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A novel image enhancement method using fuzzy Sure entropy

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A B S T R A C T

Image enhancement is a very significant issue in image processing and analysis. In practice, many images (e.g.images captured from X-ray systems) are of low quality, such a slow-luminance and low-contrast, which must be enhanced before further processing. Fuzzy set theory is a useful tool for handling the ambiguity or uncertainty. Many researchers use the maximum Shannon entropy and fuzzy complement for image enhancement. But these methods are easy to be over-enhanced or under-enhanced or time-consuming. In this paper, a flexible method is proposed, which utilizes the maximum fuzzy Sure entropy, fuzzy c-partition and fuzzy complement (MSRM). Furthermore, a positive threshold value selection algorithm is developed to tune the enhancement performance of the proposed method. A variety of highly degraded images have been experimented by the proposed method. The comparisons of those experimental results show that the performance of our method overwhelms those of the existing ones.

1. Introduction

In practice, the quality of images is easy to be effected by several factors, such as the shooting angle, the shooting condition and the capturing approaches. Generally, the original images, which are captured by charging coupled device image sensors or by other digital image sensors in non-uniform illumination conditions, are unclear or blurred. Thus, the image without enhancement is impossible to exhibit the vivid details to observers directly. For instance, the X-rays images from direct digital radiography(DDR) system are not only blurred by noise, but also captured in non-uniform and low illumination conditions. So image enhancement is essential to these digital images and thus is an important task in image processing. The fuzzy set theory is a powerful tool for developing new and robust techniques in image processing [1–7]. A number of researchers have aimed for enhancing the low-contrasted image and many developed methods which perform quite well [8–10,13,19,20]. However, when applying on the low-luminance and low-contrasted images, those existing methods presented in the previous literatures cannot work well and most of the time lack compatibility and flexibility. For these reasons, we need to look for a suitable and flexible function to modify the intensity distribution of the image so as to fitting to human eyes. In this paper, a new method for image enhancement is proposed, which is based on the maximum fuzzy Sure entropy (MSRM). MSRM uses the fuzzy set theory [11], the fuzzy c-partition, the involutive fuzzy complements and the maximum fuzzy Sure entropy. We develop a new class of membership functions as well as a new measure of fuzziness. In our study, we design a new method to select a suitable positive threshold value to control the enhancement performance. To date, the Sure entropy principle has been rarely used in the literature [12]. We tested the proposed method and other existing four methods using various images of different types. The experimental results show that the proposed method achieves better performance in image enhancement, especially when the images are extremely low contrasted and low illuminated.

The rest of this paper is organized as follows. Section 2 describes an image in the form of fuzzy set theory, as well as the maximum fuzzy-Sure entropy method, fuzzy c-partition and the involutive membership functions. In Section 3, we describe the proposed method and the threshold value selection in full detail. The family of functions to modify the membership values of the gray levels is also introduced in this section. Section 4 presents the experimental results obtained using the proposed method and other existing methods. Comparisons and discussions of the proposed method and the other four contrast enhancement techniques are carried out. Finally, conclusions are made in Section 5.

2. Background

For image enhancement, we present the framework concerning fuzzy set theory, fuzzy entropy, fuzzy c-partition and involutive membership functions. Following, we describe the details.

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2.1. Image description in fuzzy set theory

In this paper, an image A of size $M \times N$ pixels, having L gray-levels ranging from $L_{\min}$ to $L_{\max}$, can be viewed as an array of fuzzy singletons [10,11]. Each element in the array is the membership value representing the degree of brightness of the gray level $g(g \in [L_{min}, L_{max}])$. In fuzzy set theory, image A can be written as below:

$$A = \{\mu_A(g_{ij})/g_{ij}, i = 1, 2, \cdots, M, j = 1, 2, \cdots, N\} \tag{1}$$

where $\mu_A(g_{ij})$ denotes the degree of brightness possessed by the gray level intensity g_{ij} corresponding to the (i, j) pixel. The histogram of the image is described as $h_A(g)$, $(g \in [L_{min}, L_{max}])$and denotes the frequency of occurrence of the gray level g. We introduce a membership function $P_A(g)$ of fuzzy set [7], which is the probability measure of the occurrence of gray-levels and described as $P_A(g) = \mu_A(g)\bullet \tilde{h}_A(g)$. Here $\tilde{h}_A(g)$denotes the probability of the gray level g by normalizing histogram $h_A(g)$. In our study, we write $\tilde{h}_A(g)$as below:

$$\tilde{h}_A(g) = \frac{h_A(g)}{M \times N} \tag{2}$$

So the probability of this fuzzy event can be calculated by:

$$P(A) = \sum_{g=0}^{L-1} \mu_A(g)\tilde{h}_A(g) \tag{3}$$

2.2. Fuzzy c-partition

The fuzzy c-partitions can be represented by partition matrices. It is defined as [13]. Let $T = \{t_1, t_2, \cdots, t_n\}$, Q_{cn} is a set of real $c \times n$ matrices, and c is an integer, $2 \le c \le n$. Fuzzy c-Partition space for T is the set:

$$M_C = \left\{ U \in Q_{cn} \middle| \mu_{ik} \in [0, 1];\ \sum_{i=1}^{c} \mu_{ik} = 1\ \forall\ k;\ 0 < \sum_{k=1}^{n} \mu_{ik} < n\ \forall\ i \right\} \tag{4}$$

2.3. Maximum entropy of fuzzy c-partition principle

In this section, we discuss two different measures of fuzziness for fuzzy c-partition, i.e. Shannon entropy and Sure entropy. The Shannon entropy principle was used for the maximum fuzzy entropy and fuzzy c-partition in early studies [14]. Let $U = \{A_1, A_2, \cdots, A_n\}$ be a finite partition of fuzzy sets. The Shannon entropy $H(U)$ [15] is defined as below:

$$H(\mathrm{U}) = -\sum_{i=0}^{c} P(A_i)\log P(A_i) \tag{5}$$

In this paper, we have investigated image enhancement performance of Sure entropy principle according to the Shannon entropy's for the maximum fuzzy entropy and fuzzy c-partition. The Sure entropy $H(U)$ presented in [16] is given as below:

$$|P(A_i)| \le \varepsilon \Rightarrow H(U) = \sum_{i=0}^{c} \min(P(A_i)^2, \varepsilon^2) \tag{6}$$

where $\varepsilon$is a positive threshold value. At the same time, since $P(A_i) \ge 0$, thus the Sure entropy $H(U)$ can be described as follows:

$$|P(A_i)| \le \varepsilon \Rightarrow H(U) = \sum_{i=0}^{c} \min(P(A_i), \varepsilon) \tag{7}$$

In this, we can tune the enhancement performance of the image by changing the value of ε.

2.4. Involutive membership functions

Image enhancement plays a key role in digital image processing, and there are a lots of literatures concerning this topic. When the image is improved, The gray-levels of the image histogram will be modified in some respects, e.g. by histogram equalization or appropriate gray-level transformation. In a word, the selection of a suitable function for the gray-level modification is an important step. In this study, we introduce Sugeno class of involutive fuzzy complements and present an involutive memberships [2,17]. Here, the membership values $\mu_A(g)(g \in [L_{min}, L_{max}])$ of image A denote the degree of compatibility of the gray level g with a relational image property (e.g. brightness, edginess etc.). Then we can define the involutive fuzzy complements as follows.

Definition 1. Let $\mu \in F(x)$ and $\alpha \in (0, 1)$. Then the complement of μ is the fuzzy set μ^*defined for all $g \in X$ by the membership function [17]:

$$\mu_A^*(g) = \frac{1 - \mu_A(g)}{1 + \lambda\mu_A(g)} \tag{8}$$

where

$$\lambda = \frac{1 - 2\alpha}{\alpha^2} \tag{9}$$

Properties of μ^*:

1. $\lambda \in (-1, \infty)$, so μ^*belongs to Sugeno's class of involutive complements.
2. $\mu_A^*(g) = \mu_A(g)$ if and only if $\mu_A(g) = \alpha$. Therefore, $\alpha$is the equilibrium of μ^*.
3. For $\alpha = 0.5$ the complement becomes the standard fuzzy complement, i.e., $\mu_A^*(g) = 1 - \mu_A(g)$.
4. Let $\mu_A, \mu_B \in F(X)$, and μ_A is α-sharper than μ_B. Then $\mu_A^*$is α-sharper than μ_B^*.
   In this study, for the image A, the membership function $\mu_A(ij)$ is initialized as follows:

$$\mu_A(ij) = \frac{L_{ij} - L_{\min}}{L_{\max} - L_{\min}} \tag{10}$$

where $L_{ij} \in [L_{min}, L_{max}]$,$i = 1, 2, \cdots, M$, and $j = 1, 2, \cdots, N$. Referring to the literature [17], we have new involutive memberships as below:

$$\hat{\mu}_A(ij) = \begin{cases} \dfrac{1}{\alpha}\mu_A(ij)^2, & \text{if}\quad \mu_A(ij) \in [0, \alpha] \\ \dfrac{1 - B(\mu_A(ij))}{1 + \lambda B(\mu_A(ij))}, & \text{if}\quad \mu_A(ij) \in [\alpha, 1] \end{cases} \tag{11}$$

where

$$B(\mu_A(ij)) = \frac{1}{\alpha}\left(\frac{1 - \mu_A(ij)}{1 + \lambda\mu_A(ij)}\right)^2 \tag{12}$$

The relationship of α and λ is depicted in (9), we can calculate λ and replace it into (12) and (11). Then, we obtain the involutive memberships $\mu_A(ij)$, called α-involutive memberships as depicted in Fig. 1.

The α-involutive fuzzy class begins with a high-contrast image and by increasing α the image changes itself into a dim image. In this paper, we obtain the optimal parameter α using the exhausted search method.

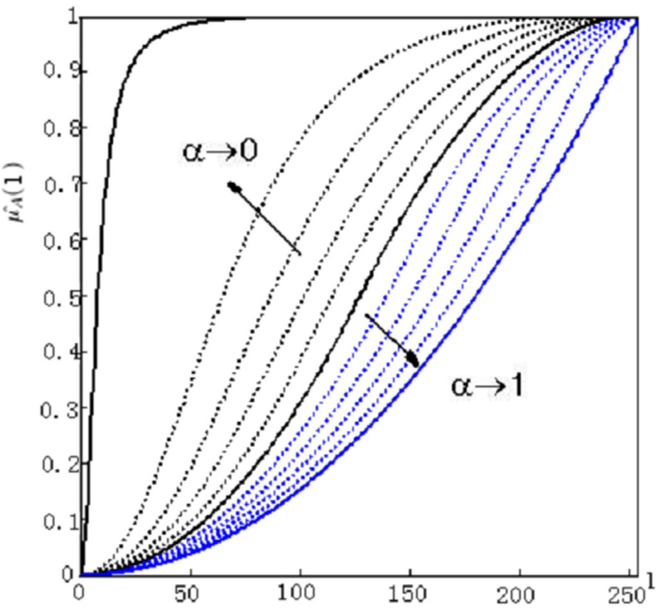

Fig. 1. The α-involutive fuzzy memberships.

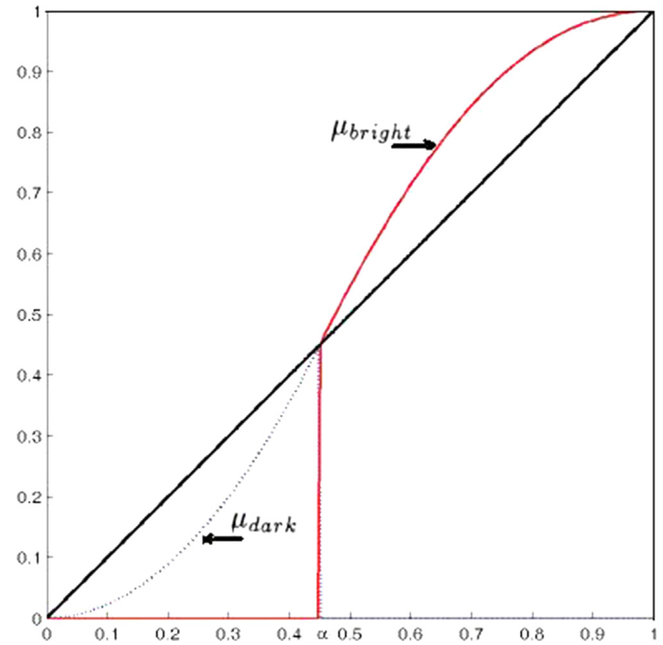

Fig. 2. The membership functions, μ_{dark} and μ_{bright}.

3. Proposed algorithm

In this section, a fuzzy 2-partition method is presented first. Next, the selection of the threshold value ε is discussed. Then, the maximum entropy is found out for the fuzzy 2-partition by the exhausted search method, as well as the equilibrium α of the image A is obtained.

3.1. Fuzzy 2-partition method

The fuzzy 2-partition method is usually used to look for the gray-level threshold in image processing. In this study, we can obtain the equilibrium $\alpha$by this method. Here, we assume that the images have 256 gray-levels ranging from 0 to 255 and normalize these values to $W \in \left[0, \frac{1}{255}, \frac{2}{255}, ..., \frac{255}{255}\right]$. The bi-level threshold is used to classify pixels to dark group or bright group. There for, two fuzzy sets described as dark and bright may be considered, in W whose membership functions are defined as below:

$$\mu_{dark} = \begin{cases} \frac{1}{\alpha}x^2, & \text{if} \quad x \in [0, \alpha] \\ 0, & \text{if} \quad x \in (\alpha, 1] \end{cases} \tag{13}$$

$$\mu_{bright} = \begin{cases} 0, & \text{if} \quad x \in [0, \alpha] \\ \frac{1 - B(x)}{1 + \lambda B(x)}, & \text{if} \quad x \in (\alpha, 1] \end{cases} \tag{14}$$

where

$$B(x) = \frac{1}{\alpha}\left(\frac{1 - x}{1 + \lambda x}\right)^2$$

where $x \in W$ is the independent variable, and $\alpha \in (0, 1)$ is the crossover point.

The relationship of α and λ refer to (9). μ_{dark} and μ_{bright} are shown in Fig. 2, which are decided by λ and α.

Fig. 2 distinctly shows these two fuzzy sets, dark and bright, partition W into two parts in a fuzzy manner. Actually, for gray-level images, the membership function of fuzzy set dark in W can be regarded as the membership of "black pixel group", while the membership function of the fuzzy set bright denotes the membership of the "white pixel group".

3.2. Selection of the threshold value

The parameter ε is a positive threshold value of fuzzy-Sure entropy. In our method, the experimental results prove that the value of ε directly connects with the image enhancement performance, i.e. the quantity of modified image can be controlled by adjusting ε. Here, for selecting a suitable ε, we firstly define the first-order fuzzy moments m and $P(A_c)_{max}$ (c is the number of partitions) as follows:

$$m = \sum_{i=0}^{L-1} t(i)\tilde{h}_A(i) \tag{15}$$

and

$$P(A_c)_{\max} = \max(P_\alpha(A_c)|\alpha \in (0, 1)) \tag{16}$$

where L is gray levels, $\tilde{h}_A(i)$ denotes the probability of the normalized histogram $P(\text{dark})_{\max} = \max(P_\alpha(\text{dark})|\alpha \in (0, 1))$ corresponding to gray-level i of the image A and refers to (2), and

$$t(i) = \frac{i}{L - 1} \tag{17}$$

In (16), $P(A_c)_{max}$ denotes the maximal value of the partitions $P_\alpha(A_c)$ with respect to α ranging from 0 to 1. In our study, $P(dark)_{max}$ and $P(bright)_{max}$ are defined as the maximal values of $P(dark)$ and $P(bright)$, respectively, that is

$$P(\text{dark})_{\max} = \max(P_\alpha(\text{dark})|\alpha \in (0, 1)) \tag{18}$$

$$P(\text{bright})_{\max} = \max(P_\alpha(\text{bright})|\alpha \in (0, 1)) \tag{19}$$

Then we can assign ε the value as below:

$$\varepsilon = \begin{cases} \frac{P(bright)_{\max}}{2}, & \text{if} \quad m \le 0.5 \\ \frac{P(dark)_{\max}}{2}, & \text{if} \quad m > 0.5 \end{cases} \tag{20}$$

In our study, obviously, when the original image appears under-exposure, the histogram of the image is close to the left of the abscissa, moreover, m is less than 0.5. While the image is over-exposed, the corresponding histogram is close to the right of the abscissa, and m is greater than 0.5. In our experience, when m is less than 0.5, the image looks dark, then we can select εranged from 0 to $P(bright)_{max}$ in order to obtain a satisfying image. Here, the greater ε is, the brighter the enhanced image looks. However, when ε is bigger than $P(bright)_{max}$, the enhanced image no longer alters along with ε's change. On the other hand, when m is bigger than 0.5 and the image looks bright, we select ε ranged from 0 to $P(dark)_{max}$. Moreover, the less ε is, the darker the enhanced image looks. In this paper, we firstly select the midpoints of the two intervals, respectively.

3.3. Our algorithm implement

In this paper, the exhausted search method is used to calculate the optimal α and λ. The details are described as below:

Step A: Input the image , set $L=256$, normalize gray level and initialize H_{max},α_{opt} , λ_{opt},$\alpha = 0.3027$, and $P(bright)_{max}$.

Step B: Compute the histogram and obtain m according to (15).

Step C: Compute the membership function $\mu_A(ij)$ according to (10) and calculate the histogram.

Step D: Compute the probability of the occurrence of the gray-levels and normalize the histogram according to (2).

Step E: Initialize $\varepsilon = 1$-m.

Step F: Use the exhausted search approach to attain the pair 2.1186 and λ_{opt}, which form a fuzzy 2-partition that has the maximum fuzzy-Sure entropy:

for $\alpha = 1$ to $\frac{254}{255}$:

(i) For given α, according as (9), (13) and (14), obtain λ and compute new membership functions, $\mu_{dark}(g)$ and $\mu_{bright}(g)(g = 0, 1, \cdots, L-1)$.

(ii) Compute the probabilities of the fuzzy events of dark and bright by (3):

$P(dark) = \sum_{g=0}^{L-1}\mu_{dark}(g)\tilde{h}_A(g)$

$P(bright) = \sum_{g=0}^{L-1}\mu_{bright}(g)\tilde{h}_A(g)$

(iii) Compute the Sure entropy of this partition according to (7):

$$|P(A_i)| \le \varepsilon \Rightarrow H(U, \alpha) = \sum_{i=1}^{2}\min(P(A_i), \varepsilon)$$
$$= \min(P(dark), \varepsilon) + \min(P(bright), \varepsilon)$$

(iv) If current computed $P(dark)$ is greater than $P(dark)_{max}$, replace $P(dark)_{max}$ with current computed $P(dark)$. In the same way, when current computed $P(bright)$is greater than $P(brigth)_{max}$, replace $P(brigth)_{max}$ with current computed $P(bright)$. Similarly, if current computed H is greater than H_{max}, replace H_{max} with current computed H. At the same time, replace α_{opt}, and λ_{opt} with current computed $\alpha$and λ, respectively.

end for α.

Step G: Modify λ according to (20).

Step H: Repeat step(F) and obtain the ultimate α_{opt} and λ_{opt}.

Step I: Obtain the involutive memberships $\hat{\mu}_A(ij)$according to (10), (12) and (11).

Step J: Get the improved image $F = (L-1) \times \hat{\mu}_A(ij)$.

If the improved image is not perfect, we can appropriately decrease or increase the value of ε and repeat steps from (F) to (I), until it is satisfying.

When using the method based on the maximum Shannon entropy, the calculational methods are all same as the above mentioned steps except the steps (E), (F), (G) and (H), of which (E), (G) and (H) are removed and in step (F) Shannon entropy is computed as (5):

$$H(U, \alpha) = -\sum_{i=1}^{2} P(A_i)\log P(A_i) = -P(dark)\log P(dark) - P(bright)\log P(bright)$$

4. Experimental results and discussion

In this section, we first tested the proposed method using different types of images with different kinds of degradations. e.g. contrast degradation, luminance degradation, and contrast-luminance degradation, etc. Then, we compared our results with those produced using the histogram equalization method(HEM), the maximizes the parametric index of fuzziness (PIFM) [1], the λ-enhancement method based on optimization of image fuzziness (λM) [2], and the maximum Shannon entropy principle of fuzzy events (MSNM) [18]. In our study, each image has 256($L=256$) gray levels from 0 (the darkest) to 255(the brightest) except the X-ray image “hand” with 65536 gray levels. The experimental comparison is in the environment of Matlab7.0, CPU is Intel Core i5 3.30 GHz, RAM is 4.00 GB.

4.1. Experimental results of the proposed method

The first tested image "pout-lowluminance-lowcontrast" with size 240×291 pixels and 256 gray-levels ranging from 0.0784 to 0.2196. The luminance and contrast of this image were badly degraded as shown in Fig. 3(a1). Then this image was enhanced by our proposed method. The first-order fuzzy moment m is equal to 0.1094 which is less than 0.5, and this accords with the histogram of the original image as shown Fig. 3(a2). After applying our method to this image with $\varepsilon = 1 - m = 0.8906$, we got the first enhanced image as shown in Fig. 3(b1) and $\alpha = 0.0039$. But Fig. 3 (b1) , (b2) and (b6) show that the image was too much over enhanced and many content and texture details are lost. At the same time, from Fig. 3(b3)–(b5), we discovered that λ has no influence on fuzzy Sure entropy, with the reason that λ is greater than not only $P(dark)_{max}$ ($P(dark)_{max} = 0.1321$), but also $P(bright)_{max}$ ($P(bright)_{max} = 0.8209$). Thus the parameter λ needed a modification and we assigned 0.41045 to λ according to (20) , after which we obtained the second enhanced image as shown in Fig. 3(c1). It is obvious that this λ is a suitable value for the enhancement as shown in Fig. 3(c1)–(c6). Therefore, the finally obtained optimal α is equal to 0.0824. The computational cost for the first enhancement is about 0.367 s, and the total computational cost is about 0.3952 s. It is evident that the luminance and contrast of the original image are highly enhanced by the proposed method. Furthermore, the final image looks clear and uniform.

In order to verify that the proposed method can work well on other kinds of images, we tested it on the image "flowers-low-luminance-lowcontrast" and the results were given in Fig. (4). The luminance and contrast of this image (size 500×362 pixels , 256 gray-levels ranging from 0.0118 to 1) were extremely degraded as shown in Fig. 4(a1), and the corresponding histogram is shown in Fig. 4(a2). The first and second enhanced results are given in Fig. 4 (b1)–(b6) and in Fig. 4(c1)–(c6), respectively. In this test, for $m = 0.1458$, then the first $\lambda = 0.8542$, the obtained $\alpha = 0.0039$ and computational cost is about 0.5058 s. Similarly, for modified $\lambda = 0.3269$, then we obtain $\alpha = 0.0353$ and the total computational cost about is 0.5342 s. The enhanced image looks much better than the original degraded one and this shows the effectiveness of our proposed method in various low-luminance and low-contrasted images.

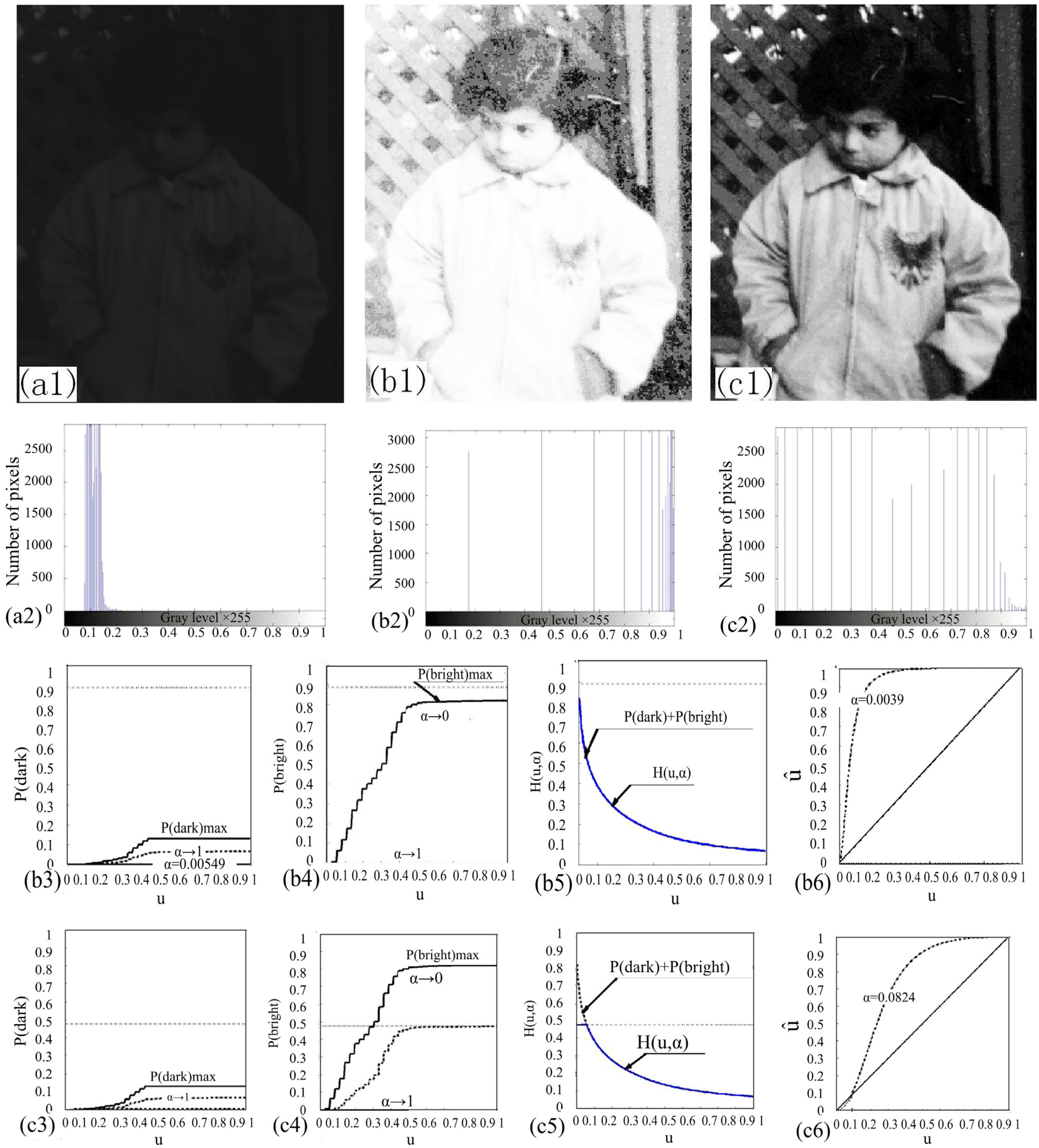

Fig. 3. "pout-lowluminance-lowcontrast" :(a1) the original image, (a2)the histogram of the original image, (b1) the first enhanced image by the proposed method , (b2) the histogram of the first enhanced image, (b3) P(dark) of the first enhancement processing ,(b4) P(bright) of the first enhancement processing, (b5) the first Sure entropy,(b6) the first membership function, (c1) the second enhanced image, (c2) the histogram of the second enhanced image, (c3) P(dark) of the second enhancement processing , (c4) P(bright) of the second enhancement processing, (c5) the second Sure entropy, (c6) the second membership function.

Next we test the image "pout-lowcontrast" with low contrast as shown in Fig. 5(a1) and (a2). Firstly, for m = 0.4654, $\lambda = 0.5346$,and $\alpha = 0.0353$, the computational cost is about 0.3670 second and the processed results are shown in Fig. 5(b1)–(b6). Secondly, for $\lambda = 0.4160$, and $\alpha = 0.0667$, the total computational cost is about 0.3970 s and the results are shown in Fig. 5(c1)–(c6). Fig. 5(a1) is a low-contrasted and vague image. It is obvious that the image obtained by the proposed method is more distinct as shown in Fig. 5(c1).

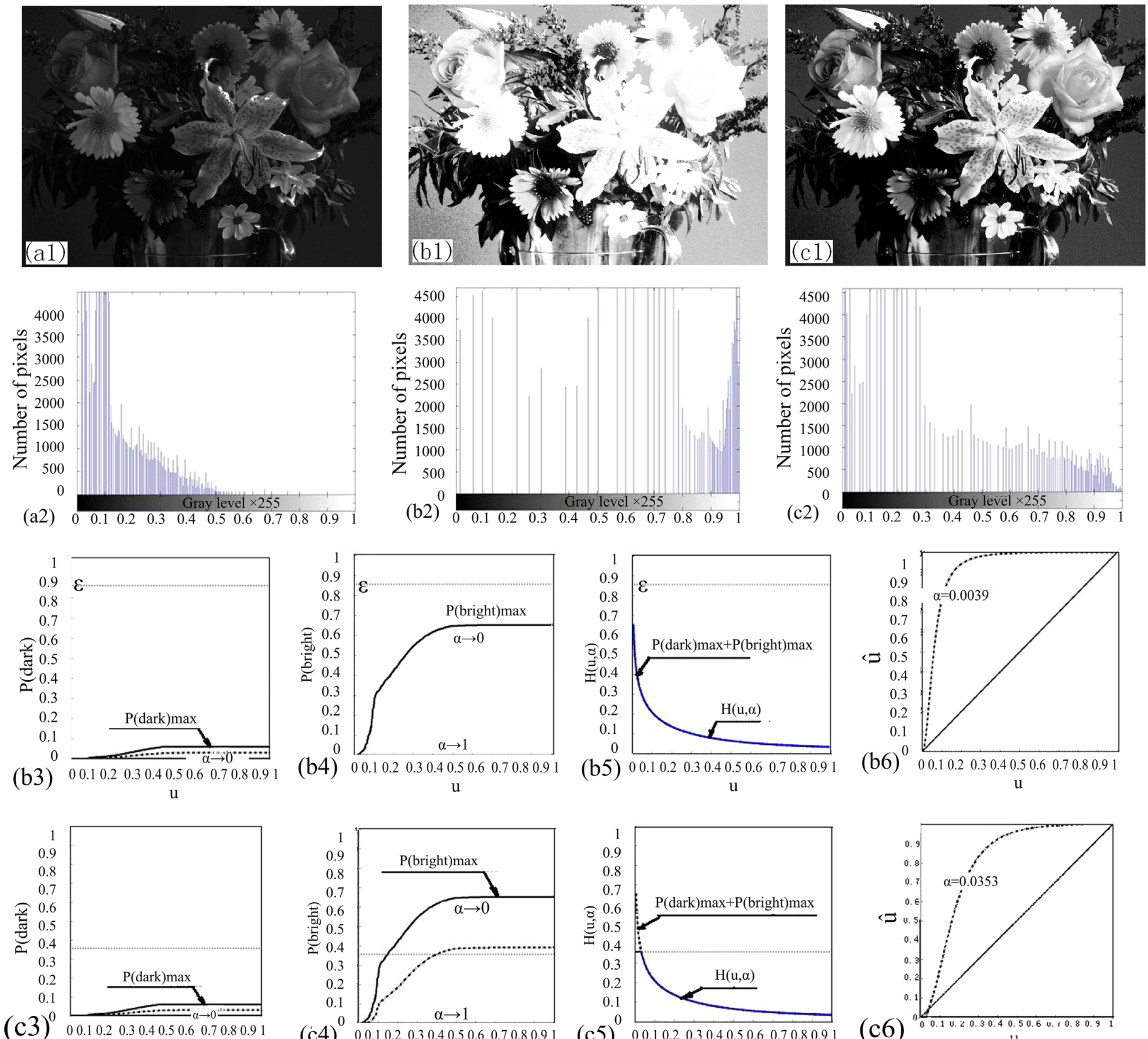

Fig. 4. "flowers-lowluminance-lowcontrast" :(a1)the original image, (a2)the histogram of the original image,(b1) the first enhanced image by the proposed method , (b2) the histogram of the first enhanced image, (b3) P(dark) of the first enhancement processing ,(b4) P(bright) of the first enhancement processing,(b5) the first Sure entropy,(b6)the first membership function,(c1)the second enhanced image ,(c2)the histogram of the second enhanced image, (c3) P(dark) of the second enhancement processing ,(c4)P (bright) of the second enhancement processing,(c5) the second Sure entropy,(c6)the second membership function.

4.2. Comparisons of algorithms

In order to evaluate the performance of the proposed method, we compared our results with those obtained by other four methods: the histogram equalization method (HEM), the maximizes the parametric index of fuzziness (PIFM), the λ-enhancement method (λM) , and the maximum Shannon entropy principle (MSNM). Tested those methods on many images, e.g. "lena", "tire", "pout-lowluminance", "flowers-lowluminance-lowcontrast", "lena-low-contrast" and "hand". The original images are shown in Fig. 7(a)–Fig. 12(a) and the corresponding histograms are given in Fig. 7(b)–Fig. 12(b).The enhanced images obtained by the HEM, PIFM, λM and MSNM are shown in (c), (d), (e) and (f) of Figs. 7–12, respectively. The final images improved by the proposed method are depicted in Figs. 7(g)–12(g) and the corresponding histograms are shown in Fig. 7(h)–Fig. 12(h). The comparative results of computational costs are depicted in Tables 1–6.The relational curves of the involutive membership functions $\lambda$and the gray level of the original images are shown in Fig. 6(a)–(f). The curves of the PIF method are depicted with dash and dot line(PIFM), λ-enhancement method described with solid line (λM), the maximum Shannon entropy principle depicted with dashed line (MSNM), and the maximum Sure entropy shown with dotted line (MSRM).

In Fig. 7, the original image "lena" (Fig. 7(a)) is vague and blurred. The histogram equalization method directly modifies the histogram of the original image (Fig. 7(b)), with the defect of amplifying the noise,the obtained image(Fig. 7(c)) looks unnatural. The contrast of the image (Fig. 7(d)) obtained employing the PIFM is enhanced, but there are some over-enhanced portions, e.g. hair and the computational cost is the most as illustrated in Table 1.

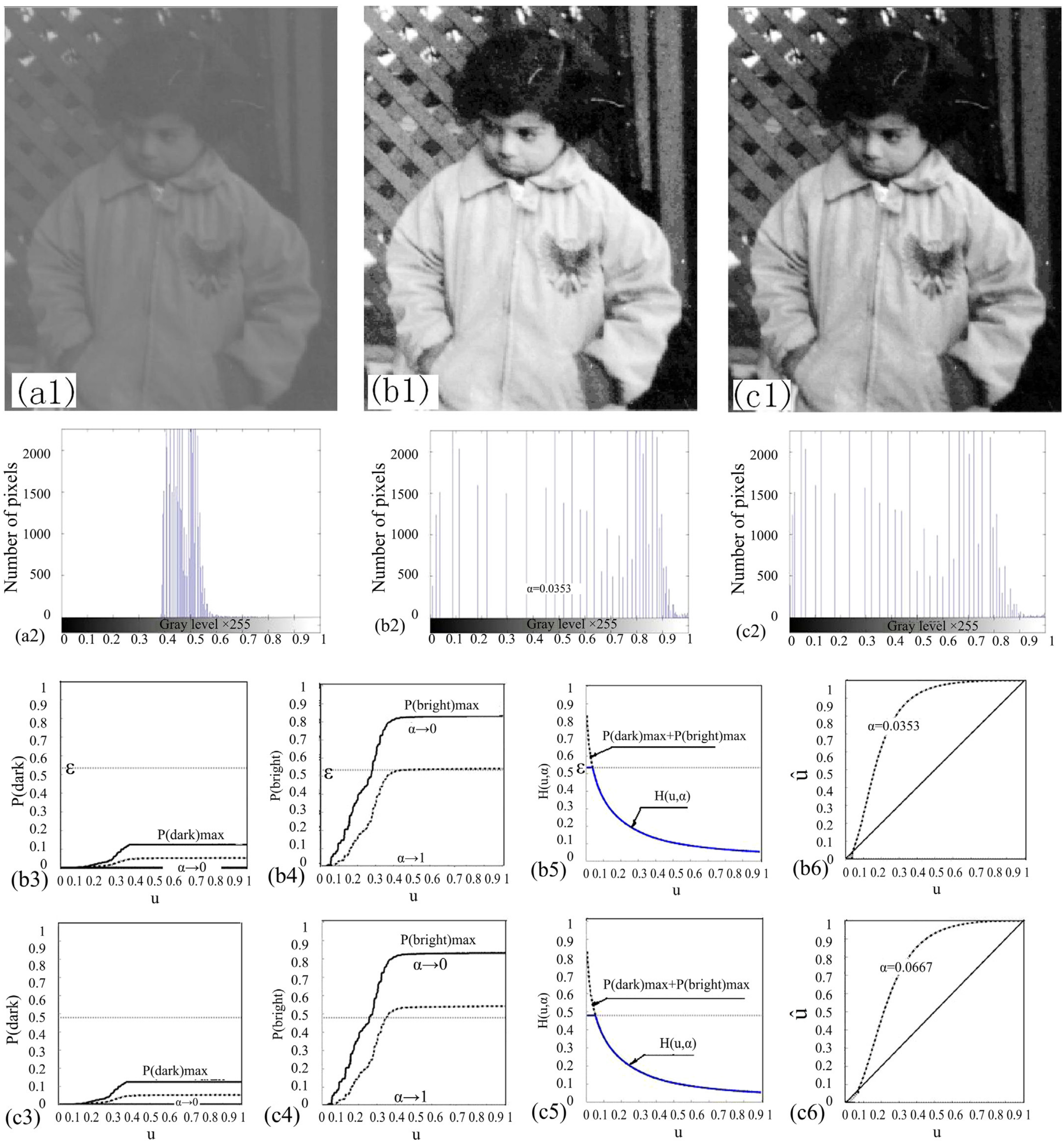

Fig. 5. "pout-lowcontrast" :(a1)the original image, (a2)the histogram of the original image,(b1) the first enhanced image by the proposed method , (b2) the histogram of the first enhanced image,(b3) P(dark) of the first enhancement processing,(b4)P(bright) of the first enhancement processing,(b5) the first Sure entropy,(b6)the first membership function,(c1)the second enhanced image ,(c2)the histogram of the second enhanced image, (c3) P(dark) of the second enhancement processing ,(c4)P(bright) of the second enhancement processing,(c5) the second Sure entropy,(c6)the second membership function.

The Fig. 7(e) shows that the λM improves the contrast of the original image and makes it clear and natural. But comparing this result with the image(Fig. 7(g)) by the MSRM, this method is inconspicuous. The MSNM reforms the contrast, but the view picture looks darker as shown in Fig. 7(f). In this test on "lena", we can discover that the MSRM has achieved a better performance over the other methods. the final image looks not only distinct and uniform, but also natural. The corresponding histogram is shown in Fig. 7(h). From Fig. 6(a), we can see that, when gray level is less than 70, the enhancement by the PIFM is the most, but the enhancement by the MSNM is the least. While gray level is greater than 180,the enhancement by the MSRM is the most, but the enhancement by the λM is the least. Fig. 7(b) shows that the gray levels of the first peak are less than 70, corresponding to the hair and tyre, so there are some over-enhancements when employing the PIFM, but some under-enhancements when employing MSNM.

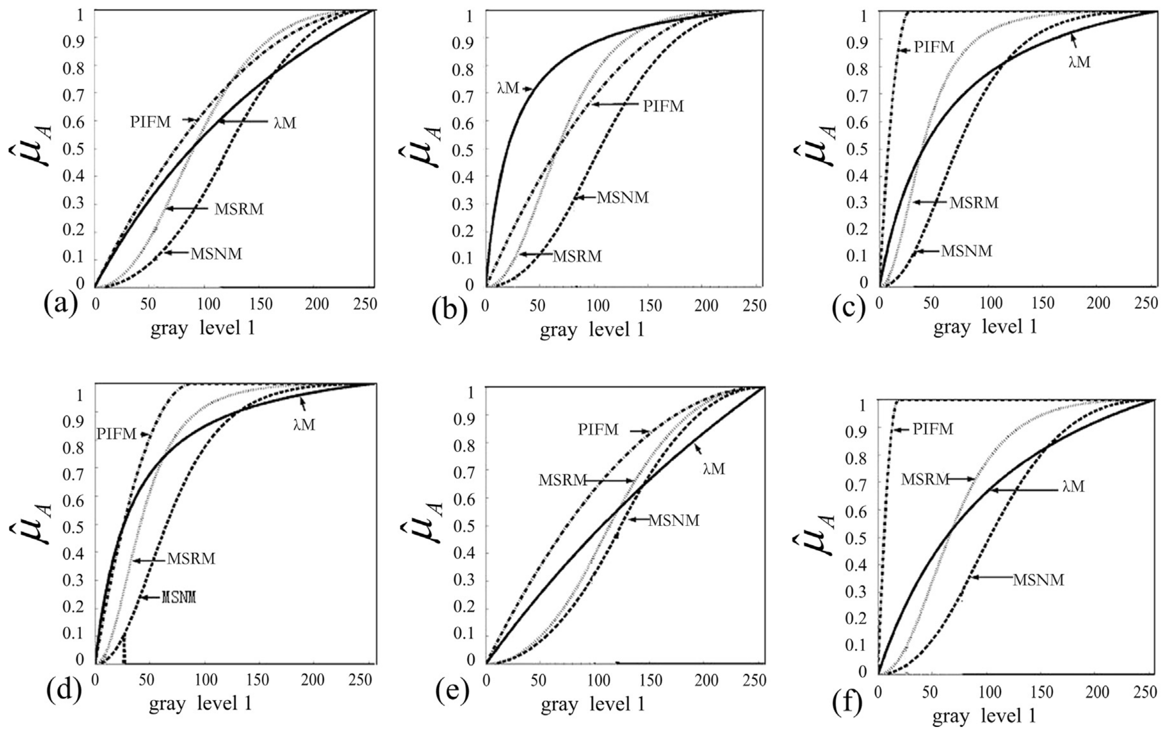

Fig. 6. "the curves of membership functions" :(a) "lena", (b) "tire",(c) "pout-lowluminance", (d) "flower-lowluminance-lowcontrast", (e) "lena-lowcontrast",(f) "hand", for the PIF method, depicted with dash and dot line, λ-enhancement method described with solid line, the maximum Shannon entropy principle depicted with dashed line,and the maximum Sure entropy shown with dotted line.

There are only a few pixels with gray levels greater than 180, so the effects by the existing four methods are similar.

In Fig. 8, the vague and blurred image "tire" is shown in Fig. 8(a), and the corresponding histogram is depicted in Fig. 8(b). It is obvious that, the HEM and λM bring over-enhancements as illustrated in Fig. 8(c) and (e), respectively. There are some under-enhanced portions in the image obtained by the MSNM as depicted in Fig. 8(f). In Fig. 8(d), there are a few over-enhanced portions such as the tyre, while the hub of the wheel is a little enhanced by the MSRM as show in Fig. 8(g). Even then, the MSRM is preferred. Firstly, the image enhanced by the MSRM is more natural than by the PIFM. Secondly, the computational cost of the MSRM is less than the PIFM's as shown in Table 2. Finally, we can obtain a satisfying image through decreasing ε (ε=0.2991) to 0.2891. Fig. 6(b) also shows that the λM causes over-enhancements and the MSNM brights under-enhancements.

In Fig. 10, the original image "flowers-lowluminance-lowcontrast" with low luminance and low contrast is depicted in Fig. 10(a), and the corresponding histogram is shown in Fig. 10(b). We can see that, the images obtained by the HEM (Fig. 10(c)),the PIFM (Fig. 10(d)), and the λM (Fig. 10(e)) all have some over-enhanced portions: the stamens and leaves of these flowers, which make them unclear and unnatural. But there are under-enhancements in Fig. 10(f), obtained by the MSNM. In this test, the proposed method get ahead of the other methods. The image enhanced by the MSRM looks not only distinct but also natural, and the computational cost is less as shown in Table 4. In addition, From Fig. 6(d), we also discover that, when gray levels are less than 50, the enhancements of the PIFM and λM are far more than the MSRM's(more than the MSNM's). We also see that a majority of gray levels are less than 50 as shown in Fig. 10(b), and the enhancement about this part is very important to improve this image, thus MSRM is a proper method.

In Fig. 11, we can see that the contrast of the original image (Fig. 11(a)) "lena-lowcontrast" was extremely degraded, and this image is very vague and blurred. The HEM can improve the contrast as shown in Fig. 11(c), but make the image awfully unnatural. The image (Fig. 11(d)) obtained by the PIFM has some over-enhancements, we also obtain this verity from Fig. 6(e). The image (Fig. 11(e)) obtained from λM is better than the original image, but it is vague and unclear. The performance of MSNR is almost similar to MSNM. The final images by the MSRM and MSNM are distinct and natural, but the image enhanced by MSRM looks brighter than by MSNM. In addition, considering the computational cost (as shown in Table 5), the proposed method is preferable.

In Fig. 12, the original image (Fig. 12(a)) "hand" was captured from DDR system directly. the image is very vague and looks dark. The obtained images by the HEM, λM and MSNM are given in Fig. 12 (c),(e) and (f), respectively, but they are inferior to the enhanced image by the MSRM as shown in Fig. 12(g). The image obtained by the PIFM has so many over-enhancements that most parts of the image are lost. From Fig. 6(f), we can discover that the PIFM has the most enhancement(leading to over-enhancement), the λM has the most even change(leading to the inconspicuous contrast enhancement). The computational cost of the MSRM is only more than the HEM's as depicted in Table 6. Thus our proposed method have the advantage over the other four methods.

5. Conclusions

This paper proposes a newly developed image enhancement method based on the maximum fuzzy Sure entropy. This method offers two major contributions. Firstly, it is very efficient and effective when applying our proposed method on low-quality images, especially on low contrast and low illuminance images. Secondly, the proposed method is quite robust as the threshold value can be selected in a relatively very large range until satisfying results. We conducted experiments on different kinds of low-quality images with our method and other existing methods, and

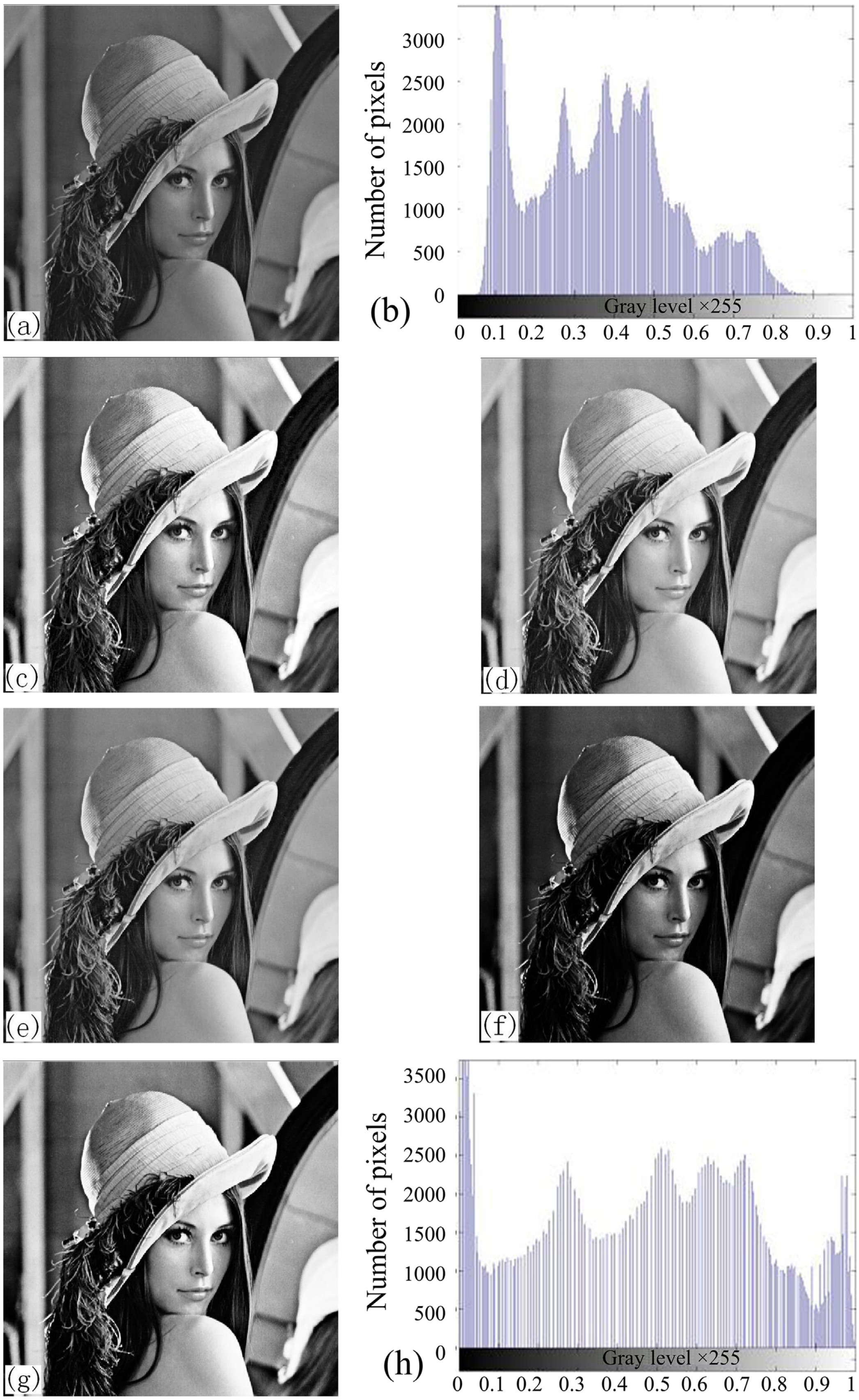

Fig. 7. "lena" :(a) the original image, (b) the histogram of the original image,(c)the enhanced image using HEM,(d)the enhanced image using PIFM, (e) the enhanced image using λM, (f)the enhanced image using MSNM,(g) the enhanced image using the proposed method(MSRM) ,(h)the histogram of the image enhanced by MSRM.

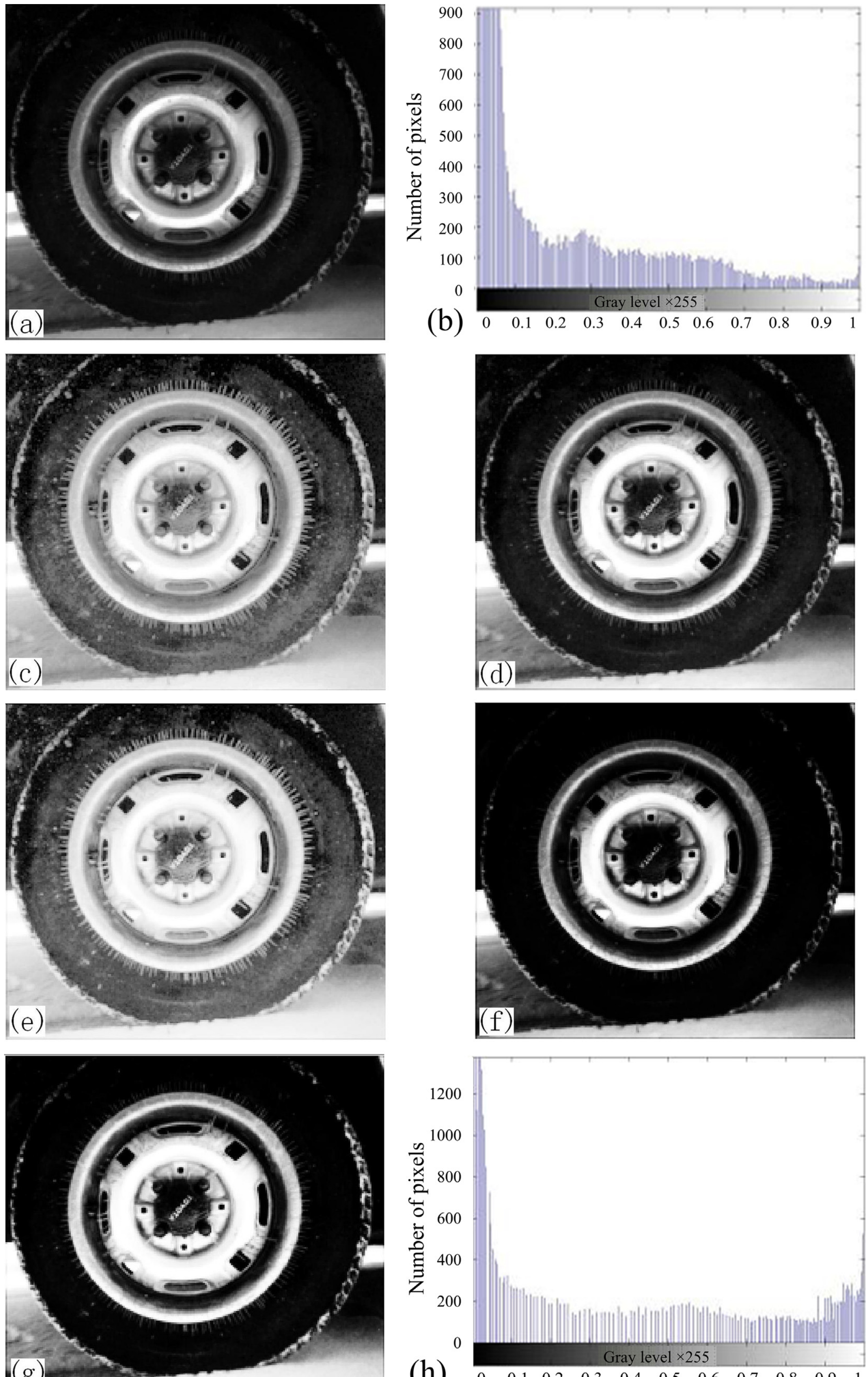

Fig. 8. "tire" :(a) the original image, (b) the histogram of the original image,(c)the enhanced image using HEM,(d)the enhanced image using PIFM, (e) the enhanced image using λM, (f)the enhanced image using MSNM,(g) the enhanced image using the proposed method(MSRM) ,(h)the histogram of the image enhanced by MSRM.

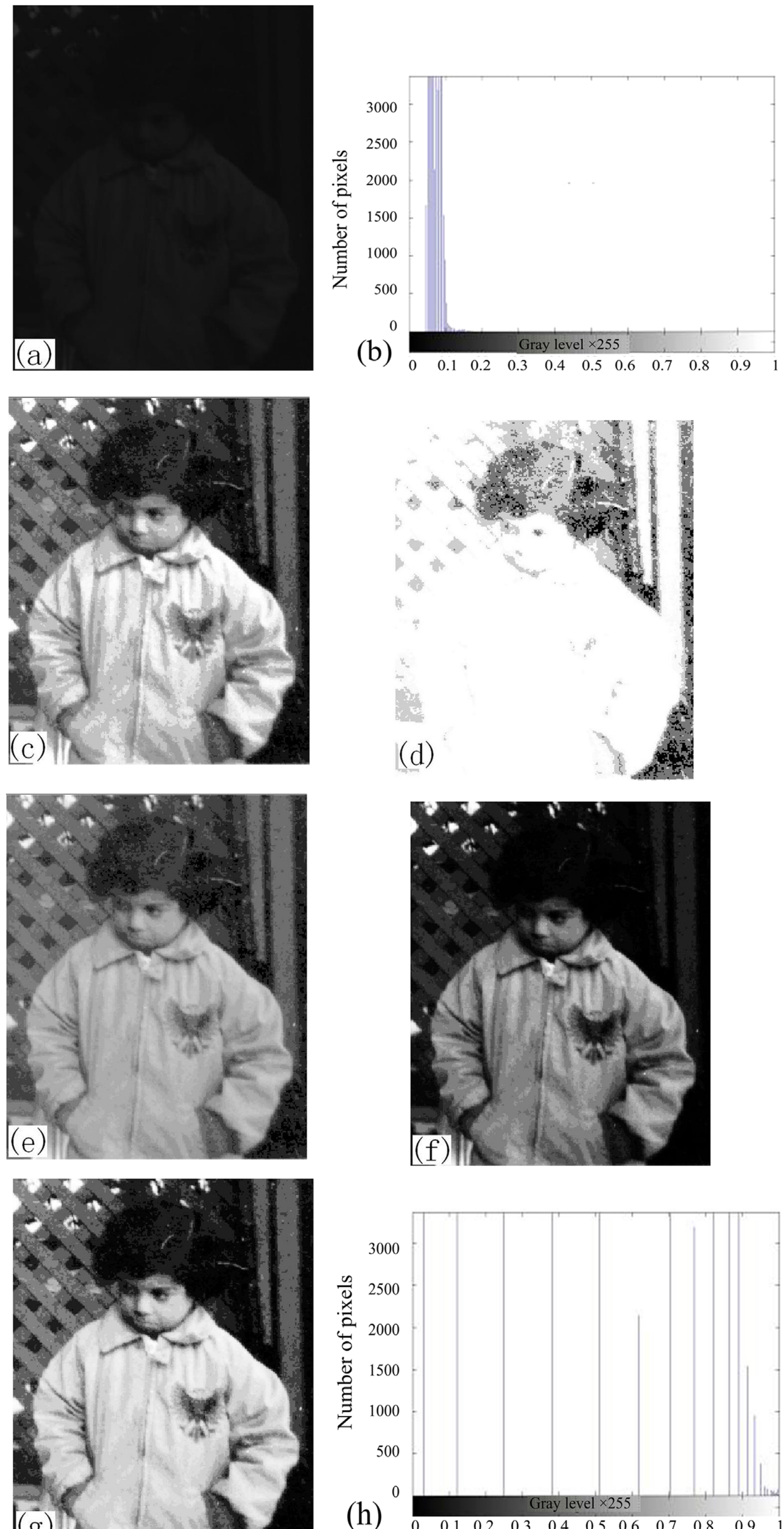

Fig. 9. "pout-lowluminance" :(a) the original image, (b) the histogram of the original image,(c)the enhanced image using HEM,(d)the enhanced image using PIFM, (e) the enhanced image using λM, (f)the enhanced image using MSNM,(g) the enhanced image using the proposed method(MSRM) ,(h)the histogram of the image enhanced by MSRM.

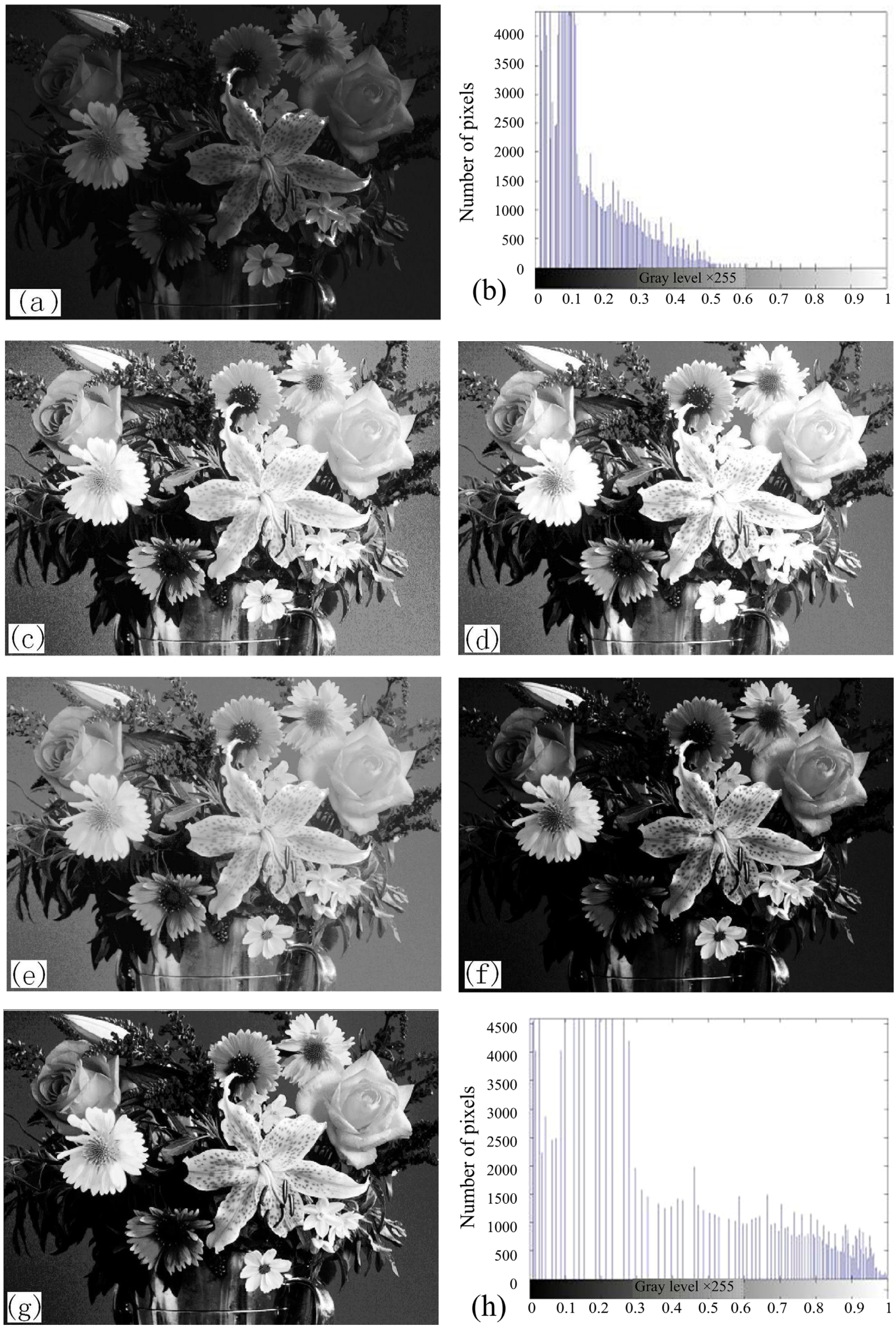

Fig. 10. "flowers-lowluminance-lowcontrast" :(a) the original image, (b) the histogram of the original image,(c)the enhanced image using HEM,(d)the enhanced image using PIFM, (e) the enhanced image using λM, (f)the enhanced image using MSNM,(g) the enhanced image using the proposed method(MSRM) ,(h)the histogram of the image enhanced by MSRM.

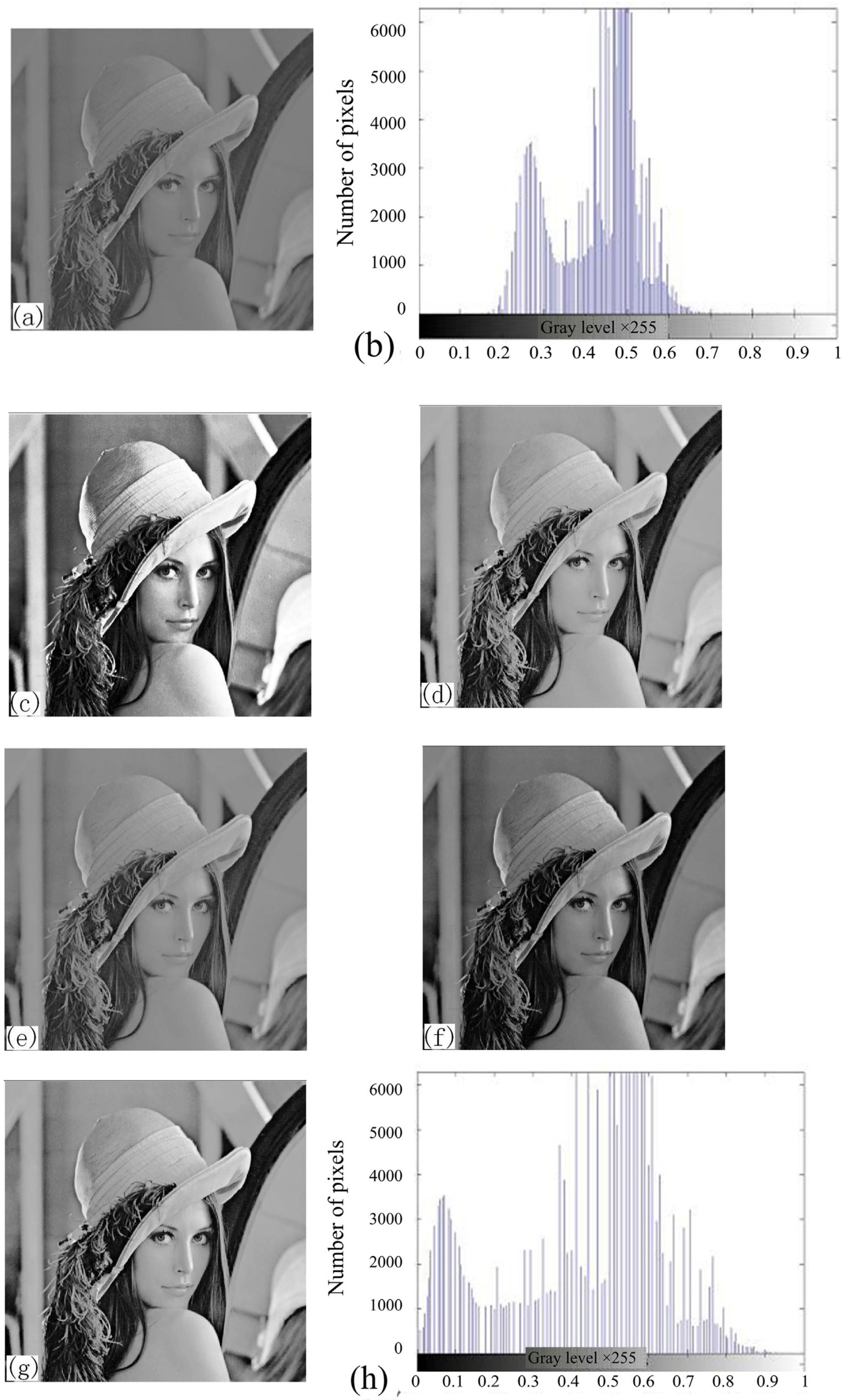

Fig. 11. "lena-lowcontrast" :(a) the original image, (b) the histogram of the original image,(c)the enhanced image using HEM,(d)the enhanced image using PIFM, (e) the enhanced image using λM, (f)the enhanced image using MSNM,(g) the enhanced image using the proposed method(MSRM) ,(h)the histogram of the image enhanced by MSRM.

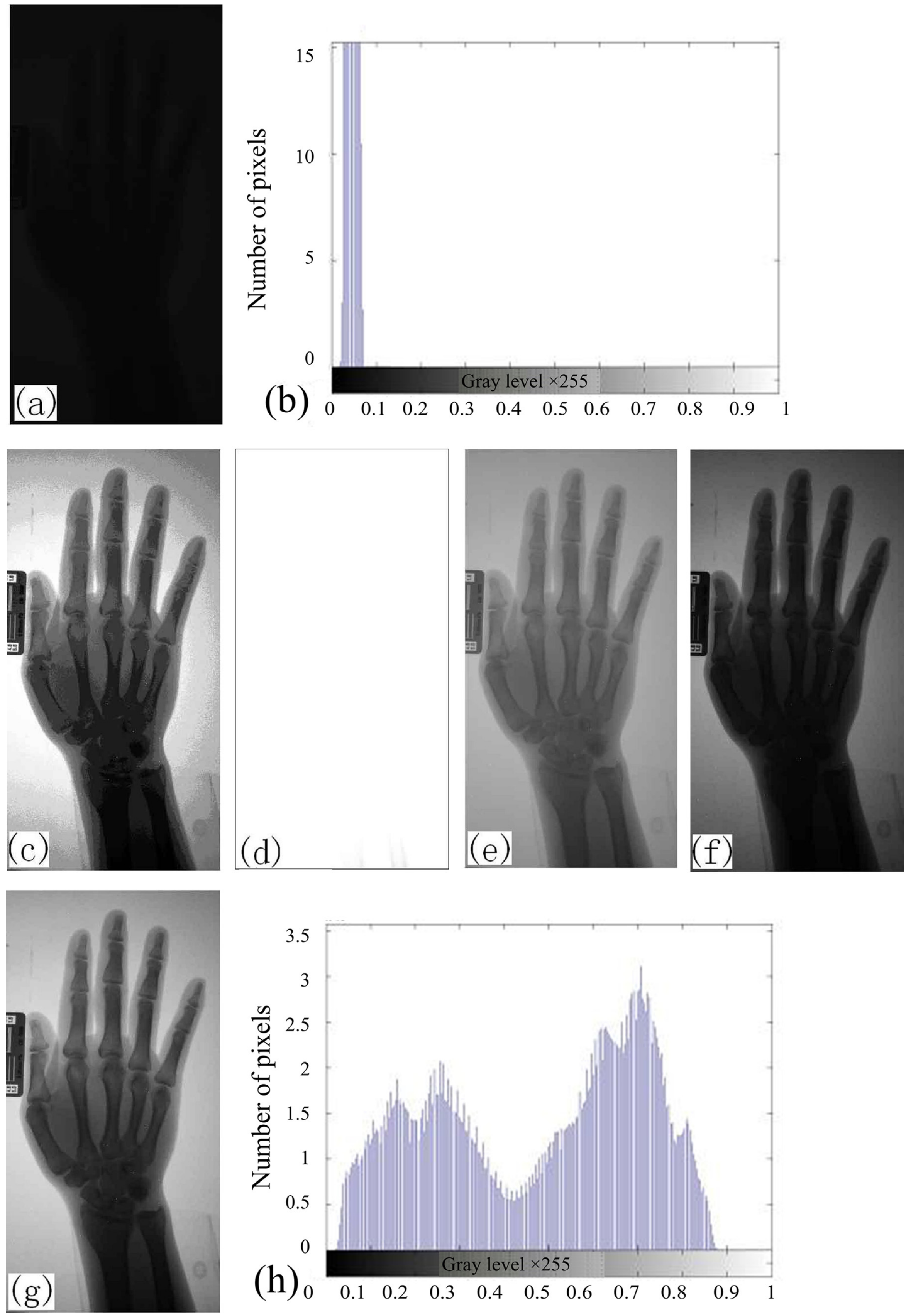

Fig. 12. the x-ray image "hand" with 2469 × 1247: (a) the original image, (b) the histogram of the original image,(c)the enhanced image using HEM,(d)the enhanced image using PIFM, (e) the enhanced image using λ M, (f)the enhanced image using MSNM,(g) the enhanced image using the proposed method(MSRM) ,(h)the histogram of the image enhanced by MSRM.

Table 1
Comparisons of algorithms on "lena".

Algorithm	HEM	PIFM (a, b, c)	λM	MSNM	MSRM
Parameter α,λ		$\left(0, \frac{1}{255}, \frac{255}{255}\right)$, $\lambda = -0.65088$	$\alpha = 0.4228$	$\alpha = 0.4549$	$\alpha = 0.2235$
Computational cost (s)	0.2396	102.3812	3.2314	2.4782	0.7782

Table 2
Comparisons of algorithms on"tire".

Algorithm	HEM	PIFM (a, b, c)	λM	MSNM	MSRM
Parameter α,λ		$\left(0, \frac{1}{255}, \frac{228}{255}\right)$, $\lambda = -0.929$	$\alpha = 0.2275$	$\alpha = 0.3255$	$\alpha = 0.1059$
Computational cost (s)	0.156	87.2888	1.3178	0.4034	0.3593

Table 3
Comparisons of algorithms on "pout-lowluminance".

Algorithm	HEM	PIFM (a, b, c)	λM	MSNM	MSRM
Parameter α,λ		$\left(0, \frac{1}{255}, \frac{28}{255}\right)$, $\lambda = -0.9945$	$\alpha = 0.3027$	$\alpha = 0.1255$	$\alpha = 0.0314$
Computational cost (s)	0.169167	90.8124	1.4948	0.5365	0.38567

Table 4
Comparisons of algorithms on "flowers-lowluminance-lowcontrast".

Algorithm	HEM	PIFM (a, b, c)	λM	MSNM	MSRM
Parameter α,λ		$\left(0, \frac{1}{255}, \frac{89}{255}\right)$, $\lambda = -0.9709$	$\alpha = 0.2539$	$\alpha = 0.102$	$\alpha = 0.0353$
Computational cost (s)	0.2184	88.797	2.1186	1.2654	0.544167

Table 5
Comparisons of algorithms on "lena-lowcontrast".

Algorithm	HEM	PIFM (a, b, c)	λM	MSNM	MSRM
Parameter α,λ		$\left(0, \frac{56}{255}, \frac{255}{255}\right)$, $\lambda = -0.3732$	$\alpha = 0.459$	$\alpha = 0.4706$	$\alpha = 0.3882$
Computational cost (s)	0.2374	83.0158	3.2876	2.5188	0.7562

Table 6
Comparisons of algorithms on "hand".

Algorithm	HEM	PIFM (a, b, c)	λM	MSNM	MSRM
Parameter α,λ		$\left(0, \frac{1}{255}, \frac{19}{255}\right)$, $\lambda = -9976$	$\alpha = 0.3642$	$\alpha = 0.3059$	$\alpha = 0.102$
Computational cost (s)	1.5784	135.39	130.947	129.92	3.331

the comparisons of those experiment results show that the performance of our method overwhelms those of the existing ones.

Acknowledgments

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References

[1] G.J. Klir, T.A. Folger, Fuzzy Sets, Uncertainty, and Information, Prentice-Hall, Englewood Cliffs, NJ, 1988.
[2] H. Li, S. Yang, Fast and reliable image enhancement using fuzzy relaxation technique, IEEE Trans. Syst. Man Cybern. 19 (5) (1989) 1276–1281.
[3] H.T. Wu, J.W. Huang, Y.Q. Shi, A reversible data hiding method with contrast enhancement for medical images, J. Vis. Commun. Image Represent. 31 (2015) 146–153.
[4] Zh.Y. Bian, J. Huang, J.H. Ma, H. Zhang, Zh.R. Liang, W.F. Chen, Iterative image reconstruction for low-dose x-ray CT using a sinogram restoration induced edge-preserving prior, in: Proceedings of the IEEE 11th International Symposium on Biomedical Imaging (ISBI), 2014, pp. 1160–1163.
[5] Yue Gao, Meng Wang, Zhengjun Zha, Jialie Shen, Xuelong Li, Xindong Wu, Visual-textual joint relevance learning for tag-based social image search, IEEE Trans. Image Process. 22 (1) (2013) 363–376.

[6] Yue Gao, Rongrong Ji, Peng Cui, Qionghai Dai, Gang Hua, Hyperspectral Image classification through bilayer graph based learning, IEEE Trans. Image Process. 23 (7) (2014) 2769–2778.
[7] Shaoyi Du, Nanning Zheng, Shihui Ying, Jianyi Liu, Affine iterative closest point algorithm for point set registration, Pattern Recognit. Lett. 31 (9) (2010) 791–799.
[8] Ioannis K. Vlachos, George D. Sergiadis, Parametric indices of fuzziness for automated image enhancement, Fuzzy Sets Syst. 157 (2006) 1126–1138.
[9] H.R. Tizhoosh, G. Krell, B. Michaelis, λ-Enhancement: contrast adaptation based on optimization of image fuzziness, Proc. FUZZ-IEEE 98 (1998) 1548–1553.
[10] H.D. Cheng, H. Xu, A novel fuzzy logic approach to contrast enhancement, Pattern Recognit. 33 (2000) 799–819.
[11] L.A. Zadeh, Probability measures of fuzzy events, J. Math. Anal. Appl. 23 (1968) 421–427.
[12] Engin Avci, Derya Avci, An expert system based on fuzzy entropy for automatic threshold selection in image processing, Expert Syst. Appl. (2008), http://dx.doi.org/10.1016/j.eswa.2008.01.02.
[13] H.D. Cheng, J. R, J. Li, Threshold selection based on fuzzy c-partition entropy approach, Pattern Recognit. 31 (7) (1998) 857–870.
[14] I. Turkoglu, An Intelligent Pattern Recognition for Nonstationary Signals Based on the Time Frequency Entropies (Ph.D. thesis), Firat University Graduate School of Natural and Applied Sciences, Elazig 2002, pp. 40–47.
[15] J.C. Bezdek, Partition Structures: A Tutorial, Analysis of Fuzzy Information, Vol. III, CRC Press, Boca Raton, 1987.
[16] MATLAB 5.3 version Image Processing Toolbox, 2006. MathWorks Company.
[17] S. Bodjanova, Alpha-bounds of fuzzy numbers, Inf. Sci. 152 (2003) 237–266.
[18] H.D. Cheng, J.R. Chen, Automatically determine the membership function based on the maximum entropy principle, Inf. Sci. 96 (1997) 163–182.
[19] H.D. Cheng, et al., A novel fuzzy entropy approach to image enhancement and thresholding, Signal Process. 75 (1999) 277–301.
[20] Leung Chung-Chu, et al., A new approach for image enhancement applied to low-contrast low-illumination IC and document images, Pattern Recognit. Lett. 26 (2005) 769–778.

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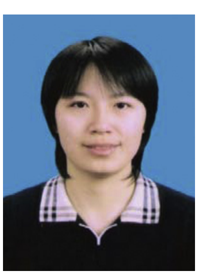

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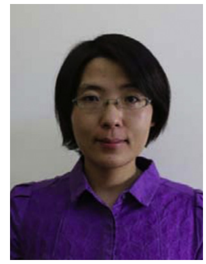

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