i \tag{20}
$$

t on the derivative of $V(t)$, get:

$$\begin{aligned}
\dot{V}(t) &= \sum_{i=1}^{N} e_i^T(t)\dot{e}_i(t) + \sum_{i=1}^{N} \delta e_t^T(t)e_i(t) - \sum_{i=1}^{N} \delta e_t^T(t-\tau_1)e_i(t-\tau_1) \\
&\quad + \alpha\sigma \sum_{i=1}^{N} e_t^T(t)e_i(t) - \alpha\sigma \sum_{i=1}^{N} e_t^T(t-\tau_2)e_i(t-\tau_2) + \sigma \sum_{i=1}^{N} \frac{1}{\partial_i} D_i^T \dot{\mathrm{D}}_i \\
&= \sum_{i=1}^{N} e_i^T(t)[Pe_i(t) + Pe_i(t-\tau_1) + \sigma \sum_{i=1}^{N} a_{ij}(\Phi(t,x_j(t)) - \Phi(t,S(t))) \\
&\quad + \sigma \sum_{j=1}^{N} b_{ij}(\Phi(t,x_j(t-\tau_2)) - \Phi(t,S(t))) + Qe_i(t) + Qe_i(t-\tau_1) \\
&\quad + \sigma \sum_{j=1}^{N} a_{ij}(\Phi(t,y_j(t)) - \Phi(t,S(t))) \\
&\quad + \sigma \sum_{j=1}^{N} a_{ij}(\Phi(t,y_j(t-\tau_2)) - \Phi(t,S(t))) + \sigma u_i(t)] \\
&\quad + \sum_{i=1}^{N} \delta e_t^T(t)e_i(t) - \sum_{i=1}^{N} \delta e_t^T(t-\tau_1)e_i(t-\tau_1) + \alpha\sigma \sum_{i=1}^{N} e_t^T(t)e_i(t) \\
&\quad - \alpha\sigma \sum_{i=1}^{N} e_t^T(t-\tau_2)e_i(t-\tau_2) + \sigma \sum_{i=1}^{N} \frac{1}{\partial_i} D_i^T \overset{\bullet}{\underset{i}{D}} \\
&\le \sum_{i=1}^{N} e_i^T(t)\left[2\delta e_i(t) + 2\delta e_i(t-\tau_1) + 2\sigma\alpha \sum_{j=1}^{N} a_{ij}e_j(t) + 2\sigma\alpha \sum_{j=1}^{N} b_{ij}e_j(t-\tau_2) + \sigma \sum_{i=1}^{N} K_i e_i(t)\right] \\
&\quad + \sum_{i=1}^{N} \delta e_t^T(t)e_i(t) - \sum_{i=1}^{N} \delta e_t^T(t-\tau_1)e_i(t-\tau_1) + \alpha\sigma \sum_{i=1}^{N} e_t^T(t)e_i(t) \\
&\quad - \alpha\sigma \sum_{i=1}^{N} e_t^T(t-\tau_2)e_i(t-\tau_2) + \sigma \sum_{i=1}^{N} \frac{1}{\partial_i} D_i^T \dot{\mathrm{D}}_i \\
&\le 4\delta \sum_{i=1}^{N} e_t^T(t)e_i(t) + 2\alpha\sigma \sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)a_{ij}e_j(t) + \alpha\sigma \sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)b_{ij}^2 e_j(t) \\
&\quad + \alpha\sigma \sum_{i=1}^{N} e_t^T(t)e_i(t) - \sigma d \sum_{i=1}^{N} e_i^T(t)e_i(t) - \sigma \sum_{i=1}^{l} e_i^T(t)d_i e_i(t) \\
&= e_i^T(t)\left[(4\delta - \sigma d)I_N + \sigma(\alpha I_N + 2\alpha\hat{A} + \alpha\hat{B}^2 - \bar{\mathrm{D}})\right]e_i(t)
\end{aligned} \tag{21}$$

In which $D{=}diag\{\underbrace{d_1, d_2, d_3, \ldots, d_l}_{l}, \underbrace{0, 0, \ldots, 0}_{n-l}\}$,

$$\alpha I_N + 2\alpha\hat{A} + \alpha\hat{B}^2 - \bar{\mathrm{D}} = \begin{pmatrix} I_l\alpha + 2\alpha\hat{A}_u + \alpha\hat{B}_u^2 - \bar{D}_u & 2\alpha\hat{A}_{l(n-l)} + \alpha\hat{B}_{l(n-l)}^2 \\ 2\alpha\hat{A}_{l(n-l)}^T + \alpha(\hat{B}_{l(n-l)}^2)^T & I_{n-l}\alpha + 2\alpha\hat{A}_{(n-l)(n-l)} + \alpha\hat{B}_{(n-l)(n-l)}^2 \end{pmatrix}$$

Let $\lambda_{\min}$ be the smallest eigenvalue of $(2\hat{A}_{l(n-l)} + \hat{B}_{l(n-l)}^2)(I_{n-l}\alpha + 2\alpha\hat{A}_{(n-l)(n-l)}^{-1} + \alpha\hat{B}_{(n-l)(n-l)}^{-2})(2\hat{A}_{l(n-l)}^T + (\hat{B}_{l(n-l)}^2)^T)$, $\lambda_{\max}$ be the smallest eigenvalue of $(2\hat{A}_u + \hat{B}_u^2)$. According to Lemma 3, $\alpha I_N + 2\alpha\hat{A} + \alpha\hat{B}^2 - \bar{D} < 0$ is equivalent to $d_i > \alpha + \alpha\lambda_{\max} - \alpha^2\lambda_{\min}$. When d is sufficiently large and δ is sufficiently small, $(4\delta - \sigma d)I_N + \sigma(\alpha I_N + 2\alpha\hat{A} + \alpha\hat{B}^2 - \bar{D}) < 0$ is gotten. Set the maximum constant set $M = \{e_i(t) \in R^n / e_i(t) = 0, i = 1, \ldots, n\}$, $E = \{e_i(t) \in R^n / \dot{V}(t) = 0, i = 1, \ldots, n\}$, get

$M \subseteq E$. According to the principle of invariance of *Lasalle*, the trajectory of the error system from any initial value will gradually converge to M.The theorem is true.

4 Numerical Simulation

In the simulation, consider the complex network system with N = 100 nodes. Take the Lorentz system with delay and Chen system with time delay:

$$\begin{cases} \dot{x}_{i1} = -ax_{i1} + ax_{i2} \\ \dot{x}_{i2} = cx_{i1} + (k-1)x_{i2} - x_{i1}x_{i3} + kx_{i2}(t-\tau_1) \\ \dot{x}_{i3} = -bx_{i3} + x_{i1}x_{i2} \end{cases} \tag{22}$$

when $a = 10,\ b = 8/3,\ c = 28$, k = 5, $\tau_1 = 0.1$, Fig. 1 shows the time-delayed Lorentz system is chaotic.

$$\begin{cases} \dot{y}_{i1} = -ay_{i1} + ay_{i2} \\ \dot{y}_{i2} = (c-a)y_{i1} + (k+c)y_{i2} - y_{i1}y_{i3} + ky_{i2}(t-\tau_1) \\ \dot{y}_{i3} = -by_{i3} + y_{i1}y_{i2} \end{cases} \tag{23}$$

When $a = 35, b = 3, c = 28$, k = 3, $\tau_1 = 0.1$, Fig. 2 shows the time-delayed Chen system is chaotic.

Then the corresponding complex network dynamics system is described as:

$$\begin{cases} \dot{X}_i = f(x(t), x(t-\tau_1)) + \sigma \sum_{j=1}^{N} a_{ij}x_j(t) + \sigma \sum_{j=1}^{N} b_{ij}x_j(t-\tau_2) + \sigma u_i(t) \\ \dot{Y}_i = g(y(t), y(t-\tau_1)) + \sigma \sum_{j=1}^{N} a_{ij}y_j(t) + \sigma \sum_{j=1}^{N} b_{ij}y_j(t-\tau_2) + \sigma u_i(t) \end{cases} \tag{24}$$

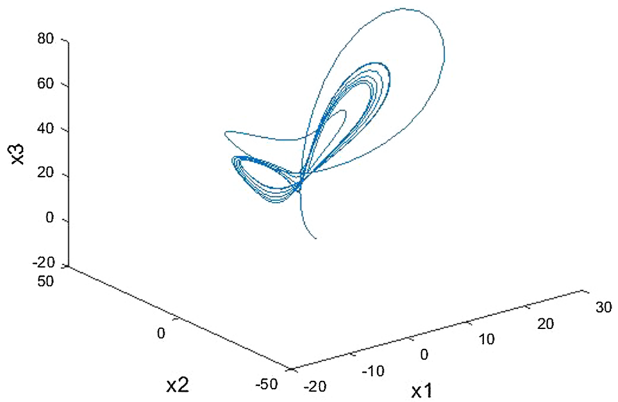

Fig. 1 Time delay chaotic Lorentz system

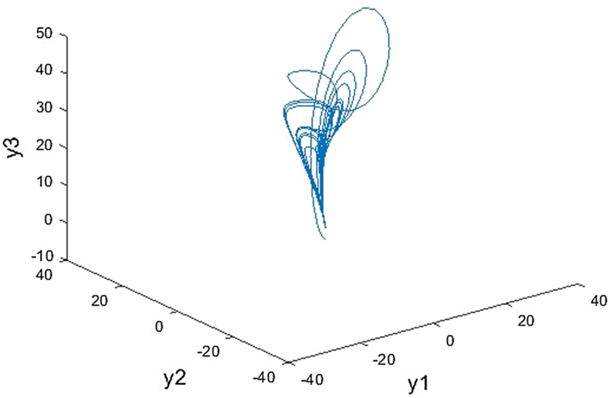

Fig. 2 Time delay chaotic Chen system

Here take $S1 = 4, S2 = 4, S3 = 8, \sigma = 5, \tau_2 = 0.5$, the initial value of $x(i,1) = 10 * rand$, $x(i,2) = 10 * rand$, $x(i,3) = -10 * rand$, $y(i,1) = 10 * rand$, $y(i,2) = 10 * rand$, $\mathrm{y}(i,3) = -10 * rand$, where $rand \in [0,10]$ is any number, taken from the adaptive rate as the identity matrix.

Case Consider the coupling matrix of the star network and the nearest neighbor network of A and B respectively.

And in Fig. 3a–c represent the error curves of e1, e2 and e3. It shows that the evolution of error in the process of complex network operation is always changing obviously, and finally tends to be stable state. The network system realizes hybrid synchronization under the action of adaptive containment controller.

5 Conclusions

In this paper, the hybrid synchronization problem of chaotic systems over complex networks with double delays is studied. The network model is constructed by adding two independent chaotic systems to complex network nodes with node delay and coupled time delay. The adaptive synchronization control strategy is designed to realize the hybrid synchronization of the network under certain conditions. The reliability of the method is demonstrated through numerical simulation.

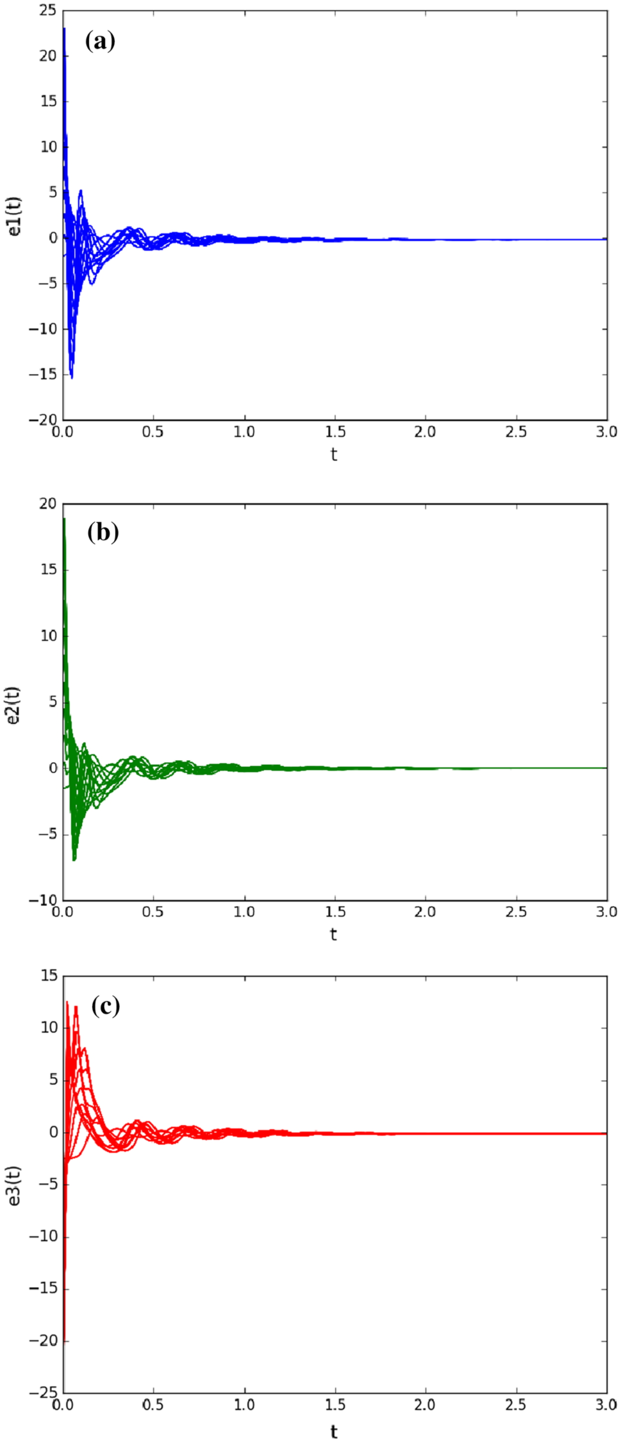

Fig. 3 Error curves

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References

1. Kim, J., & Hastak, M. (2018). Social network analysis: Characteristics of online social networks after a disaster. *International Journal of Information Management, 38*(1), 86–96.
2. Tian, Z., Jia, L., Dong, H., et al. (2016). Analysis of urban road traffic network based on complex network. *Procedia Engineering, 137,* 537–546.
3. Kontorinaki, M., Karafyllis, I., & Papageorgiou, M. (2017). Nonlinear feedback control for general acyclic traffic networks. *IFAC-PapersOnLine, 50*(1), 5281–5288.
4. Song, Q., Yu, Q., Zhao, Z., et al. (2018). Dynamics of complex-valued neural networks with variable coefficients and proportional delays. *Neurocomputing, 275*, 2762–2768.
5. Zarefard, M., & Effati, S. (2017). Adaptive synchronization between two non-identical BAM neural networks with unknown parameters and time-varying delays. *International Journal of Control, Automation and Systems, 1,* 1–11.
6. Ally, A. F., & Zhang, N. (2018). Effects of rewiring strategies on information spreading in complex dynamic networks. *Communications in Nonlinear Science and Numerical Simulation, 57*, 97–110.
7. Ma, K., Li, W., Guo, Q., et al. (2018). Information spreading in complex networks with participation of independent spreaders. *Physica A: Statistical Mechanics and Its Applications, 492*, 21–27.
8. Caprihan, R., Kumar, A., & Stecke, K. E. (2013). Evaluation of the impact of information delays on flexible manufacturing systems performance in dynamic scheduling environments. *International Journal of Advanced Manufacturing Technology, 67*(1–4), 311–338.
9. Tkachenko, M., Yamshinin, A., Lyubimov, N., et al. (2016). Language identification using time delay neural network D-vector on short utterances. In *International conference on speech and computer* (pp. 443–449). Cham: Springer.
10. Furhad, H., Haque, M. A., Kim, C. H., et al. (2013). An Analysis of reducing communication delay in network-on-chip interconnect architecture. *Wireless Personal Communications, 73*(4), 1403–1419.
11. Cabaniss, R., Kumar, V., & Madria, S. (2015). *Multi-party encryption (MPE): Secure communications in delay tolerant networks*. New York: Springer.
12. Hoang, T. M., Palit, S. K., Mukherjee, S., et al. (2016). Synchronization and secure communication in time delayed semiconductor laser systems. *Optik - International Journal for Light and Electron Optics, 127*(22), 10930–10947.
13. Yuan, X., Li, J., & Li, J. (2017). Adaptive synchronization of unknown complex dynamical networks with derivative and distributed time-varying delay couplings. *International Journal of Fuzzy Systems, 6825,* 1–10.
14. Gong, D., Dai, X., Song, J., et al. (2017). Synchronization analysis for complex networks with interval coupling delay. In *International symposium on neural networks* (pp. 329–336). Cham: Springer.
15. Ahmed, M. A. A., Liu, Y., Zhang, W., et al. (2017). Exponential synchronization for a class of complex networks of networks with directed topology and time delay. *Neurocomputing, 266*, 274–283.
16. Qiu, S. H., Huang, Y. L., & Ren, S. Y. (2016). Finite-time synchronization of multi-weighted complex dynamical networks with and without coupling delay. *Neurocomputing, 275*, 1250–1260.
17. Xu, M., Wang, J. L., Huang, Y. L., et al. (2017). Pinning synchronization of complex dynamical networks with and without time-varying delay. *Neurocomputing, 266*, 263–273.
18. Khelifa, M. A., & Boukabou, A. (2016). Design of an intelligent prediction-based neural network controller for multi-scroll chaotic systems. *Applied Intelligence, 45*(3), 1–15.
19. Bagheri, P., & Shahrokhi, M. (2016). Neural network-based synchronization of uncertain chaotic systems with unknown states. *Neural Computing and Applications, 27*(4), 945–952.
20. Oooka, K., & Oguchi, T. (2015). Synchronization patterns in network-of-networks of chaotic systems via cartesian product. *IFAC-PapersOnLine, 48*(18), 13–18.
21. Ling, L., Chen, L., Han, C., et al. (2017). Synchronization transmission of spatiotemporal chaotic signal in the uncertain time-varying network. *Physica A: Statistical Mechanics and Its Applications, 467,* 74–84.

22. Li, X., Fang, J. A., & Li, H. (2017). Exponential adaptive synchronization of stochastic memristive chaotic recurrent neural networks with time-varying delays. *Neurocomputing, 267*, 396–405.
23. Nian, F., & Liu, W. (2016). Hybrid synchronization of two independent chaotic systems on complex network. *Pramana, 86*(6), 1209–1222.
24. Chen, T., Liu, X., & Lu, W. (2007). Pinning complex networks by a single controller. *IEEE Transactions on Circuits and Systems I: Regular Papers, 54*(6), 1317–1326.
25. Min, H., Zhang, Y. M., & Meng, Z. (2014). Outer synchronization analysis of two time-varying networks with double delays based on pinning control. *Acta Physica Sinica, 64*(7), 70506–070506.
26. Ren, G., & Yu, Y. (2017). Pinning synchronization of fractional general complex dynamical networks with time delay. **This work is supported by the National Nature Science Foundation of China (No. 11371049). *50*(1), 8058–8065.
27. Liang, Y., & Wang, X. (2013). Chaotic synchronization in complex networks with delay nodes by non-delay and delay couplings. *Acta Physica Sinica, 62*(1), 709–712.
28. Wen, B., & Zhao, M. (2015). Pinning synchronization control of drive and response network with adaptive coupling strength. *Computer Engineering, 41*(9), 126–130.
29. Boyd, S., El Ghaoui, L., Feron, E., et al. (1994). Linear matrix inequalities in system and control theory. *Philadelphia: Society for Industrial and Applied Mathematics, 86*(12), 2473–2474.

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