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ORIGINAL PAPER

The evolution of multidimensional shock structures in magnetized dusty plasmas with arbitrary dust size distribution

L P Zhang* and **H M Du**

School of Sciences, LanZhou University of Technology, Lanzhou 730050, China

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Abstract: A theoretical investigation on the evolution of multidimensional nonlinear shock structures has been carried out in magnetized dusty plasmas with nonadiabatic charge fluctuation and arbitrary dust size distribution in present work. A Korteweg–de Vries (KdV) Burgers equation for the multidimensional shock structures is obtained by Reductive Perturbation Method, and the effects of arbitrary dust size distribution, nonadiabatic charge fluctuation and external magnetic field on the multidimensional shock structures' evolution are illustrated. The results show that there are two types of shock structures, namely, oscillatory shock structures and monotone shock structures. Furthermore, the boundary condition between oscillatory shock structures and monotone shock structures is discussed in great detail.

Keywords: Nonlinear multidimensional shock structures; Magnetized dusty plasmas; Charge fluctuation; Dust size distribution

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1. Introduction

Nonlinear structures have received significant attention and excited many research activities in dusty plasmas, where the mass and the charge of dust grains are variant due to collisions with ions and electrons in this system. The different collective motion of the plasma and some new eigen modes will be introduced [1–4] in the scope of low frequency and very low frequency oscillations due to the exiting of charged dust grains. Its dynamic characteristic is very important in investigating linear and nonlinear structures in dusty plasmas. In the linear theory, the non-adiabatic (τ_{ch}/τ_d is small but finite, where τ_{ch} represents the charging time scale, and τ_d represents the hydrodynamical time scale.) charge fluctuation leads to the damping of the linear structures [5–9]. However, in the nonlinear theory, the non-adiabatic charge fluctuation results in an abnormal dissipation which leads to the emergence of nonlinear shock structures in dusty plasmas [10–12]. The nonlinear shock structures with dust charge fluctuation have been investigated by some researchers. Wang investigated the nonlinear propagation of shock structures in dusty plasma with the strongly coupled dust particles and nonadiabatic dust charge fluctuation [13]. Dissipative shock structures with trapped electrons and an oblique magnetic field in a varying charge electronegative magnetized dusty plasma are examined and it is shown that the shock structures are more dispersive by a decrease of its magnitude or an increase of the magnetic field obliqueness [14]. The propagations of dust ion acoustic shock structures with varying charge have been investigated due to nonextensive electrons and the results show that both steepness and strength of shock waves emergence by the electrons evolve that is remote from their thermodynamic equilibrium with parameter ranges which are consistent with Saturn's rings [15]. Shah investigated propagation of spherical and cylindrical shock structures in a multispecies plasma with consisting of Boltzmann light ions, adiabatic positively charged inertial heavy ions, and inertialess kappa-distributed superthermal electrons [16].

Furthermore, the sizes of dust particles are different in a real circumstance such as space plasmas, laboratory experiments, gas-discharge plasmas, and so on. For space plasmas in cometary environments, F and G rings of Saturn, the dust grains are distributed by power law and for laboratory dusty plasmas they have a Gaussian distribution.

*Corresponding author, E-mail: zhanglp@lut.cn

In general, the dust size distribution can be an arbitrary function which depends on the experimental conditions and the environment in the laboratory plasmas or in space plasmas. Since the dust size distribution is an arbitrary function in the laboratory, even in the space plasma, the size of the dust particles are asymmetrical. For an arbitrary dust size distribution dusty plasma, Ma et al. [17] observed the propagation of dust negative ion acoustic solitary waves in a magnetized multi-ion dusty plasma containing hot isothermal electron, ions (light positive ions and heavy negative ions) and extremely massive charge fluctuating dust grains by employing the reductive perturbation method. The research results can be applied to dusty plasmas with these asymmetric dust size distribution both in the laboratory and in space plasmas. It has been found that there has been of great interest in study of the propagation of plasma waves in dusty plasmas with dust size distribution. Recently, the transportation of mass, heat and charges of dust grains in a weakly ionized plasma has been investigated [18]. Banerjee [19] studied the properties of dust acoustic solitary waves in dusty plasmas with dust grains having power law size distribution and kappa-distributed ions. The dust particles with different size play an important role in plasma collective behavior. It is necessary to study the effect of dust size distribution on the propagation of waves in dusty plasmas, either in the space or laboratory plasmas. The research results can be applied to these asymmetric dust size distribution dusty plasmas both in the laboratory and in space plasmas.

Due to the above factors, the combined effects of dust size distribution and the nonadibatic charge fluctuation on the propagation of shock structures in dusty plasmas with external magnetized field are investigated under the assumption that τ_{ch}/τ_d is small but finite in present study. We obtain a Korteweg–de Vries (KdV) Burgers equation by reductive perturbation methods. The propagation of dust acoustic shock structures and the boundary condition between the monotone shock structures and the oscillatory shock structures are discussed with the dust charge variation, the external magnetized field and dust size distribution in great detail.

2. Theoretical model

We consider nonlinear multidimensional shock structures in magnetized dusty plasmas with stationary dust particles with N different species, nonthermal ions and Boltzmann distributed electrons. The external static magnetic field directed along z axis $B_0 \parallel z_0$, where z_0 is a unit vector along z direction. The quasi neutrality equilibrant condition is given by $n_{i0} = \sum_{j=1}^{N} Z_{dj0} n_{dj0} + n_{e0}$, where n_{i0}, n_{dj0} and n_{e0} are the unperturbed ion, jth dust grain, and electron densities, respectively, and Z_{dj0} represents the jth dust charge number. The basic set of equations governing the dynamics of dust acoustic shock structures in dusty plasmas are

$$\begin{cases} \dfrac{\partial n_{dj}}{\partial t} + \nabla \cdot (n_{dj}\mathbf{V}_{dj}) = 0 \\ \dfrac{\partial \mathbf{V}_{dj}}{\partial t} + \mathbf{V}_{dj} \cdot \nabla \mathbf{V}_{dj} = \dfrac{1}{m_{dj}} (Z_{dj0} - q_{dj}) \left[\nabla \phi - \omega_{cd} (\mathbf{V}_{dj} \times z_0) \right] \\ \nabla^2 \phi = \displaystyle\sum_{j=1}^{N} (Z_{dj0} - q_{dj}) n_{dj} + n_e - n_i \end{cases} \tag{1}$$

where $\mathbf{V}_{dj} = (u_{dj}, v_{dj}, w_{dj})$ is the jth dust grains' velocity nondimensionalize by the dust acoustic speed $C_d = \sqrt{\overline{Z}_{d0} T_i/\overline{m}_d}$ (T_i is the ion-temperature), $\overline{m}_d = \sum_{j=1}^{N} m_{dj} n_{dj0}/N_{tot}$ is the average dust grains' mass, and $\overline{Z}_{d0} = \sum_{j=1}^{N} Z_{dj0} n_{dj0}/N_{tot}$ is the average dust charged number. n_{dj0} and n_{i0} are number densities of dust grain and ions normalized by N_{tot} and n_{i0}. ϕ is the dimensionless electrostatic plasma potential. The space x and time t coordinates are normalized by the Debye length $\lambda_{Dd} = C_d/\omega_{pd}$ and the inverse of dust plasma period $\omega_{pd}^{-1} = \sqrt{\overline{m}_d/4\pi N_{tot} \overline{Z}_{d0}^2 e^2}$, respectively.

Because observation of space plasmas very often indicates the existence of ion distribution that is far from thermodynamic equilibrium, the occurrence of nonthermal ions is considered to be a very common feature in such environments. Nonthermal ions from the Earth's bow-shock have been observed by the Vela satellites [20]. The research results show that the existence of non-thermal ions provides a possibility for the coexistence of the large amplitude and the compressional solitary waves, whereas both of them are decoupled in the small amplitude limit [21]. The effect of nonthermal ions on nonlinear structures can also provide a guideline for interpreting the most recent numerical simulation results, which exhibit the simultaneous presence of non-thermal ion distributions and associated dust acoustic localized wave packets. For the non-thermal ions, the distribution function is: $n_i = \frac{1}{1-\mu}[1 + \beta(\phi + \phi^2)] \exp(-\phi)$, where $\mu = n_{e0}/n_{i0}$, $\beta = \frac{4\alpha}{1+3\alpha}$, α represents the number of nonthermal ions. The Boltzmann distribution of electrons is: $n_e = \frac{\mu}{1-\mu} \exp(\sigma_i \phi)$, where $\sigma_i = T_i/T_e$, with T_e being the temperature of electrons.

When $\tau_{ch} \gg \tau_d$, the dust charge is regarded as constant. However, when $\tau_{ch} \simeq \tau_d$, the dust charge variation will be considered. The dust charge $Q_{dj} = -Z_{dj0}e + q_{dj}$, where q_{dj} is the dimensionless dust charge variation of the the jth dust grain. In order to calculate the dimensionless charge variable q_{dj}, the dimensionless orbital motion-limited [22] charge current balance equation is

$$\frac{\tau_{ch}}{\tau_d}\left(\frac{\partial q_{dj}}{\partial t}+\mathbf{V}_{dj}\nabla q_{dj}\right)=\frac{\tau_{ch}}{Z_{dj0}e}(I_e+I_i) \tag{2}$$

where I_e and I_i are the electron and ion current, respectively. For spherical dust particles with radius a, the dimensionless expressions of the electron and nonthermal ion currents are [23]

$$I_e=-\ \pi a^2 e\sqrt{\frac{8T_e}{\pi m_e}}n_{e0}(x)\ \exp\ (\sigma_i\phi)\ \exp\ [z(q_{dj}-Z_{dj0})] \tag{3}$$

$$\begin{aligned} I_i=&\pi a^2 e\sqrt{\frac{8T_i}{\pi m_i}}\frac{n_{i0}}{(1+3\alpha)}\left[\left(\left(1+\frac{24\alpha}{5}\right)+\frac{16\alpha}{3}\phi+4\alpha\phi^2\right)\right.\\ &\left.-\frac{z(q_{dj}-Z_{dj0})}{\sigma_i}\left(\left(1+\frac{8\alpha}{5}\right)+\frac{8\alpha}{3}\phi+4\alpha\phi^2\right)\right]\exp\ (-\phi) \end{aligned} \tag{4}$$

$$\begin{cases} \frac{\partial n_{dj1}}{\partial \tau}-v_0\frac{\partial n_{dj2}}{\partial \xi}+l_x n_{dj0}\frac{\partial u_{dj2}}{\partial \xi}+l_y n_{dj0}\frac{\partial v_{dj2}}{\partial \xi}+l_z n_{dj0}\frac{\partial w_{dj2}}{\partial \xi}+l_z\frac{\partial n_{dj1}u_{dj1}}{\partial \xi}=0 \\ \frac{\partial w_{dj1}}{\partial \tau}-v_0\frac{\partial w_{dj2}}{\partial \xi}+l_z w_{dj1}\frac{\partial w_{dj1}}{\partial \xi}-\frac{Z_{dj0}}{m_{dj}}l_z\frac{\partial \phi_2}{\partial \xi}+\frac{l_z}{m_{dj}}q_{dj1}\frac{\partial \phi_1}{\partial \xi}=0 \\ \frac{\partial^2\phi_1}{\partial \xi^2}-\sum_{j=1}^N Z_{dj0}n_{dj2}+\sum_{j=1}^N n_{dj1}q_{dj1}+\sum_{j=1}^N n_{dj0}q_{dj2}-\frac{\mu\sigma_i^2-1}{2(1-\mu)}\phi_1^2+\frac{\beta-\mu\sigma_i-1}{1-\mu}\phi_2=0 \\ -\beta_1\phi_2+q_{dj2}+\beta_2 q_{dj1}\phi_1+\beta_3\phi_1^2+\beta_4 q_{dj1}^2-\nu v_0\frac{\partial q_{dj1}}{\partial \xi}=0 \end{cases} \tag{6}$$

where $z=Z_{dj0}e^2/4\pi\varepsilon_0 aT_e$, $4\pi\varepsilon_0 a$ is the spherical dust particles' capacitance. The charging time scale $\tau_{ch}[\approx(dQ_d/dt)^{-1}]$ is

$$\tau_{ch}=\left[\frac{a}{\sqrt{2\pi}}\frac{\omega_{pi}^2}{V_{thi}}\frac{5+8\alpha}{5+15\alpha}\left(1+z+\frac{5+24\alpha}{5+8\alpha}\sigma_i\right)\right]^{-1} \tag{5}$$

where V_{thi} and ω_{pi} are the ion thermal velocity and the ion plasma frequency, respectively.

3. Weakly nonlinear theory

The KdV Burgers equation is obtained using the standard reductive perturbation technique [24] in order to study the propagation of shock structures in dusty plasma. The stretched coordinates are $\xi=\epsilon^{1/2}(l_x x+l_y y+l_z z-v_0 t)$ and $\tau=\epsilon^{3/2}t$, where ϵ is a small parameter determining the intensity of non-linearity, v_0 is the unknown phase velocity, l_x, l_y and l_z are the directional cosines of the wave vector $\mathbf{k}$ along $x-$, $y-$, and $z-$axes, respectively, so that $l_x^2+l_y^2+l_z^2=1$. Substituting $n_{dj}=n_{dj0}+\epsilon n_{dj1}+\epsilon^2 n_{dj2}+\cdots, u_{dj}=\epsilon^{3/2}u_{dj1}+\epsilon^2 u_{dj2}+\cdots, v_{dj}=\epsilon^{3/2}v_{dj1}+\epsilon^2 v_{dj2}+\cdots, w_{dj}=\epsilon w_{dj1}+\epsilon^2 w_{dj2}+\cdots, \phi=\epsilon\phi_1+\epsilon^2\phi_2+\cdots,$ $q_{dj}=\epsilon q_{dj1}+\epsilon^2 q_{dj2}+\cdots$, $\tau_{ch}/\tau_d=\nu\epsilon^{1/2}$(For nonadiabatic charge fluctuation, τ_{ch}/τ_d is small but finite [25]) into Eqs. (1) and (2) and gathering the terms in the different powers of ϵ, we obtain at the lowest order: $v_{dj1}=\frac{l_x}{\omega_{cd}}\frac{\partial\phi_1}{\partial\xi}$, $u_{dj1}=\frac{l_y}{\omega_{cd}}\frac{\partial\phi_1}{\partial\xi}$, $w_{dj1}=\frac{v_0}{n_{dj0}l_z}n_{dj1}$, $\sum_{j=1}^N n_{dj1}=\frac{B}{Z_{dj0}}\phi_1$, $q_{dj1}=\beta_1\phi_1$, $v_0^2=-l_z^2\sum_{j=1}^N\frac{n_{dj0}Z_{dj0}^2}{m_{dj}}/B$, where $B=\beta_1\sum_{j=1}^N n_{dj0}+\frac{\beta-\mu\sigma_i-1}{1-\mu}$ and $\beta_1=\frac{\frac{40\alpha}{15+24\alpha}(2\sigma_i+zZ_{dj0})-(\delta\sigma_i+zZ_{dj0})(1+\sigma_i)}{zZ_{dj0}(1+zZ_{dj0}+\delta\sigma_i)}$ $(\delta=\frac{5+24\alpha}{5+8\alpha})$. The dust charge fluctuation is determined by the variable parameterβ_1.

We gain a set of equations in the second lowest order:

where $\beta_2=\frac{\sigma_i(\delta\sigma_i+zZ_{dj0})-\frac{15-16\alpha}{15+24\alpha}}{1+zZ_{dj0}+\delta\sigma_i}$, $\beta_3=\frac{(zZ_{dj0}+\delta\sigma_i)(\sigma_i^2-1)+\frac{40\alpha}{15+24\alpha}(\sigma_i-zZ_{dj0})}{2Z_{dj0}(1+zZ_{dj0}+\delta\sigma_i)}$, $\beta_4=\frac{zZ_{dj0}(\delta\sigma_i+zZ_{dj0})}{2(1+zZ_{dj0}+\delta\sigma_i)}$. Now, the KdV Burgers equation is obtained

$$\frac{\partial\phi_1}{\partial\tau}+A\phi_1\frac{\partial\phi_1}{\partial\xi}+C\frac{\partial^3\phi_1}{\partial\xi^3}-D\frac{\partial^2\phi_1}{\partial\xi^2}=0 \tag{7}$$

where $\phi\equiv\phi_1$ and

$$\begin{aligned} A=&\frac{3l_z^4}{2Bv_0^3}\sum_{j=1}^N\frac{Z_{dj0}^3 n_{dj0}}{m_{dj}}+\frac{3\beta_1 l_z^2}{2Bv_0}\sum_{j=1}^N\frac{Z_{dj0}n_{dj0}}{m_{dj}}\\ &+\frac{v_0}{B}\frac{\mu\sigma_i^2-1}{1-\mu}+\frac{v_0}{B}\sum_{j=1}^N n_{dj0}(\beta_3+\beta_1\beta_2+\beta_1^2\beta_4) \end{aligned} \tag{8}$$

$$C=-\frac{v_0}{2B}-\frac{v_0(1-l_z^2)}{2B\omega_{cd}^2}\sum_{j=1}^N\frac{Z_{dj0}^2 n_{dj0}}{m_{dj}} \tag{9}$$

$$D=\frac{v_0^2\beta_1\nu\sum_{j=1}^N n_{dj0}}{2B} \tag{10}$$

The KdV Burgers equation (7) contains both dispersive and dissipative terms. The KdV Burgers equation has a specific solution representing the monotonic shock structures.

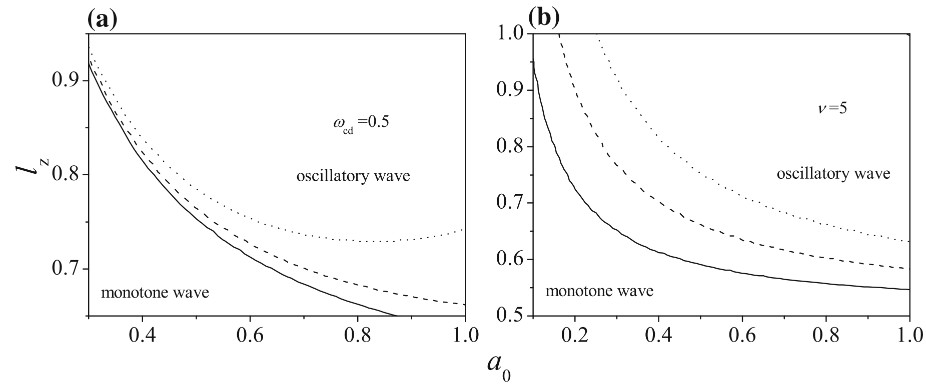

Fig. 1 The boundary condition between oscillatory structures and monotone structures ((a) solid line ($\nu = 5$), Dash line ($\nu = 10$), dot line ($\nu = 15$); (b) solid line ($\omega_{cd} = 0.3$), Dash line ($\omega_{cd} = 0.4$), dot line ($\omega_{cd} = 0.5$))

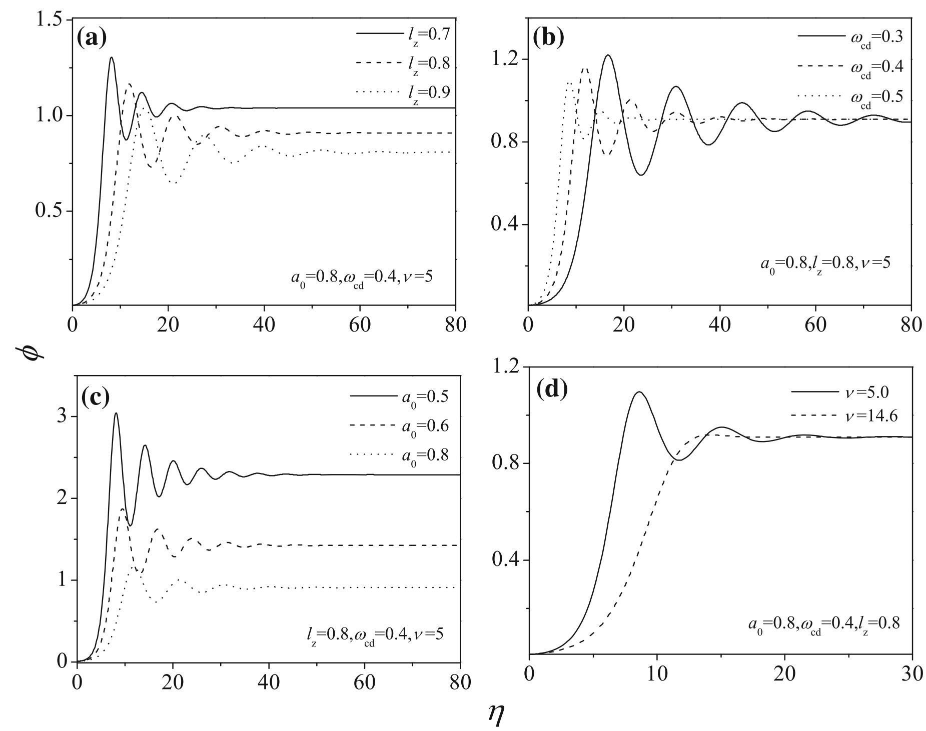

Fig. 2 Profile of the shock structures for different l_z, ω_{cd}, a_0 and ν

Whereas, when the combined action of dispersion and dissipation balance the nonlinear effects resulting in wave breaking a dispersive shock wave appears in plasma. If dissipation disappear, the KdV Burgers equation reduce to the KdV equation exhibiting soliton structures. From Eq. (7), it is seen that the Burgers term is proportional to the term ν, arising due to the nonadiabatic dust charge variation. D is also modified by dust size distribution. We can expect that the shock structures will be modified by the dust charge variation, the external magnetized field, the presence of nonthermally distributed ions, the dust size distribution, the temperature of ion, and the ration of electron to ion density.

Due to the nonadiabatic dust charge fluctuation, the Burgers term in (7) indicates probability of the existence of shock structures. From the KdV Burgers equation (7), a second-order equation can be obtained by the transforming to wave frame: $\eta = V\tau - \xi$

$$\frac{d^2\phi}{d\eta^2} = \frac{V}{C}\phi - \frac{A}{2C}\phi^2 - \frac{D}{C}\frac{d\phi}{d\eta} \tag{11}$$

From the second-order equation (11), a set of two first-order equations can be obtained

$$\frac{d\phi}{d\eta} = \Phi \tag{12}$$

$$\frac{d\Phi}{d\eta} = \frac{V}{C}\phi - \frac{A}{2C}\phi^2 - \frac{D}{C}\Phi \tag{13}$$

The system governed by Eqs. (12) and (13) has two fixed points $(\phi_1^*, \Phi_1^*) = (0, 0)$ and $(\phi_2^*, \Phi_2^*) = (\frac{2V}{A}, 0)$. It is clear that the first singular point (ϕ_1^*, Φ_1^*) is a saddle point but the second singular point (ϕ_2^*, Φ_2^*) is a stable focus or a stable node on the basis of $D^2 < 4VC$ or $D^2 > 4VC$.

This means that the system governed by Eqs. (12), (13) has a heteroclinic orbit interconnecting the saddle-node or

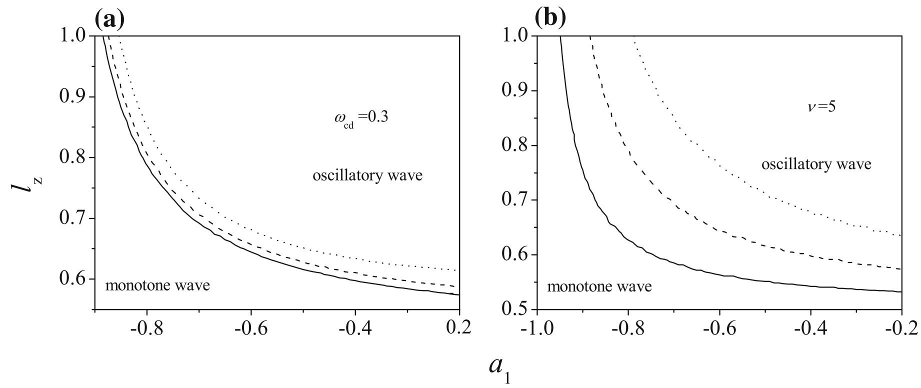

Fig. 3 The boundary condition between oscillatory structures and monotone structures ((a) solid line ($\nu = 5$), Dash line ($\nu = 10$), dot line ($\nu = 15$); (b) solid line ($\omega_{cd} = 0.2$), Dash line ($\omega_{cd} = 0.3$), dot line ($\omega_{cd} = 0.4$))

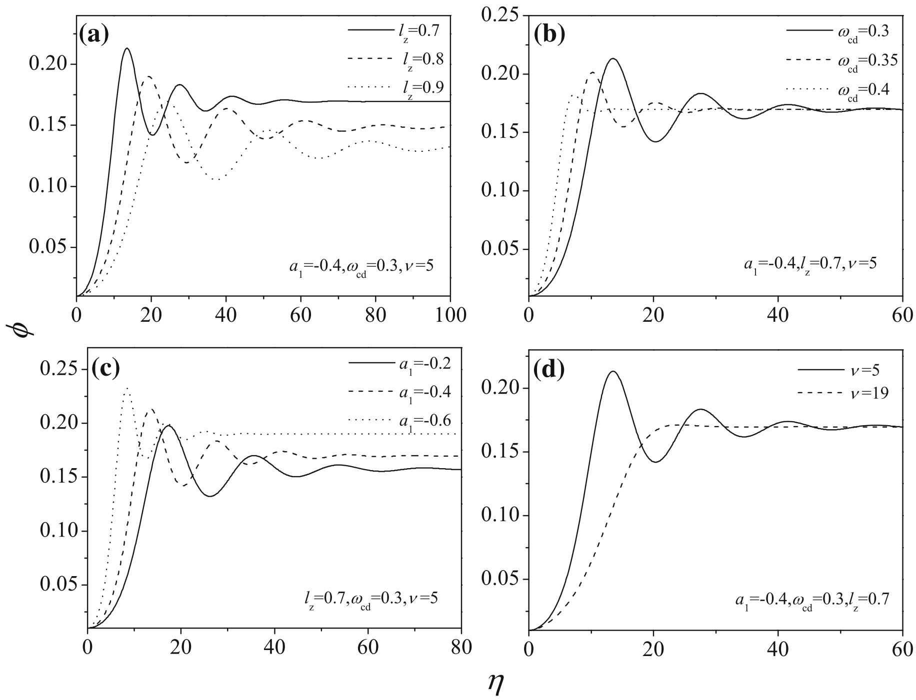

Fig. 4 Profile of the shock structures for different l_z, ω_{cd}, a_1 and ν

saddle-focus point. Furthermore, oscillatory shock structures always corresponds to the stable focus but monotone shock structures corresponds to the stable node. To explore the nonlinear shock structures clearly, we integrate Eq. (11) with respect to η subject to the boundary conditions $\phi \to 0$ at $\eta \to -\infty$, then $\phi(\eta)$ is obtained. Therefore, the potential ϕ increases from zero value $t \to \infty(\eta \to \infty)$ to a stable value $2V\,/\,A$ at long past at any x.

When we chose that all dust grain radius $a \ll \lambda_{Dd}$, the dust grains' mass is $m_{dj} = k_m a_j^3$ and the dust grains' charge is $Q_{dj0} = k_q a_j^{\gamma}$ or $Z_{dj0} = k_z a_j^{\gamma}$ [26], where $k_m \approx 4/3\pi\rho_d$, $k_q \approx 4\pi\varepsilon_0\Phi_0$, $k_z \approx 4\pi\varepsilon_0\Phi_0/e$, are approximately equal to constants. ρ_d is the dust grains' mass density (considered to be equal and constant for all particles), ε_0 is the vacuum permittivity, Φ_0 is the electric balance surface potential.

4. Results and discussion

We consider a polynomial expressed distribution function in dusty plasmas. For dust particles with radius r in a given scope $[r_{min}, r_{max}]$, the form of the differential polynomial expressed distribution function is [27],

$$n_d(r)dr = (a_0 + a_1 r + a_2 r^2 + a_3 r^3 + \cdots)dr \tag{14}$$

where r is the radius of the dust grains, $a_0, a_1, a_2, a_3, \cdots$ are all constants which should be decided by the following equation:

$$N_{tot} = \int_{r_{\min}}^{r_{\max}} n(r)dr \tag{15}$$

where $N_{\rm tot}$ is the total number density of dust grains. Then we can obtain

$$\begin{cases} \sum_{j=1}^{N} n_{dj0} = a_0(r_{max} - r_{min}) + \frac{a_1}{2}\left(r_{max}^2 - r_{min}^2\right) + \frac{a_2}{3}\left(r_{max}^3 - r_{min}^3\right) + \frac{a_3}{4}\left(r_{max}^4 - r_{min}^4\right) + \cdots \\ \sum_{j=1}^{N} \frac{Z_{dj0}^3 n_{dj0}}{m_{dj}} = \frac{k_z^3}{k_m^2}\left[\frac{a_0}{3\gamma-5}\left(r_{max}^{3\gamma-5} - r_{min}^{3\gamma-5}\right) + \frac{a_1}{3\gamma-4}\left(r_{max}^{3\gamma-4} - r_{min}^{3\gamma-4}\right) + \frac{a_2}{3\gamma-3}\left(r_{max}^{3\gamma-3} - r_{min}^{3\gamma-3}\right) \right. \\ \qquad \left. + \frac{a_3}{3\gamma-2}\left(r_{max}^{3\gamma-2} - r_{min}^{3\gamma-2}\right) + \cdots \right] \\ \sum_{j=1}^{N} \frac{Z_{dj0}^2 n_{dj0}}{m_{dj}} = \frac{k_z^2}{k_m}\left[\frac{a_0}{2\gamma-2}\left(r_{max}^{2\gamma-2} - r_{min}^{2\gamma-2}\right) + \frac{a_1}{2\gamma-1}\left(r_{max}^{2\gamma-1} - r_{min}^{2\gamma-1}\right) + \frac{a_2}{2\gamma}\left(r_{max}^{2\gamma} - r_{min}^{2\gamma}\right) \right. \\ \qquad \left. + \frac{a_3}{2\gamma+1}\left(r_{max}^{2\gamma+1} - r_{min}^{2\gamma+1}\right) + \cdots \right] \\ \sum_{j=1}^{N} \frac{Z_{dj0} n_{dj0}}{m_{dj}} = \frac{k_z}{k_m}\left[\frac{a_0}{\gamma-2}\left(r_{max}^{\gamma-2} - r_{min}^{\gamma-2}\right) + \frac{a_1}{\gamma-1}\left(r_{max}^{\gamma-1} - r_{min}^{\gamma-1}\right) + \frac{a_2}{\gamma}\left(r_{max}^{\gamma} - r_{min}^{\gamma}\right) \right. \\ \qquad \left. + \frac{a_3}{\gamma+1}\left(r_{max}^{\gamma+1} - r_{min}^{\gamma+1}\right) + \cdots \right] \end{cases} \tag{16}$$

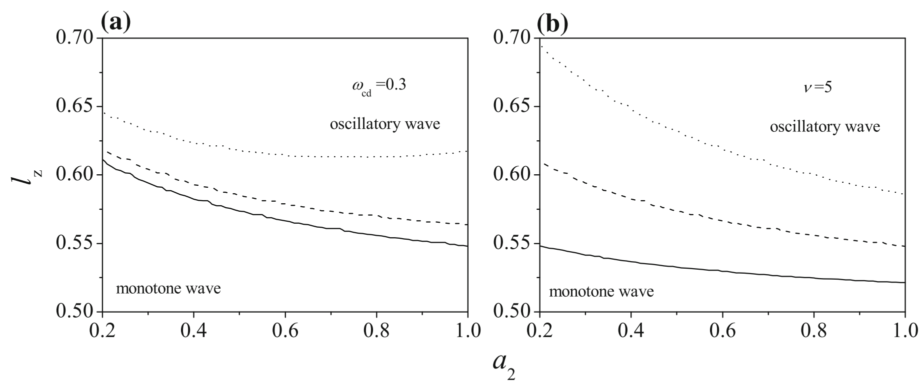

Fig. 5 The boundary condition between oscillatory structures and monotone structures ((a) solid line ($\nu = 5$), Dash line ($\nu = 25$), dot line ($\nu = 45$); (b) solid line($\omega_{cd} = 0.2$), Dash line ($\omega_{cd} = 0.3$), dot line ($\omega_{cd} = 0.4$))

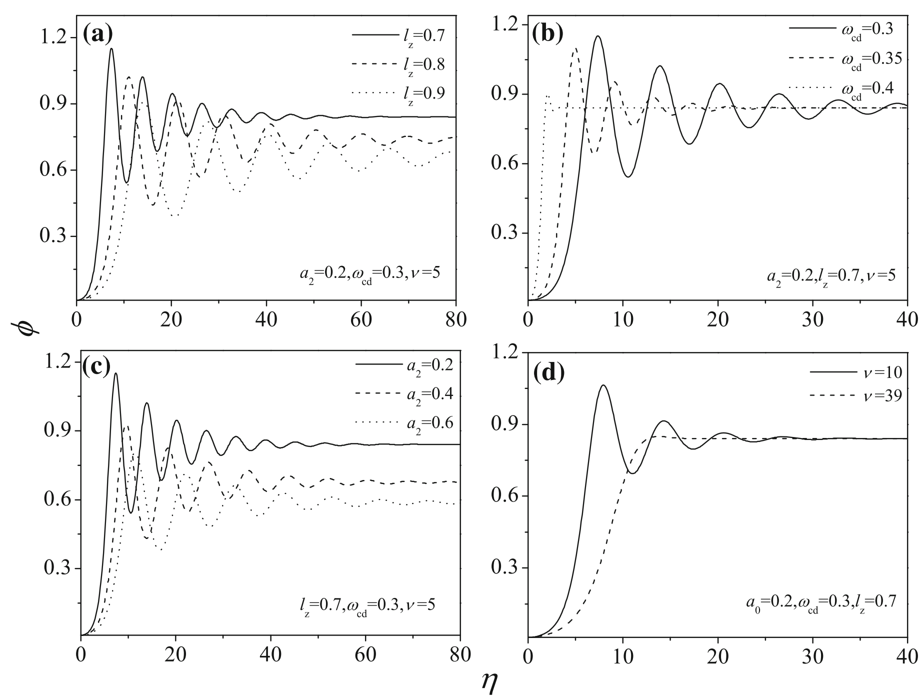

Fig. 6 Profile of the shock structures for different l_z, ω_{cd}, a_2 and ν

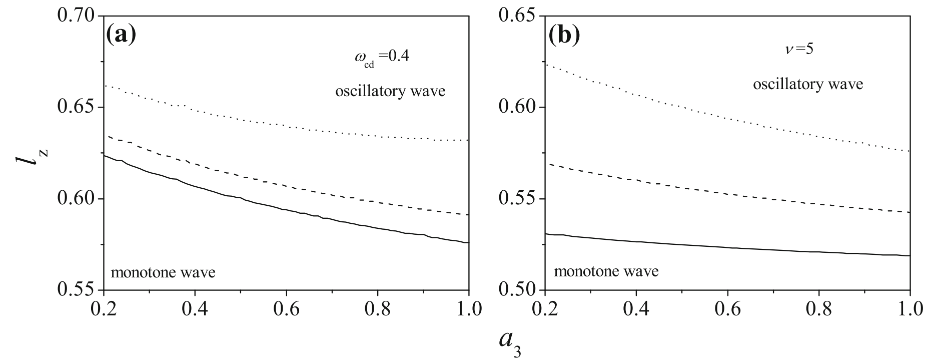

Fig. 7 The boundary condition between oscillatory structures and monotone structures ((a) solid line ($v = 5$), Dash line ($v = 15$), dot line ($v = 25$); (b) solid line ($\omega_{cd} = 0.2$), Dash line ($\omega_{cd} = 0.3$), dot line ($\omega_{cd} = 0.4$))

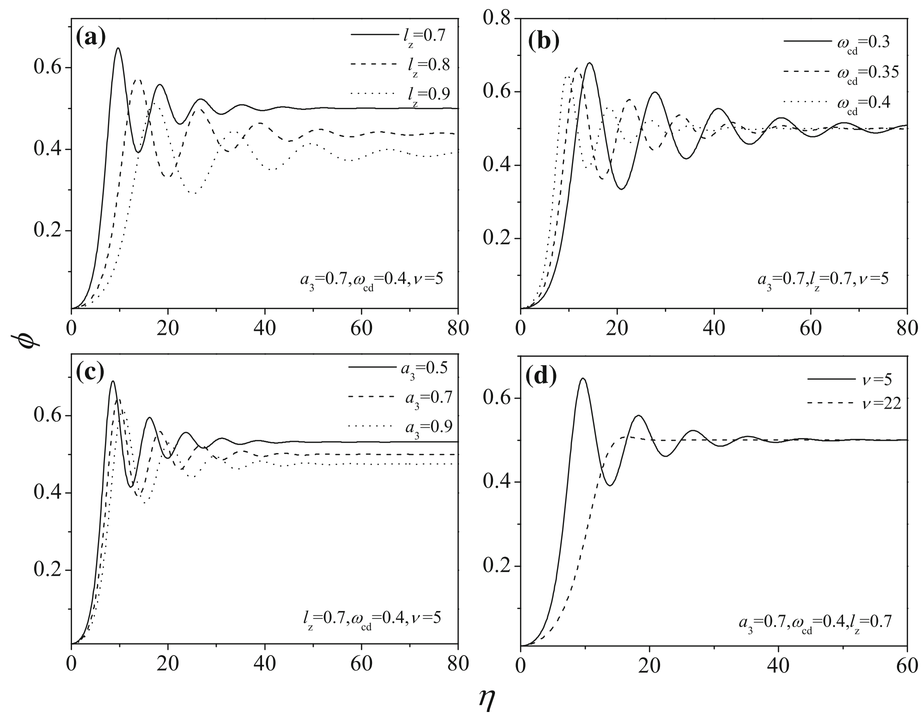

Fig. 8 Profile of the shock structures for different l_z, ω_{cd}, a_3 and v

We study the following special circumstances:

1. $n_d(r) = a_0$

It is clear that a_0 represents the value of total number density of dust grains and is positive. Now, we explore the nonlinear propagation of the shock structures in dusty plasmas with external magnetized field, nonadiabatic charge fluctuation and dust size distribution numerically. There are oscillatory shock structures and monotone shock structures in this system according to theoretical analysis. Furthermore, when dissipation dominates, the shock front exhibits a monotonic transition of the plasma density, while the shock transition is of oscillatory nature when the dissipation is weak. Figure 1 indicates two regions of monotone shock structures and oscillatory shock structures for different w_{cd} and v in (a_0, l_z) plane for $a_{min} = 0.01$, $a_{max} = 1.0$, $k_z = 2$, $k_m = 4$, $z = 2$, $\sigma_i = 0.1$, $\alpha = 0.5$, $\mu = 0.3$, $\gamma = 1.7$. It is clear from Fig. 1 that the region of monotone shock structures increases with ω_{cd} and v, that is, the dissipation induced by the external magnetized field and the dust change fluctuation weakens the oscillation of shock structures results in the transition from oscillatory shock structures to monotone shock structures. It is also indicated in Fig. 1 that for a fixed l_z, when a_0 increases there is a transition from the monotone shock structures to the oscillatory shock structures, in turn, for a fixed a_0, when l_z increases there is a transition from the monotone shock structures to the oscillatory shock structures. In order to determine the influence of nonadiabatic charge fluctuation, dust size distribution and the external magnetized field on the propagation of shock structures, we have simulated the behavior of shock structures with different values of a_0,

ω_{cd}, l_z and v by numerical simulation of the Eqs. (12) and (13). Figures 2 (a) and (c) show that the amplitude of the oscillatory waves decreases with the value of total number density of dust grains and the obliqueness of the magnetic field. Furthermore, Fig. 2(b) shows that the amplitude does not vary with the values of the external magnetized field, ω_{cd}. It is shown from Fig. 2(d) that when the values of v increase, oscillatory wave structure decreases and completely disappears at $v = 14.6$ leaving only laminar shock front, namely, the oscillatory shock structures are turned into the monotone shock structures.

2. $n_d(r) = a_0 + a_1 r$

In this case, it is noted that the properties of shock structures are dependent of a_1. He and Duan suggested that $a_1 > 0$ represents that the larger dust grains' number density is greater than the smaller dust grains' number density, which is the unfrequent condition of dusty plasma. For another, $a_1 < 0$ represents that the larger dust grains' number density is smaller than the smaller dust grains' number density, which is the widespread condition of dusty plasma. Here, we discuss the characteristics of the shock structures in the widespread condition of dusty plasma with $a_{min} = 0.01$, $a_{max} = 1.0$, $k_z = 2$, $k_m = 4$, $z = 2$, $a_0 = 0.8$, $\sigma_i = 0.1$, $\alpha = 0.8$, $\mu = 0.2$, $\gamma = 1.8$. Fig. 3 illustrates the boundary condition between the monotone shock structures and the oscillatory shock structures in this case. It can be seen from Fig. 3 that the effect of the external magnetized field and the dust charge fluctuation on the boundary condition is the same as that in Fig. 1. As also shown in Fig. 3 that for a fixed l_z, when a_1 increases there is a transition from the monotone shock structures to the oscillatory shock structures. The effect of a_1, l_z, ω_{cd} and v on the characteristics of shock structures is shown in Fig. 4. It is clear that from Fig. 4 that the oscillatory structures' amplitude decreases with a_1 and l_z. Figure 4(d) displays that as the increase of values of v, the oscillation of shock structures decreases and the oscillatory shock structures are turn into the monotone shock structures at $v = 19$.

3. $n_d(r) = a_0 + a_1 r + a_2 r^2$

We show the characteristics of the shock structures with $a_{min} = 0.01$, $a_{max} = 1.0$, $k_z = 2$, $k_m = 4$, $z = 2$, $a_0 = 0.6$, $a_1 = -0.4$, $\sigma_i = 0.1$, $\alpha = 0.5$, $\mu = 0.3$, $\gamma = 1.8$ in this case. In Fig. 5, we show the two regions of monotone shock structures and oscillatory shock structures for different w_{cd} and v in (a_2,l_z) plane. From Fig. 5, it is observed that the variation of boundary condition between monotone shock structures and oscillatory shock structures with a_2 and l_z is not obvious. Furthermore, compared with the front two cases, the effect of the dissipation due to dust charge fluctuation on this boundary condition is relatively weak. It is clear from Fig. 6 that the oscillatory shock structures' amplitude decrease with a_2 and l_z. Figure 6 tells that when the values of v increase, the oscillatory shock structures' amplitude decreases and the oscillatory shock structures are turn into the monotone shock structures at $v = 39$.

4. $n_d(r) = a_0 + a_1 r + a_2 r^2 + a_3 r^3$

In this case, we also investigate the dependence of characteristics of the shock structures on the parameters such as a_0, a_1, a_2 , a_3, and so on. In order to determine the influence of the parameter a_3, we let $a_{min} = 0.01$, $a_{max} = 1.0$, $k_z = 2$, $k_m = 4$, $z = 2$, $a_0 = 0.6$, $a_1 = -0.4$, $a_2 = 0.4$, $\sigma_i = 0.1$, $\alpha = 0.5$, $\mu = 0.3$, $\gamma = 1.8$. The numerical results illustrate that the variation of boundary condition between the oscillatory shock structures and the monotone shock structures shown in Fig. 7 is the same as that in Fig. 5. We have simulated the behavior of shock structures for different values of a_3, l_z w_{cd} and v in Fig. 8. The results indicate that the variation of the oscillatory shock structures' amplitude with l_z and ω_{cd} is same as in case 3. When the values of a_3 increase, the oscillatory shock structures' amplitude increases insignificantly. It is observed from Fig. 8(d) that as v increase, the oscillatory shock structures' amplitude decreases and the oscillatory shock structures are turn into the monotone shock structures at $v = 22$.

5. Conclusion

In this paper, the nonlinear propagation characteristics of shock structures descried by the KdV Burgers equation are discussed in dusty plasmas with nonadiabatic charge fluctuation, the external magnetized field and dust size distribution. The correlation between the propagation of shock structures and the dust size distribution, the magnetized field and charge variation in four special cases is discussed separately. The research results show the effect of different relevant plasma parameters on propagation characteristics of shock structures and how oscillatory shock structures turns into monotonic shock structures. Numerical results reveal that the transition from oscillatory shock structures to monotonic shock structures occurs when the dissipation effect due to the nonadiabatic charge fluctuation overwhelms the dispersion effect exists. The existence of external magnetized field weaken oscillatory of shock structures, that is, the the shock front exhibits a monotonic transition when the intensity of the magnetic field reaches a certain value. However, the magnetic field has no effect on the shock strength. It is also shown that the characteristics of shock structures are changed significantly by dust size distribution. The number of dust grains increases, the shock strength and the oscillatory of shock front become weak by dissipation. In accordance with the present study, we have more profound understanding of the characteristics of shock structures.

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