



Synchronization between neurons coupled by memristor



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ABSTRACT

Synapse plays an important role in signal exchange and information encoding between neurons. Electric and chemical synapses are often used to investigate the synchronization in electrical activities of neurons. In this paper, memristor is used to connect two neurons and the phase synchronization in electrical activities is discussed. Inter-spike interval (ISI) is calculated from the sampled time series for membrane potential, and the dependence of coupling intensity on phase synchronization of neuron is investigated and the effect of electromagnetic induction is considered. Furthermore, the synchronization stability of network is detected under noise; a statistical synchronization factor is also calculated. It is found synchronization can be enhanced under memristor coupling and appropriate noise is also helpful for synchronization stability.

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1. Introduction

The release of neurotransmitter from synapse can carry important information and synapse coupling is effective to realize signal exchange between neurons [1–11]. In complex biological nerve system composed of a large number of neurons, each neuron can connect with many other neurons synchronously thus many dendrons can be developed to receive different signals from different channels [12], for example, a spinal motoneuron can receive more than 10 thousands synapse inputs, while Purkinje cell cerebellum can hold more than 150 thousands Synaptic sites. For most of the researchers in the field of computational neuroscience, setting more reliable neuron models and practical neuronal circuits could be more attractive. For a brief review about these neuron models, readers can refer to the survey [13–16] and relevant references therein. For isolate neuron, artificial circuits and digital circuits [17–30] are proposed to produce the main properties in electrical activities of different types of neurons. In fact, some realistic factors and anatomical structure should be considered in setting neuron model and neuronal circuits. For example, autapse [31] is a specific synapse which can connect to its body via a close loop, in dynamical view, autapse connection can impose time-delayed feedback on the membrane potential and thus the electrical activities of neuron can be modulated [32,33], as a result, the self-adaptation of neuron can be enhanced [34–38]. Furthermore, appro-

appropriate distribution of autapse in neuronal network can enhance synchronization behavior and pattern formation by generating continuous pulse or wave fronts [39–42]. That is, autapse connection can play important biological role in regulating the electrical activities in neuron and neuronal networks [43]. As a result, Wang et al. discussed the possible formation mechanism of autapse [44], and confirmed that formation of autapse could be associated with injury in axon of neuron and auxiliary loop can be helpful to bridge the injured parts of axon thus the blocked signals can be transmitted completely [45]. In fact, the nerve system contains a large number neurons and astrocytes [43,46], complex connection and mutual modulation make neurons trigger multiple modes in electrical activities, as a result, super-large scale integrated circuit (VLSIC) is suggested that artificial synapse can be produced [47].

In model setting for neurons, another important physical factors, electromagnetic induction and radiation should be considered [48–54]. In most of the current neuron models, transmembrane and channel current are considered for contribution to the change of membrane potentials. However, in the molecular level, the effect of electromagnetic induction in cell should be considered because the distribution and exchange of charged ions (Na^+ , K^+ , Ca^{2+}) can induce complex electromagnetic field in cell thus the membrane potential can be modulated. As a result, the author in this paper suggested magnetic flux [55,56] can be introduced into the neuron model because the magnetic flux is associated with electromagnetic field. Based on the proposed neuron model [55], electromagnetic radiation is imposed to trigger multiple modes in electrical activities [56] and these results are consistent with biological

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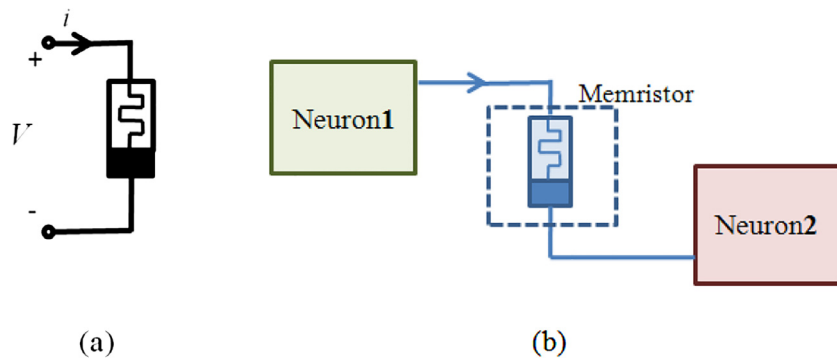


Fig. 1. Diagram for two neurons coupled by memristor, the current across the memristor can be calculated as $\rho(\varphi)(x_2 - x_1)$, x_2 , x_1 is membrane potential for two neuron, $\rho(\varphi)$ is memductance of memristor, φ is magnetic flux.

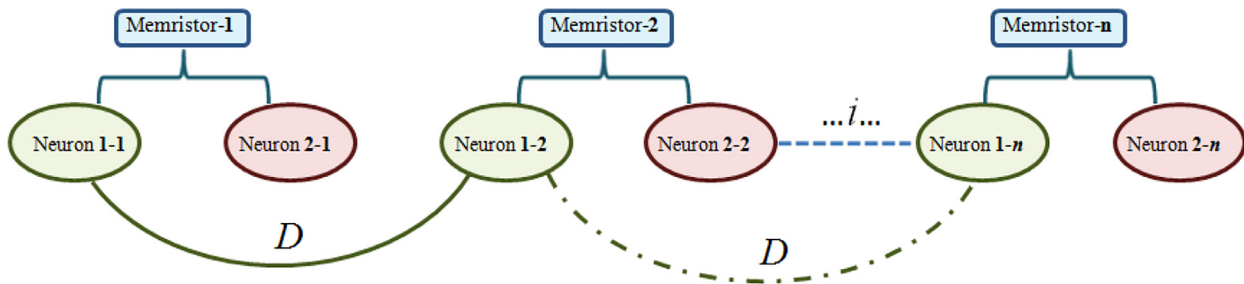


Fig. 2. Diagram for the chain network coupled by memristor, each pair of neurons are coupled via gap junction.

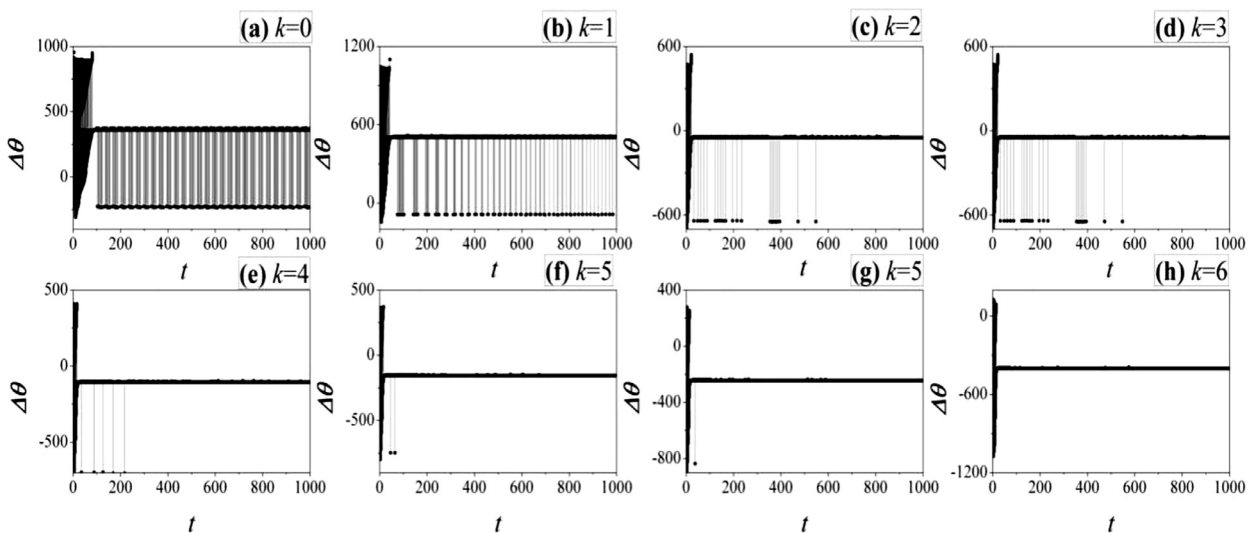


Fig. 3. Evolution of phase error between two neurons is calculated via memristor coupling by applying different induction coefficients.

results though many researchers used to produce the same result by using two-parameter bifurcation. To be consistent with physical units, memristor [57] is used to realize coupling between membrane potential and magnetic flux, thus the induced current from electromagnetic induction is approached. Memristor [58,59] is a specific electric device and the dependence of charge flux on magnetic flux is under nonlinear property, and the memductance of memristor is dependent on the inputs current thus memory is kept when external current stimuli is removed. As a result, memristor is used as nonlinear device and chaotic circuits can be designed to discuss new behaviors in dynamical systems [60–64]. Particularly, Wang et al [65] found that the accumulation and release of Calcium between presynaptic membrane and postsynaptic membrane can be great contribution to the synaptic plasticity, and then Ag-oxide memristor is used to confirm the same processing, and it is

found that diffusive memristor could be effective to describe short and long period synaptic plasticity of neurons, this progress could be helpful for further investigating the morphological features of nerve system. Furthermore, memristor coupling is also used to set an improve cardiac model, which can accounts two kinds of death mechanism of heart subjected to electromagnetic radiation [66,67].

Indeed, reliable neuron models are much helpful for further investigation in electrical activities, synchronization behaviors, and possible emergence of neuronal diseases [68–73]. Pattern formation [74–79] and synchronization [80–82] are two different aspects associated with collective behaviors of networks. Regular spatial patterns can be induced and developed to occupy the network when synchronization instability occurs due to intrinsic cooperation and self-organization. In fact, regular spatial distribution and ordered pattern formation could mean cooperation and coexis-

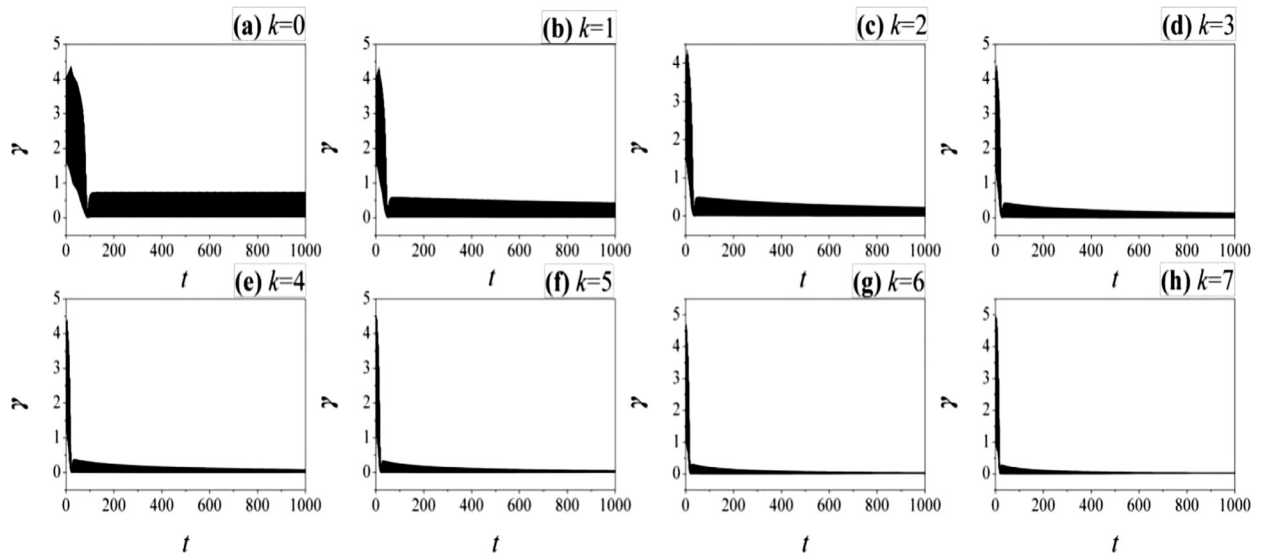


Fig. 4. Evolution of variable error between two neurons is calculated via memristor coupling by applying different induction coefficients.

tence, as a result, prediction for breakup or collapse [78,83] in network become important and thus it is worthy of investigating the stability of networks. As mentioned above, memristor holds specific property as memory and often used as reliable electric device, therefore, it is interesting to discuss the synchronization behavior of neurons coupled by memristor.

2. Model and scheme

For most of the neuron models, appropriate variables and parameters are set to trigger appropriate time series to be consistent with the biological data observed from experiments. For simplicity, FitzHugh suggested that relaxed Van der Pol equation [84] can be improved for setting a two-variable neuron model (FitzHugh–Nagumo) [85,86], and it reads as follows

$$\begin{cases} \varepsilon \frac{dx}{dt} = \left(x - \frac{x^3}{3} - y\right) + I \\ \frac{dy}{dt} = ax + by + d \end{cases} \quad (1)$$

Where the variable x, y describes the activator (fast variable) and inhibitor (slow variable), respectively, and I denotes the external stimuli or synapse current, ε defines the scale for fast and slow variable. Signal exchange can be induced when two neurons are connected via memristor, the diagram can be illustrated in Fig. 1.

According to the dynamical equation and nonlinear property for memristor, the collective behaviors of neurons coupled by memristor can be described by

$$\begin{cases} \frac{dx_1}{dt} = 20\left(x_1 - \frac{x_1^3}{3} - y_1\right) - k\rho(\varphi)(x_1 - x_2) \\ \frac{dy_1}{dt} = x_1 + a_1 \\ \frac{dx_2}{dt} = 20\left(x_2 - \frac{x_2^3}{3} - y_2\right) - k\rho(\varphi)(x_2 - x_1) \\ \frac{dy_2}{dt} = x_2 + a_2 \\ \frac{d\varphi}{dt} = k(x_1 - x_2) \end{cases} \quad (2)$$

Where the nonlinear term $\rho(\varphi)(x_2 - x_1)$ denotes induced current, k is induction coefficient, $\rho(\varphi) = \alpha + 3\beta\varphi^2$ defines the memductance

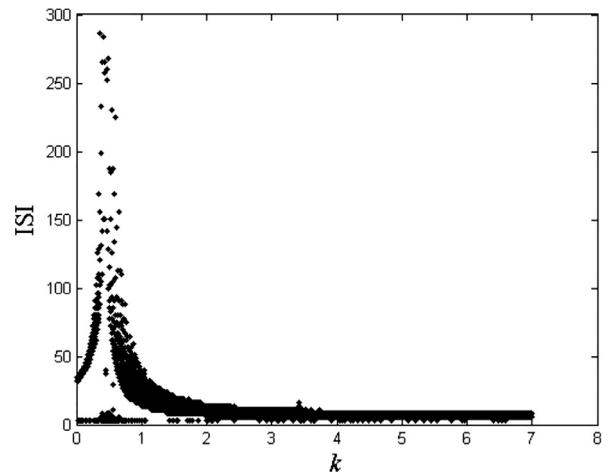


Fig. 5. Bifurcation diagram for inter-spike interval (ISI) is calculated by changing induction coefficient k . The subthreshold for ISI is set as $x_{1i} > -1$.

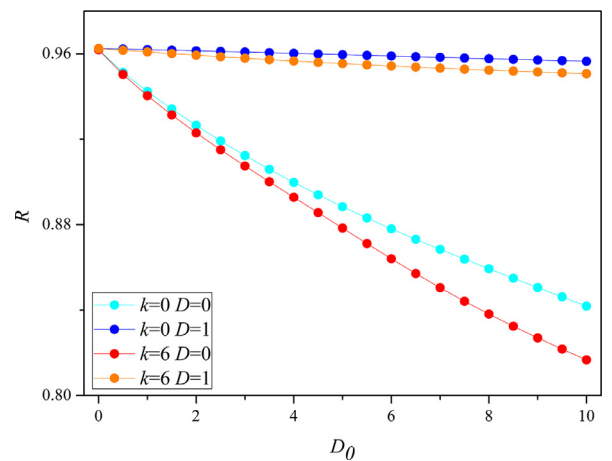


Fig. 6. Distribution for synchronization factors is calculated by setting different noise on the network.

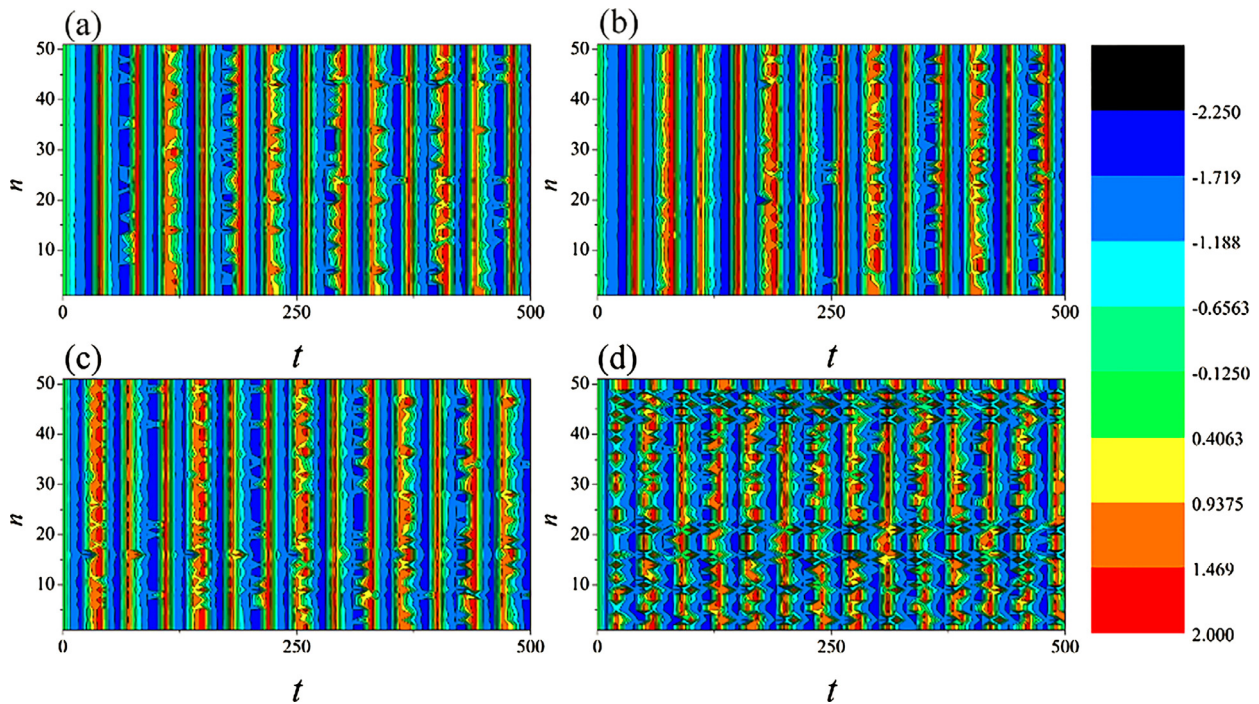


Fig. 7. Developed spatial patterns of network driven by noise and modulated by memristor. Coupling intensity between nodes $D=0$, noise intensity $D_0=6$, for (a) $k=6$; (b) $k=6.5$; (c) $k=7$; (d) $k=7.5$.

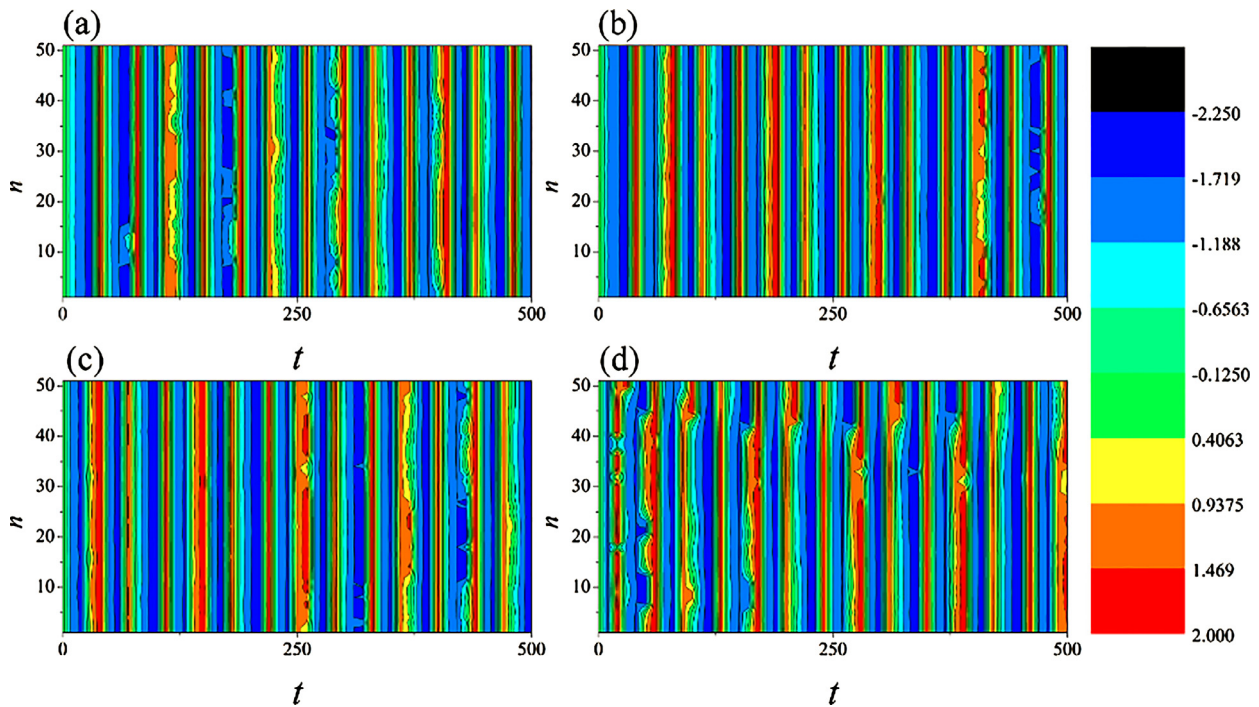


Fig. 8. Developed spatial patterns of network driven by noise and modulated by memristor. Coupling intensity between nodes $D=0.5$, noise intensity $D_0=6$, for (a) $k=6$; (b) $k=6.5$; (c) $k=7$; (d) $k=7.5$.

of memristor. For simplicity, $\alpha=0.1$, $\beta=0.03$, $a_1=0.5$, $a_2=0.51$, and two neurons are selected with different parameters because neurons in realistic nerve system could be much different. As well known, coupling can induce complete synchronization between two identical neurons, while phase synchronization could be available for non-identical neurons or oscillators. For neurons, rhythm in electrical activities or phase could be better to describe the important encoding information than amplitude; therefore, the phase

[87] is often calculated by using oscillation extremum method. That is, detected the time (t_1, t_2, \dots, t_n) for reaching maximal values from the sampled time series $x(t)$, it reads as follows

$$\theta(t) = 2\pi \frac{t - t_n}{t_{n+1} - t_n} + 2\pi n, \quad t_n < t < t_{n+1} \quad (3)$$

As a result, the calculated phase is segment piecewise-linear and the fluctuation between successive maximal values is left out.

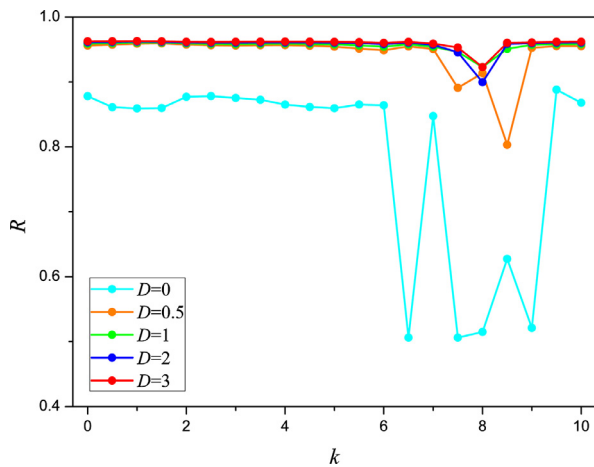


Fig. 9. Distribution for factor of synchronization is calculated under different induction coefficients, the noise intensity is fixed at $D_0=6$, and the transient period for calculating is 1000 time units.

To detect the synchronization approach, the phase error and variable error is respectively defined as follows

$$\begin{cases} \Delta\theta = \theta_1 - \theta_2 \\ \gamma(x, y) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \end{cases} \quad (4)$$

Furthermore, the synchronization of neurons on chain network is also investigated, and noise is also considered. The diagram for the chain network is described by

According to Fig. 2, the network can be described by dynamical equations as follows

$$\begin{cases} \frac{dx_{1i}}{dt} = 20\left(x_{1i} - \frac{x_{1i}^3}{3} - y_{1i}\right) - k\rho(\varphi)(x_{1i} - x_{2i}) + D(x_{1i+1} + x_{1i-1} - 2x_{1i}) + \xi(t) \\ \frac{dy_{1i}}{dt} = x_{1i} + a_1 \\ \frac{dx_{2i}}{dt} = 20\left(x_{2i} - \frac{x_{2i}^3}{3} - y_{2i}\right) - k\rho(\varphi)(x_{2i} - x_{1i}) + \xi(t) \\ \frac{dy_{2i}}{dt} = x_{2i} + a_2 \\ \frac{d\varphi_i}{dt} = k(x_{1i} - x_{2i}) \end{cases} \quad (5)$$

Where D represents the coupling intensity between adjacent pair of neurons, $\xi(t)$ is Gaussian white noise, its statistical property can be approached by $\langle \xi(t) \rangle = 0, \langle \xi(t)\xi(t') \rangle = 2D_0\delta(t-t')$, and D_0 denotes the noise intensity. To approach and discuss the collective behavior, statistical factor of synchronization (R) is defined in the chain network by using mean field theory, it reads as follows

$$\begin{cases} F = \frac{1}{n} \sum_{i=1}^n x_i \\ R = \frac{\langle F^2 \rangle - \langle F \rangle^2}{\frac{1}{n} \sum_{i=1}^n (\langle x_i^2 \rangle - \langle x_i \rangle^2)} \end{cases} \quad (6)$$

Where n is node number of neuronal network, $\langle * \rangle$ describes the average of variable over time, for simplicity, a transient period $T = 1000$ time units will be used for numerical studies. It indicates perfect synchronization when R is approached close to 1, while non-perfect synchronization is reached at $R \sim 0$. In the numerical section, the fourth order Runge–Kutta algorithm is used

to approach solutions for the dynamical equations with time step $h = 0.01$ being used.

3. Numerical results and discussion

The initial values are selected as (0.3, 0.1, 5.0, 0, 0.2), transient period for calculation is about 1000 time units. The parameters are fixed at $a_1 = 0.5, a_2 = 0.51$, firstly, the induction coefficient k is changed to observe the synchronization degree between two non-identical neurons coupled by memristor, the results are shown in Fig. 3.

Synapse coupling, chemical synapse and electric synapse, could be much effective to propagate signals between neurons and realize information encoding for neurons. As shown in Figs. 3 and 4, memristor coupling can enhance the synchronization between two neurons with diversity in parameter. With increasing the induction coefficient, the two neurons can reach phase synchronization and the variable error is decreased completely. The potential mechanism could be that memristor coupling can exchange the magnetic flux and induced current can be imposed to drive the neuron to keep pace with another neuron. Furthermore, the inter-spike interval (ISI) is calculated for bifurcation analysis, the results are shown in Fig. 5.

That is, weak effect of electromagnetic induction and setting small induced current via memristor can induce multiple modes in electrical activities, thus two neurons is out of phase synchronization. Extensive numerical results confirmed that phase synchronization can be reached by further increasing the coupling intensity via memristor. The similar investigation is also carried out on chain network, the node number of network is set as $n=50$, that is, the network holds 100 neurons as shown in Fig. 1. The factor of synchronization will be calculated from Neuron-1(i), $i=1, 2, 3, \dots, 50$ with noise being considered and the results are shown in Fig. 6.

It is found that the network synchronization is dependent on the coupling between neurons, and memristor coupling shows slight modulation on synchronization of network. And the synchronization keeps robust to external noise even memristor coupling is considered. Otherwise, the synchronization is decreased. For further illustration for the effect of coupling intensity and induction coefficient on synchronization degree of network, the spatiotemporal patterns are calculated in Figs. 7 and 8.

The results in Fig. 7 confirmed that the network synchronization can be suppressed when connection and coupling are removed, and memristor coupling for each pair of neurons can also decrease the synchronization. It is found in Fig. 8, by further increasing the coupling between neurons, synchronization can be reached and enhanced, and the effect of memristor coupling is also suppressed in presence of noise. Furthermore, the dependence of synchronization on induction coefficient is discussed by calculated the distribution for synchronization factors when coupling intensity, noise is fixed, and the results are shown in Fig. 9.

It is found that isolate neurons ($D=0$) can reach synchronization, and increasing induction coefficient can enhance desynchronization because diversity in excitability is increased. It is interesting to find that factor of synchronization will approach a lower region at $k=7\sim 9$ even coupling intensity between adjacent nodes is selected with higher value, that is, coupling between adjacent neurons of the chain network contributes greatly for the synchronization behaviors of network and the effect of memristor can be suppressed. Furthermore, the sampled time series for node (20) and pattern stability is calculated under different coupling amplitudes, as illustrated in Figs. 10 and 11.

The sampled time series form membrane potential show distinct periodicity, and each pair of neurons coupled by memristor can reach complete synchronization, while the disconnected chain network ($D=0$) can't reach spatial synchronization even noise is

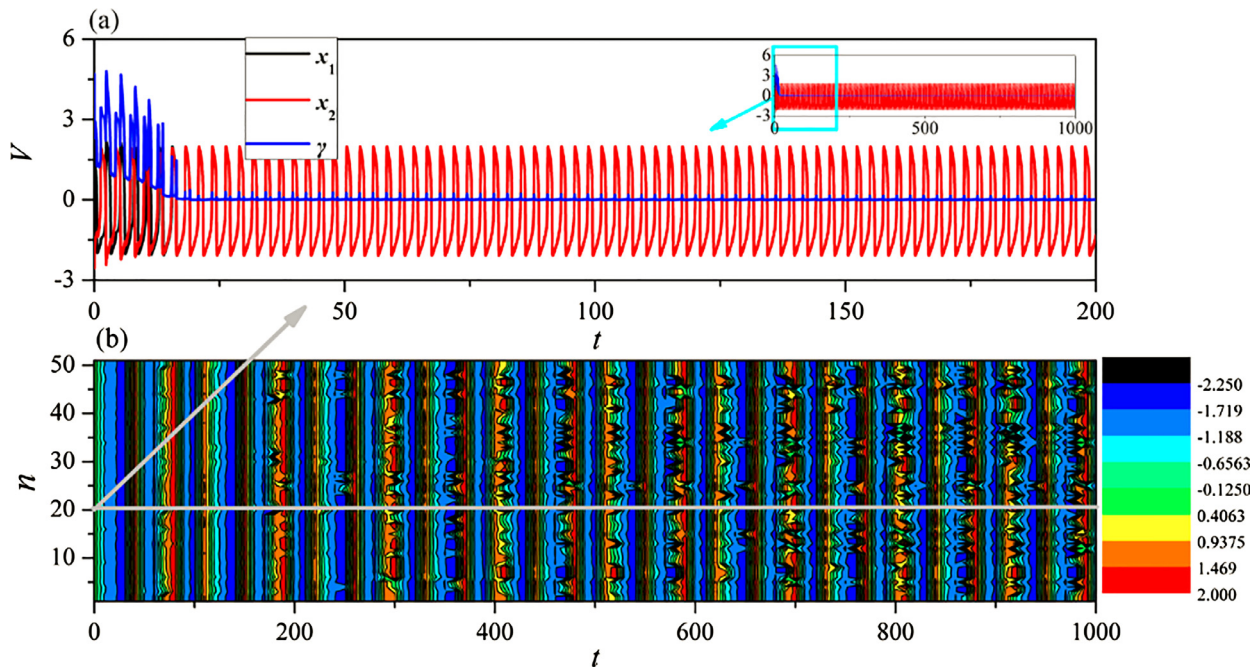


Fig. 10. Sampled time series for membrane potential from node (20), and development of spatiotemporal pattern is calculated at $D=0$, $k=6.5$, $D_0=6$, transient period is 1000 time units. The blue line describes the evolution for variable error. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

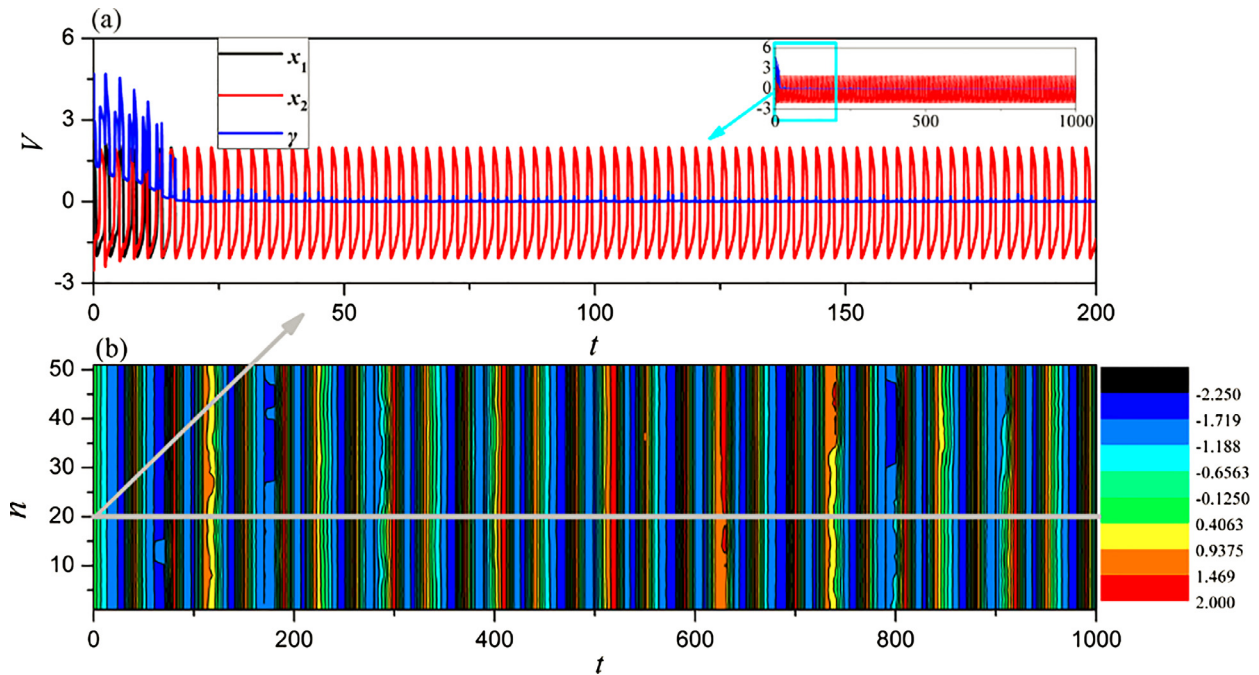


Fig. 11. Sampled time series for membrane potential from node (20), and development of spatiotemporal pattern is calculated at $D=3$, $k=6.0$, $D_0=6$, transient period is 1000 time units. The blue line describes the evolution for variable error. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

imposed on the network. Furthermore, the chain network is activated by increasing the coupling intensity between adjacent nodes, and the results are shown in Fig. 11.

The results in Fig. 11 found that the network reached complete synchronization by further increasing the coupling intensity and each pair of neurons coupled by memristor also is stabilized at synchronization state even noise is considered.

In a summary, memristor is helpful to bridge neurons and make neurons reach phase synchronization and even complete

synchronization under appropriate induction coefficient. The potential mechanism is that memristor describe the effect of field coupling and exchange of magnetic flux, the memory effect makes each neuron to keep pace with another neuron in modes of electrical modes. Furthermore, network synchronization can also be realized by setting appropriate coupling intensity between adjacent nodes though modulation from memristor coupling can decrease the synchronization degree slightly.

4. Conclusions

In this paper, the effect of electromagnetic induction is described by magnetic flux and memristor is used bridge the information exchange by generating induced current on the membrane potential. It is found synchronization can be enhanced and approached completely via memristor coupling. Furthermore, a chain network is designed to study the synchronization problem and each node is modulated by another neuron via memristor coupling. That is, the network is composed of nodes, and each node is controlled by a pair of neurons via memristor coupling. By setting appropriate coupling intensity and induction coefficient, which describes the coupling effect and electromagnetic induction via memristor, the electrical activities can present distinct periodicity and the network reach complete synchronization even noise is considered.

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