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Dynamics of an alcoholism model on complex networks with community structure and voluntary drinking ☆

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HIGHLIGHTS

- A new alcoholism model on scale-free networks with community structure and voluntary drinking is introduced.
- Stability of all the equilibria of our model with two communities for some special cases are investigated.
- Numerical simulations are also conducted to explain and extend our analytic results.

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ABSTRACT

A new alcoholism model on scale-free networks with community structure and voluntary drinking is introduced. Local and global stability of all the equilibria of our model with two communities for some special cases are investigated. Numerical simulations are also conducted to explain and extend our analytic results. Our results show that the effects of voluntary drinking and transmission rate between different community structure on dynamics of our model are important.

1. Introduction

US surveys indicate approximately 90% of college students have consumed alcohol at least once, and more than 40% of college students have engaged in binge drinking [1,2]. The people who drink a large amount of alcohol are likely to exhibit antisocial behavior [3]. About 3.8% deaths and 4.6% disability are caused by alcohol all over the world [4]. Alcoholism has become more severe, especially among college students [5]. Between 1998–2001, the number of deaths and injuries per 100 000 college students increased by 6% [6]. Thus, it is very important to study and prevent alcoholism.

Many researchers studied the binge drinking by establishing mathematical models to find some way to control the drinking behavior [7–10]. Huo and Song [11] investigated a more realistic binge drinking model with two stages, in which the youths with alcohol problems were divided into those who admitted the problem and those who did not admit it. Huo, Wang and Kong [12] proposed an objective functional which considered not only alcohol quitting effects but also the cost of controlling alcohol, and studied optimal control strategies in an alcoholism model with the method of Pontryagin Maximum

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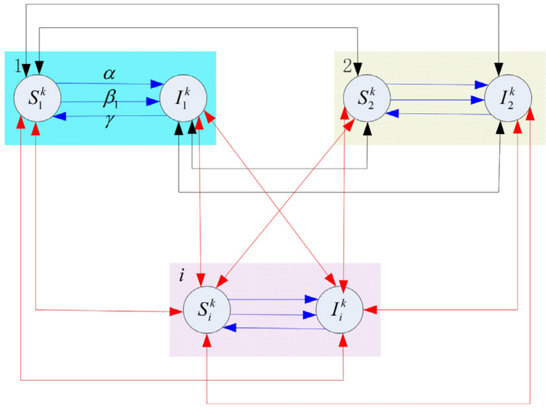


Fig. 1. Illustration of the modeling substrate.

Principle. Huo, Chen and Xiang [13] made a more realistic binge drinking model with time delay. Wang, Huo, Hattaf, et al. [14] formulated an alcohol quitting model in which they considered the impact of distributed time delay between contact and infection process by characterizing dynamic nature of alcoholism behaviors, and considered two different control strategies. Xiang, Song and Xiang [15] dealt with the global property of a drinking model with public health educational campaigns. Huo and Zhang [16] constructed a novel alcoholism model which involved impact of Twitter, and studied complex dynamics of their model. Other models about drinking or epidemic can be found in Refs. [17–23].

Complex networks are often applied to deal with the effect of contact heterogeneity on the disease transmission dynamics. A node denotes a corresponding state and an edge between two nodes represents an interaction or expression the disease transmission. Many people have studied epidemic models or drinking models on complex networks, please see [24–29] and references cited therein.

To study the effect of the community structure on the model with network is of great significance in theory and practical application. The impact of community structure on dynamics of networks causes a great deal of concern recently. Many networks have the property of community structure, such as WWW network [30], protein networks [31,32], genetic networks [33], metabolic network [34], the network financial market [35], the United States congress relationship network [36], scientists cooperation network [37], as well as a large number of social network or biological network [38–40] and so on. Pan, Sun and Jin [41] developed a complex network susceptible–infected–susceptible (SIS) model which captures the transmission between communities by short-time travelers to investigate the impact of demographic factors on disease spread. Zhang and Jin [42] studied an SEAIR epidemic network model with community structure. Other papers about the impact of community structure on dynamics of networks, please see Yan and Fu [43] and references cited therein.

Motivated by the above, we not only incorporate community structure into our model, but also consider the effect of voluntary drinking on our model. Furthermore, we study the stability of all the equilibria of our model with two communities for some special cases. The paper is organized as follows. The new alcoholism model on scale-free networks and some preliminary theorems are constructed in Section 2. In Section 3, dynamics of our alcoholism model with two communities for the special case is studied and some numerical simulations are also presented. Some conclusions and discussions are given in last section.

2. Mathematical model

2.1. System description

In this section, we propose an SIS alcoholism model on complex networks with community and voluntary drinking. Nodes represent individuals and edges (connections) represent their contacts. As shown in Fig. 1, the two-way arrow indicates that individuals can convert at different rates. The blue line indicates the individual intra-community transmission within community. The black line represents the inter-community transmission between the community 1 and the community 2, and the red line indicates the inter-community transmission between community 1, community 2 and the random community i.

Population in community i is divided into two compartments $S_i^k(t)$ and $I_i^k(t)$, where i denotes the ith community, $i = 1, 2, \ldots, m$. $S_i^k(t)$ represents the number of susceptible vertices of degree k at time t on community i; $I_i^k(t)$ represents the number of the problem alcoholic vertices of degree k at time t on community i. $N_i^k(t)$ represents the total number of vertices of degree k at time t on community i, and $N_i^k(t) = S_i^k(t) + I_i^k(t)$, $(k = 1, 2 \ldots, n; i = 1, 2 \ldots m)$, where n is the maximal degree of the complex network. When susceptible individuals contact the problem alcoholic for a certain time, some susceptible individuals can become the problem alcoholic. The problem alcoholic can recover to susceptible individuals.

Furthermore, we assume that individuals among different communities can transfer whatever they want. Then we have the following model,

$$\begin{aligned}
\frac{dS_i^k}{dt} &= -\alpha_i^k S_i^k + \gamma_i^k I_i^k - \sum_{j\neq i}\sigma_{ij}^k S_i^k + \sum_{j\neq i}\sigma_{ji}^k S_j^k - \sum_{j\neq i} k\sigma_{ij}^k S_i^k \beta_{ij}^k \frac{\sum_{k=1}^n kI_j^k}{\sum_{k=1}^n kN_j^k} + \sum_{j\neq i}\gamma_i^k\sigma_{ji}^k I_j^k - kS_i^k\beta_i^k \frac{\sum_{k=1}^n kI_i^k}{\sum_{k=1}^n kN_i^k},\\
\frac{dI_i^k}{dt} &= kS_i^k\beta_i^k \frac{\sum_{k=1}^n kI_i^k}{\sum_{k=1}^n kN_i^k} + \alpha_i^k S_i^k - \gamma_i^k I_i^k - \sum_{j\neq i}\sigma_{ij}^k I_i^k + \sum_{i\neq j}\sigma_{ji}^k I_j^k + \sum_{j\neq i} k\sigma_{ji}^k S_j^k \beta_{ji}^k \frac{\sum_{k=1}^n kI_i^k}{\sum_{k=1}^n kN_i^k} - \sum_{j\neq i}\gamma_j^k\sigma_{ij}^k I_i^k,
\end{aligned} \tag{2.1}$$

where $k = 1, 2 \dots, n$; $i = 1, 2 \dots m$. It is assumed that all the parameters are positive constants. The explanation of model (2.1) are as follows: α_i^k is the ratio that susceptible people in community i with degree k who do not get the influence of alcoholics and transfer to the problem alcoholics voluntarily, so the term $\alpha_i^k S_i^k$ is the number of susceptible people in community i with degree k who transfer to the problem alcoholics voluntarily. γ_i^k is the rate of recovery from the problem alcoholics in community i with degree k. The term $\gamma_i^k I_i^k$ is the number of individuals who recover from the problem alcoholics in community i with degree k. σ_{ij}^k $(i, j = 1, 2 \dots, m)$ is the transmission rate at which nodes with k degree leave from community i to community j per unit time. Since the σ_{ij}^k give conditional probabilities of visiting another community, $0 \le \sigma_{ij}^k \le 1$ for $i \neq j$, we can get that $\sum_{j=1}^m \sigma_{ij}^k = 1, (i = 1, 2, \dots, m)$. σ_{ji}^k $(i, j = 1, 2 \dots, m)$ means the transmission rate at which nodes leave from community j to community i, and $\sum_{i=1}^m \sigma_{ji}^k = 1, (j = 1, 2, \dots, m)$. β_{ij}^k is the contact rate from susceptible individuals in community i to the problem alcoholics in community j with degree k. The term $\sum_{j\neq i}\sigma_{ij}^k S_i^k$ is susceptible individuals with degree k who leave community i to community j. The term $\sum_{j\neq i}\sigma_{ji}^k S_j^k$ is susceptible individuals with degree k who leave community j to community i. The term $\sum_{j\neq i} k\sigma_{ij}^k S_i^k \beta_{ij}^k \frac{\sum_{k=1}^n kI_j^k}{\sum_{k=1}^n kN_j^k}$ is the number of susceptible individuals who leave community i to community j, and become the problem alcoholics when they contact with the problem alcoholics in community j. The term $\sum_{j\neq i}\gamma_i^k\sigma_{ji}^k I_j^k$ is the number of the problem alcoholics who leave community j to community i, and recover susceptible people. β_i^k is the contact rate from susceptible individuals to the problem alcoholics in community i with degree k. The term $kS_i^k\beta_i^k \frac{\sum_{k=1}^n kI_i^k}{\sum_{k=1}^n kN_i^k}$ is the number of susceptible individuals who become the problem alcoholics when they contact with the problem alcoholics in community i. The term $\sum_{j\neq i}\sigma_{ij}^k I_i^k$ is the problem alcoholics with degree k who leave community i to community j. The term $\sum_{i\neq j}\sigma_{ji}^k I_j^k$ is the problem alcoholics with degree k who leave community j to community i. The term $\sum_{j\neq i} k\sigma_{ji}^k S_j^k \beta_{ji}^k \frac{\sum_{k=1}^n kI_i^k}{\sum_{k=1}^n kN_i^k}$ is the number of susceptible individuals who leave community j to community i, and become the problem alcoholics when they contact with the problem alcoholics in community i. The term $\sum_{j\neq i}\gamma_j^k\sigma_{ij}^k I_i^k$ is the number of the problem alcoholics who leave community i to community j, and recover susceptible people in community j. For simplicity, we assume that $\beta_i^k = \beta_j^k = \beta_{ij}^k = \beta_{ji}^k = \frac{\sum_{k=1}^n kN_i^k}{\sum_{k=1}^n k\left(N_1^k + N_2^k + \cdots + N_m^k\right)}$. It means that contact rate is defined as the ratio of all individuals with k in community i account for all the individuals with k. We know

$$\langle k\rangle = \sum_{k=1}^n kp(k) = \sum_{k=1}^n k\frac{N_1^k + N_2^k + \cdots + N_m^k}{N} = \frac{1}{N}\sum_{k=1}^n k\left(N_1^k + N_2^k + \cdots + N_m^k\right), \tag{2.2}$$

where N is a constant, and represents the total number of nodes of the system (2.1), namely $N = \sum_{k=1}^n\sum_{i=1}^m S_i^k(t) + \sum_{k=1}^n\sum_{i=1}^m I_i^k(t)$. So, $\beta_i^k = \beta_j^k = \beta_{ij}^k = \beta_{ji}^k = \frac{\sum_{k=1}^n kN_i^k}{N\langle k\rangle}$. Then, we have the following model

$$\begin{aligned}
\frac{dS_i^k}{dt} &= -\alpha_i^k S_i^k + \gamma_i^k I_i^k - \sum_{j\neq i}\sigma_{ij}^k S_i^k + \sum_{j\neq i}\sigma_{ji}^k S_j^k - \sum_{j\neq i} k\sigma_{ij}^k S_i^k \frac{\sum_{k=1}^n kI_j^k}{N\langle k\rangle} + \sum_{j\neq i}\gamma_i^k\sigma_{ji}^k I_j^k - kS_i^k \frac{\sum_{k=1}^n kI_i^k}{N\langle k\rangle},\\
\frac{dI_i^k}{dt} &= kS_i^k \frac{\sum_{k=1}^n kI_i^k}{N\langle k\rangle} + \alpha_i^k S_i^k - \gamma_i^k I_i^k - \sum_{j\neq i}\sigma_{ij}^k I_i^k + \sum_{i\neq j}\sigma_{ji}^k I_j^k + \sum_{j\neq i} k\sigma_{ji}^k S_j^k \frac{\sum_{k=1}^n kI_i^k}{N\langle k\rangle} - \sum_{j\neq i}\gamma_j^k\sigma_{ij}^k I_i^k.
\end{aligned} \tag{2.3}$$

Define $\theta_i = \frac{\sum_k kI_i^k}{N\langle k\rangle}$ and $\theta_j = \frac{\sum_{k=1}^n kI_j^k}{N\langle k\rangle}$, we have

$$\begin{aligned}
\frac{dS_i^k}{dt} &= -\alpha_i^k S_i^k + \gamma_i^k I_i^k - \sum_{j\neq i}\sigma_{ij}^k S_i^k + \sum_{j\neq i}\sigma_{ji}^k S_j^k - \sum_{j\neq i} k\sigma_{ij}^k S_i^k \theta_j + \sum_{j\neq i}\gamma_i^k\sigma_{ji}^k I_j^k - kS_i^k\theta_i,\\
\frac{dI_i^k}{dt} &= kS_i^k\theta_i + \alpha_i^k S_i^k - \gamma_i^k I_i^k - \sum_{j\neq i}\sigma_{ij}^k I_i^k + \sum_{i\neq j}\sigma_{ji}^k I_j^k + \sum_{j\neq i} k\sigma_{ji}^k S_j^k\theta_i - \sum_{j\neq i}\gamma_j^k\sigma_{ij}^k I_i^k.
\end{aligned} \tag{2.4}$$

2.2. Positivity and boundedness of solutions

To show that the model (2.4) is meaningful, we will prove that all solutions of the system (2.4) with initial conditions $S_i^k(0) > 0, I_i^k(0) > 0$ are positive and bounded for $t \geq 0$. Thus, we have the following theorems

Theorem 2.2.1. *Let $\left(S_i^k(t), I_i^k(t)\right)$ be the solution of system (2.4) with $S_i^k(0) > 0, I_i^k(0) > 0$, then for $k = 1, 2, \ldots, n$, we have $S_i^k(t) > 0$ and $I_i^k(t) > 0$ for all $t > 0$.*

Proof. If the conclusion does not hold, then at least one of $S_i^k(t)$ and $I_i^k(t)$ are not always positive. Without loss of generality, we can assume that $S_i^k(t)$ is not always positive when $t > 0$. Notice that $S_i^k(0) > 0$ and $I_i^k(0) > 0$. By the first equation of the system (2.4) and the continuity of $S_i^k(t)$ and $I_i^k(t)$, there exists a first time $t_1 > 0$ such that $S_i^k(t_1) = 0$, $\frac{dS_i^k(t_1)}{dt} < 0$ and $S_i^k(t) \geq 0, I_i^k(t) \geq 0$ for $t \in (0, t_1)$. Combining with the first equation of the system (2.4), we have $\frac{dS_i^k(t_1)}{dt} > 0$, which is a contradiction with $\frac{dS_i^k(t_1)}{dt} < 0$. Thus, the solutions $\left(S_i^k(t), I_i^k(t)\right)$ with $S_i^k(0) > 0, I_i^k(0) > 0$ remain positive for all $t > 0$. This completes the proof.

Theorem 2.2.2. *$\Omega = \{(S_i^k, I_i^k) \in R_+^{2n} | 0 < S_i^k, I_i^k < 1, k = 1, 2, \ldots, n, i = 1, 2, \ldots, m\}$ is a positively invariant set of the system (2.4).*

Proof. Adding all the equations of system (2.4), we have, $\left(\sum_{i=1}^m S_i^k(t) + \sum_{i=1}^m I_i^k(t)\right)' = 0$, and, $\sum_{i=1}^m S_i^k(t) + \sum_{i=1}^m I_i^k(t) = 1$. By Theorem 2.2.1, it is easy to know $\Omega = \{(S_i^k, I_i^k) \in R_+^{2n} | 0 < S_i^k, I_i^k < 1, k = 1, 2, \ldots, n, i = 1, 2, \ldots, m\}$ is a positively invariant set of the system (2.4). This completes the proof.

3. Mathematical model with two communities

3.1. The basic reproduction number

For simplicity, we assume that $m = 2$, and $\alpha_1^k = \alpha_1, \alpha_2^k = \alpha_2, \gamma_1^k = \gamma_1, \sigma_{12}^k = \sigma_{12}, \sigma_{21}^k = \sigma_{21}$. We have following model.

$$
\begin{aligned}
\frac{dS_1^k}{dt} &= -\alpha_1 S_1^k + \gamma_1 I_1^k - \sigma_{12} S_1^k + \sigma_{21} S_2^k - k\sigma_{12} S_1^k \theta_2 + \gamma_1 \sigma_{21} I_2^k - k S_1^k \theta_1, \\
\frac{dS_2^k}{dt} &= -\alpha_2 S_2^k + \gamma_2 I_2^k - \sigma_{21} S_2^k + \sigma_{12} S_1^k - k\sigma_{21} S_2^k \theta_1 + \gamma_2 \sigma_{12} I_1^k - k S_2 \theta_2, \\
\frac{dI_1^k}{dt} &= k S_1^k \theta_1 + \alpha_1 S_1^k - \gamma_1 I_1^k - \sigma_{12} I_1^k + \sigma_{21} I_2^k + k\sigma_{21} S_2^k \theta_1 - \gamma_2 \sigma_{12} I_1^k, \\
\frac{dI_2^k}{dt} &= k S_2^k \theta_2 + \alpha_2 S_2^k - \gamma_2 I_2^k - \sigma_{21} I_2^k + \sigma_{12} I_1^k + k\sigma_{12} S_1^k \theta_2 - \gamma_1 \sigma_{21} I_2^k.
\end{aligned}
\tag{3.1}
$$

It is easy to know that $S_1^k + S_2^k + I_1^k + I_2^k = 1$. Then system (3.1) can be written as

$$
\begin{aligned}
\frac{dS_2^k}{dt} &= -\alpha_2 S_2^k + \gamma_2 I_2^k - \sigma_{21} S_2^k + \sigma_{12}\left(1 - S_2^k - I_2^k - I_1^k\right) - k\sigma_{21} S_2^k \theta_1 + \gamma_2 \sigma_{12} I_1^k - k S_2^k \theta_2, \\
\frac{dI_1^k}{dt} &= k\left(1 - S_2^k - I_2^k - I_1^k\right)\theta_1 + \alpha_1\left(1 - S_2^k - I_2^k - I_1^k\right) - \gamma_1 I_1^k - \sigma_{12} I_1^k + \sigma_{21} I_2^k + k\sigma_{21} S_2^k \theta_1 - \gamma_2 \sigma_{12} I_1^k, \\
\frac{dI_2^k}{dt} &= k S_2^k \theta_2 + \alpha_2 S_2^k - \gamma_2 I_2^k - \sigma_{21} I_2^k + \sigma_{12} I_1^k + k\sigma_{12}\left(1 - S_2^k - I_2^k - I_1^k\right)\theta_2 - \gamma_1 \sigma_{21} I_2^k,
\end{aligned}
\tag{3.2}
$$

where $\theta_i = \frac{\sum_k k I_i^k}{N\langle k\rangle}, i = 1, 2$.

When $\alpha_2 = 0$ and $\sigma_{21} = 0$, system (3.2) has the alcohol free equilibrium $E_0 = (\vec{S_2^k}, \vec{0}, \vec{0}) = (1, \ldots, 1, 0, \ldots, 0, 0, \ldots, 0)_{3n}$ Using the next generation matrix [44], we have

$$
\frac{dy}{dt} = \mathscr{F} - \mathscr{V},
$$

where

$$
y = (I_1^1, I_1^2, \ldots, I_1^n, I_2^1, I_2^2, \ldots, I_2^n, \ldots, S_2^1, S_2^2, \ldots, S_2^n)^T.
$$

$$\mathcal{F}(y)=\begin{pmatrix} 1\left(1-S_2^1-I_2^1-I_1^1\right)\theta_1+\alpha_1\left(1-S_2^1-I_2^1-I_1^1\right)\\ 2\left(1-S_2^2-I_2^2-I_1^2\right)\theta_1+\alpha_1\left(1-S_2^2-I_2^2-I_1^2\right)\\ \vdots\\ n\left(1-S_2^n-I_2^n-I_1^n\right)\theta_1+\alpha_1\left(1-S_2^n-I_2^n-I_1^n\right)\\ 1S_2^1\theta_2\\ 2S_2^2\theta_2\\ \vdots\\ nS_2^n\theta_2\\ 0\\ 0\\ \vdots\\ 0 \end{pmatrix}_{3n},$$

and

$$\mathcal{V}(y)=\begin{pmatrix} \gamma_1 I_1^1+\sigma_{12}I_1^1+\gamma_2\sigma_{12}I_1^1\\ \gamma_1 I_1^2+\sigma_{12}I_1^2+\gamma_2\sigma_{12}I_1^2\\ \vdots\\ \gamma_1 I_1^n+\sigma_{12}I_1^n+\gamma_2\sigma_{12}I_1^n\\ \gamma_2 I_2^1-\sigma_{12}I_1^1-1\sigma_{12}S_1^1\theta_2\\ \gamma_2 I_2^2-\sigma_{12}I_1^2-2\sigma_{12}S_1^2\theta_2\\ \vdots\\ \gamma_2 I_2^n-\sigma_{12}I_1^n-n\sigma_{12}S_1^n\theta_2\\ -\gamma_2 I_2^1-\sigma_{12}\left(1-S_2^1-I_2^1-I_1^1\right)-\gamma_2\sigma_{12}I_1^1+1S_2^1\theta_2\\ -\gamma_2 I_2^2-\sigma_{12}\left(1-S_2^2-I_2^2-I_1^2\right)-\gamma_2\sigma_{12}I_1^2+2S_2^2\theta_2\\ \vdots\\ -\gamma_2 I_2^n-\sigma_{12}\left(1-S_2^n-I_2^n-I_1^n\right)-\gamma_2\sigma_{12}I_1^n+nS_2^n\theta_2 \end{pmatrix}_{3n}.$$

The jacobian matrix of the $\mathcal{F}(y)$ and $\mathcal{V}(y)$ in the alcohol free equilibrium E_0 can be expressed respectively,

$$F=D\mathcal{F}(E_0)=\begin{pmatrix} A_{11} & A_{12} & A_{13}\\ A_{21} & A_{22} & A_{23}\\ A_{31} & A_{32} & A_{33}\end{pmatrix}$$

$$V=D\mathcal{V}(E_0)=\begin{pmatrix} B_{11} & B_{12} & B_{13}\\ B_{21} & B_{22} & B_{23}\\ B_{31} & B_{32} & B_{33}\end{pmatrix}$$

where

$$A_{11}=A_{12}=A_{13}=\begin{pmatrix} -\alpha_1 & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & -\alpha_n\end{pmatrix}$$

$$A_{21}=A_{23}=A_{31}=A_{32}=A_{33}=\begin{pmatrix} 0 & \cdots & 0\\ 0 & \ddots & 0\\ 0 & \cdots & 0\end{pmatrix}, A_{22}=\begin{pmatrix} \frac{1}{\langle k\rangle N} & \frac{2}{\langle k\rangle N} & \cdots & \frac{n}{\langle k\rangle N}\\ 2\frac{1}{\langle k\rangle N} & 2\frac{2}{\langle k\rangle N} & \cdots & 2\frac{n}{\langle k\rangle N}\\ \vdots & \vdots & \cdots & \vdots\\ n\frac{1}{\langle k\rangle N} & n\frac{2}{\langle k\rangle N} & \cdots & n\frac{n}{\langle k\rangle N}\end{pmatrix}$$

and

$$B_{11}=\begin{pmatrix}h_1 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & h_n\end{pmatrix}, B_{22}=\begin{pmatrix}g_1 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & g_n\end{pmatrix},$$

$$B_{12}=\begin{pmatrix}0 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & 0\end{pmatrix}, B_{21}=\begin{pmatrix}-\sigma_{12}^1 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & -\sigma_{12}^n\end{pmatrix},$$

where $h_k=\gamma_1+\sigma_{12}+\gamma_2\sigma_{12}$, $g_k=\gamma_2+\gamma_1\sigma_{12}$. Then,

$$V^{-1}=\begin{pmatrix}C_{11} & 0\\ C_{21} & C_{22}\end{pmatrix},$$

where,

$$C_{11}=\begin{pmatrix}\frac{1}{\gamma_1+\sigma_{12}+\gamma_2\sigma_{12}} & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & \frac{1}{\gamma_1+\sigma_{12}+\gamma_2\sigma_{12}}\end{pmatrix} C_{22}=\begin{pmatrix}\frac{1}{\gamma_2} & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & \frac{1}{\gamma_2}\end{pmatrix},$$

$$C_{21}=\begin{pmatrix}\frac{\sigma_{12}}{\gamma_2\left(\gamma_1+\sigma_{12}+\gamma_2\sigma_{12}\right)} & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & \frac{\sigma_{12}}{\gamma_2\left(\gamma_1+\sigma_{12}+\gamma_2\sigma_{12}\right)}\end{pmatrix}.$$

We have

$$R_0=\rho(FV^{-1})=\frac{-\alpha_1\sigma_{12}\gamma_2\left\langle k\right\rangle N+\gamma_2\left[\alpha_1\gamma_2(\gamma_1+\sigma_{12}+\gamma_2\sigma_{12})^2+\alpha_1\sigma_{12}\right]}{\left\langle k\right\rangle N\left[\alpha_1\gamma_2(\gamma_1+\sigma_{12}+\gamma_2\sigma_{12})^2+\alpha_1\sigma_{12}\right]}.$$

Remark 3.1.1. When $\alpha_2\neq 0$ and $\sigma_{21}\neq 0$, system (3.2) does not have the alcohol free equilibrium.

According to the Theorem 2 of [44], we obtain the local stability of the alcohol free equilibrium.

Theorem 3.1.1. *For the model (3.2), if $\alpha_2=0$, $\sigma_{21}=0$ and $R_0<1$, then the alcohol free equilibrium E_0 is locally asymptotically stable.*

3.2. Global stability of the alcohol free equilibrium

Using the comparison principle, we can further prove the global stability of alcohol free equilibrium when $\alpha_2=0$, $\sigma_{21}=0$ and $R_0<1$.

Theorem 3.2.1. *For the model (3.2), if $\alpha_2=0$, $\sigma_{21}=0$ and $R_0<1$, then the alcohol free equilibrium E_0 is globally asymptotically stable.*

Proof. It follows from (3.2) that

$$\begin{aligned}
\frac{dS_2^k}{dt}&\leq -\alpha_2S_2^k+\gamma_2I_2^k-\sigma_{21}S_2^k+\sigma_{12}-k\sigma_{21}S_2^k\theta_1+\gamma_2\sigma_{12}I_1^k-kS_2^k\theta_2,\\
\frac{dI_1^k}{dt}&\leq k\theta_1+\alpha_1-\gamma_1I_1^k-\sigma_{12}I_1^k+\sigma_{21}I_2^k+k\sigma_{21}S_2^k\theta_1-\gamma_2\sigma_{12}I_1^k,\\
\frac{dI_2^k}{dt}&\leq kS_2^k\theta_2+\alpha_2S_2^k-\gamma_2I_2^k-\sigma_{21}I_2^k+\sigma_{12}I_1^k+k\sigma_{12}\theta_2-\gamma_1\sigma_{21}I_2^k.
\end{aligned}\tag{3.3}$$

Consider the auxiliary system

$$\begin{aligned}
\frac{dS_2^k}{dt} &= -\alpha_2 S_2^k + \gamma_2 I_2^k - \sigma_{21} S_2^k + \sigma_{12} - k\sigma_{21} S_2^k \theta_1 + \gamma_2 \sigma_{12}^k I_1^k - k S_2^k \theta_2, \\
\frac{dI_1^k}{dt} &= k\theta_1 + \alpha_1 - \gamma_1 I_1^k - \sigma_{12} I_1^k + \sigma_{21} I_2^k + k\sigma_{21} S_2^k \theta_1 - \gamma_2 \sigma_{12} I_1^k, \\
\frac{dI_2^k}{dt} &= k S_2^k \theta_2 + \alpha_2 S_2^k - \gamma_2 I_2^k - \sigma_{21} I_2^k + \sigma_{12} I_1^k + k\sigma_{12}\theta_2 - \gamma_1 \sigma_{21} I_2^k.
\end{aligned} \tag{3.4}$$

Which can be expressed concisely as

$$\frac{dy}{dt} = (F - V)y.$$

Since $R_0 < 1$, then the eigenvalues of the matrix $(F - V)$ all have negative real parts, then every non-negative solution of (3.3) tends to 0 as $t \to \infty$. According to the comparison principle [45], we know that every non-negative solution of (3.2) also tend to 0 as $t \to \infty$. So the disease-free equilibrium E_0 is globally asymptotically stable and the proof is completed. □

Remark 3.2.1.
Let $S_2^k = 1 - S_1^k - I_1^k - I_2^k$, then system (3.1) can be written as

$$\begin{aligned}
\frac{dS_1^k}{dt} &= -\alpha_1 S_1^k + \gamma_1 I_1^k - \sigma_{12} S_1^k + \sigma_{21}(1 - S_1^k - I_1^k - I_2^k) - k\sigma_{12} S_1^k \theta_2 + \gamma_1 \sigma_{21} I_2^k - k S_1^k \theta_1, \\
\frac{dI_1^k}{dt} &= k S_1^k \theta_1 + \alpha_1 S_1^k - \gamma_1 I_1^k - \sigma_{12} I_1^k + \sigma_{21} I_2^k + k\sigma_{21}(1 - S_1^k - I_1^k - I_2^k)\theta_1 - \gamma_2 \sigma_{12} I_1^k, \\
\frac{dI_2^k}{dt} &= k(1 - S_1^k - I_1^k - I_2^k)\theta_2 + \alpha_2(1 - S_1^k - I_1^k - I_2^k) - \gamma_2 I_2^k - \sigma_{21} I_2^k + \sigma_{12} I_1^k + k\sigma_{12} S_1^k \theta_2 - \gamma_1 \sigma_{21} I_2^k.
\end{aligned} \tag{3.5}$$

For system (3.5), when $\alpha_1 = 0$ and $\sigma_{12} = 0$, similarly, we can obtain the following basic reproduction number R_0'

$$R_0' = \frac{-\alpha_2 \sigma_{21} \gamma_1 \langle k \rangle N + \gamma_1 \left[\alpha_2 \gamma_1 (\gamma_2 + \sigma_{21} + \gamma_1 \sigma_{21})^2 + \alpha_2 \sigma_{21}\right]}{\langle k \rangle N \left[\alpha_2 \gamma_1 (\gamma_2 + \sigma_{21} + \gamma_1 \sigma_{21})^2 + \alpha_2 \sigma_{21}\right]}.$$

and we can obtain the following theorems.

Theorem 3.2.2. *For the model (3.5), if $\alpha_1 = 0$, $\sigma_{12} = 0$ and $R_0' < 1$, then the alcohol free equilibrium $E_0' = (\vec{S_1^k}, \vec{0}, \vec{0})$ is locally asymptotically stable.*

Theorem 3.2.3. *For the model (3.5), if $\alpha_1 = 0$, $\sigma_{12} = 0$ and $R_0' < 1$, then the alcohol free equilibrium E_0' is globally asymptotically stable.*

When $\alpha_1 \neq 0$ and $\sigma_{12} \neq 0$, we also know that system (3.5) does not have the alcohol free equilibrium.

3.3. The existence of equilibria

First, we will discuss the existence of equilibria of system (3.2).

Theorem 3.3.1. *If $\sigma_{12} = 0$ and $R_1 = \frac{(\alpha_1+\gamma_1)^2}{\gamma_1} \frac{\langle k \rangle}{\langle k^2 \rangle} > 1$, where $\langle k^2 \rangle = \sum_k k^2 p(k)$, then system (3.2) exists a unique alcohol-present-in-community 1-only-equilibrium $\hat{E}_1 \left(\vec{0}, \hat{I}_1^k, \vec{0}\right)$.*

Proof. By (3.2), the alcohol-present-in-community 1-only-equilibrium $\hat{E}_1 \left(\vec{0}, \hat{I}_1^k, \vec{0}\right)$ satisfies the following equation

$$k\left(1 - \hat{I}_1^k\right)\theta_1 + \alpha_1\left(1 - \hat{I}_1^k\right) - \gamma_1 \hat{I}_1^k = 0, \tag{3.6}$$

then,

$$\hat{I}_1^k = \frac{k\theta_1 + \alpha_1}{k\theta_1 + \alpha_1 + \gamma_1}.$$

According to the expressions of θ_1, we have the following equation about θ_1,

$$\theta_1 = f_1(\theta_1) = \frac{1}{N\langle k \rangle} \sum_{k=1}^{n} k \hat{I}_1^k = \frac{1}{N\langle k \rangle} \sum_{k=1}^{n} k \left(\frac{k\theta_1 + \alpha_1}{k\theta_1 + \alpha_1 + \gamma_1}\right).$$

Let $F_1(\theta_1) = \theta_1 - f_1(\theta_1)$, we have

$$F_1{}'(\theta_1) = 1 - \frac{1}{N\langle k\rangle}\sum_{k=1}^{n} k\frac{k(k\theta_1+\alpha_1+\gamma_1) - k(k\theta_1+\alpha_1)}{(k\theta_1+\alpha_1+\gamma_1)^2}$$
$$= 1 - \frac{1}{N\langle k\rangle}\sum_{k=1}^{n} k^2\frac{\gamma_1}{(k\theta_1+\alpha_1+\gamma_1)^2},$$

then,

$$F_1{}'(0) = 1 - \frac{1}{N\langle k\rangle}\sum_{k=1}^{n} k^2\frac{\gamma_1}{(\alpha_1+\gamma_1)^2} = 1 - \frac{\gamma_1}{(\alpha_1+\gamma_1)^2}\frac{\langle k^2\rangle}{\langle k\rangle} > 0.$$

Similarly, we obtain

$$F_1{}''(\theta_1) = -\frac{1}{N\langle k\rangle}\sum_{k=1}^{N} k^3\left(\frac{\gamma_1}{(k\theta_1+\alpha_1+\gamma_1)^3}\right) < 0.$$

It is easy to know that $F_1(0) < 0$ and $F_1(1) > 0$, then $F_1(\theta_1) = 0$ has a unique root $\theta_1^* > 0$. So there exists a unique alcohol-present-in-community 1-only-equilibrium $\hat{E}_1\left(\vec{0}, \hat{I}_1^k, \vec{0}\right)$. The proof is completed. □

By the symmetry of system (3.1), we also have the following theorem.

Theorem 3.3.2. *If* $\sigma_{21} = 0$ *and* $R_2 = \frac{(\alpha_2+\gamma_2)^2}{\gamma_2}\frac{\langle k\rangle}{\langle k^2\rangle} > 1$, *where* $\langle k^2\rangle = \sum_k k^2 p(k)$, *then system* (3.2) *exists a unique alcohol-present-in-community* 2*-only-equilibrium* $\hat{E}_2\left(\hat{S}_2^k, \vec{0}, \hat{I}_2^k\right)$.

We know that the alcohol present equilibrium $E^*\left(S_2^*, I_1^*, I_2^*\right)$ of system (3.2) must satisfy the following equations

$$\begin{aligned} &A_1S_2^k - B_1I_1^k - C_1I_2^k + D_1 = 0,\\ &A_2S_2^k - B_2I_1^k + C_2I_2^k + D_2 = 0,\\ &A_3S_2^k + B_3I_1^k + C_3I_2^k + D_3 = 0,\end{aligned} \tag{3.7}$$

where $A_1 = -k\theta_1 - \alpha_1 + k\sigma_{21}\theta_1$, $B_1 = k\theta_1 + \alpha_1 + \gamma_1 + \sigma_{12} + \gamma_2\sigma_{12}$, $C_1 = k\theta_1 + \alpha_1 - \sigma_{12}$, $D_1 = k\theta_1 + \alpha_1$, $A_2 = -(\alpha_2 + \sigma_{21} + \sigma_{12} + k\sigma_{21}\theta_1 + k\theta_2)$, $B_2 = \sigma_{12} - \gamma_2\sigma_{12}$, $C_2 = \gamma_2 - \sigma_{12}$, $D_2 = \sigma_{12}$, $A_3 = \alpha_2 + k\theta_2 - k\sigma_{12}\theta_2$, $B_3 = \sigma_{12} - k\sigma_{12}\theta_2$, $C_3 = -\gamma_2 - \sigma_{21} - k\sigma_{12}\theta_2 - \gamma_1\sigma_{21}$ and $D_3 = k\sigma_{12}\theta_2$.

So,

$$\begin{aligned} S_2^* = &\frac{B_1\left[(A_2D_1 - A_1D_2)(A_3C_1 + A_1C_3) - (A_2C_1 + A_1C_2)(A_3D_1 - A_1D_3)\right]}{A_1\left[(A_2B_1 - B_2A_1)(A_3C_1 + A_1C_3) - (A_3B_1 - B_3A_1)(A_2C_1 + A_1C_2)\right]}\\ &- \frac{C_1\left[(A_2B_1 - A_1B_2)(A_3D_1 - A_1D_3) - (A_2D_1 - A_1D_2)(A_3B_1 - A_1B_3)\right]}{A_1\left[(A_2B_1 - B_2A_1)(A_3C_1 + A_1C_3) - (A_3B_1 - B_3A_1)(A_2C_1 + A_1C_2)\right]}\\ &- \frac{D_1\left[(A_2B_1 - B_2A_1)(A_3C_1 + A_1C_3) - (A_3B_1 - B_3A_1)(A_2C_1 + A_1C_2)\right]}{A_1\left[(A_2B_1 - B_2A_1)(A_3C_1 + A_1C_3) - (A_3B_1 - B_3A_1)(A_2C_1 + A_1C_2)\right]},\end{aligned} \tag{3.8}$$

$$I_1^* = \frac{(A_2D_1 - A_1D_2)(A_3C_1 + A_1C_3) - (A_2C_1 + A_1C_2)(A_3D_1 - A_1D_3)}{(A_2B_1 - B_2A_1)(A_3C_1 + A_1C_3) - (A_3B_1 - B_3A_1)(A_2C_1 + A_1C_2)}, \tag{3.9}$$

$$I_2^* = \frac{(A_2B_1 - A_1B_2)(A_3D_1 - A_1D_3) - (A_2D_1 - A_1D_2)(A_3B_1 - A_1B_3)}{(A_2B_1 - B_2A_1)(A_3C_1 + A_1C_3) - (A_3B_1 - B_3A_1)(A_2C_1 + A_1C_2)}. \tag{3.10}$$

Since the existence of the alcohol present equilibrium $E^*\left(S_2^*, I_1^*, I_2^*\right)$ of system (3.2) and global stability of boundary and positive equilibria are very difficult to prove, we only present the numerical results directly in next section.

3.4. Numerical simulations

In this section, we will present some numerical simulations to illustrate and extend our theoretical results. Our simulations take the scale-free networks with degree distribution is $P(k) = 2k^{-2}$.

Let $\sigma_{12} = 0.6, \sigma_{21} = 0, \alpha_1 = 0.4, \alpha_2 = 0, \gamma_1 = 1, \gamma_2 = 0.8$, by Fig. 2, we know that alcohol-free equilibrium E_0 of (3.2) is globally asymptotically stable when $R_0 < 1$.

Let $\sigma_{12} = 0, \alpha_1 = 0.2, \alpha_2 = 0.4, \gamma_1 = \gamma_2 = 0.8, \sigma_{21} = 0.6, k = 10, N = 1000\,000$. By Fig. 3(a), we know that the unique alcohol-present-in-community 1-only-equilibrium $\hat{E}_1\left(\vec{0}, \hat{I}_1^k, \vec{0}\right)$ is global asymptotically stable. Let $\sigma_{21} = 0, \alpha_1 = 0.2, \alpha_2 =$

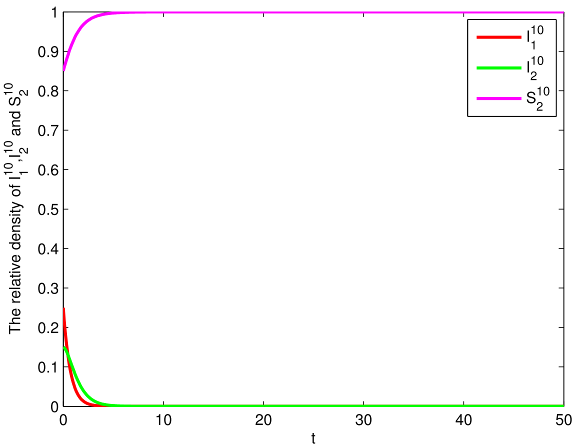
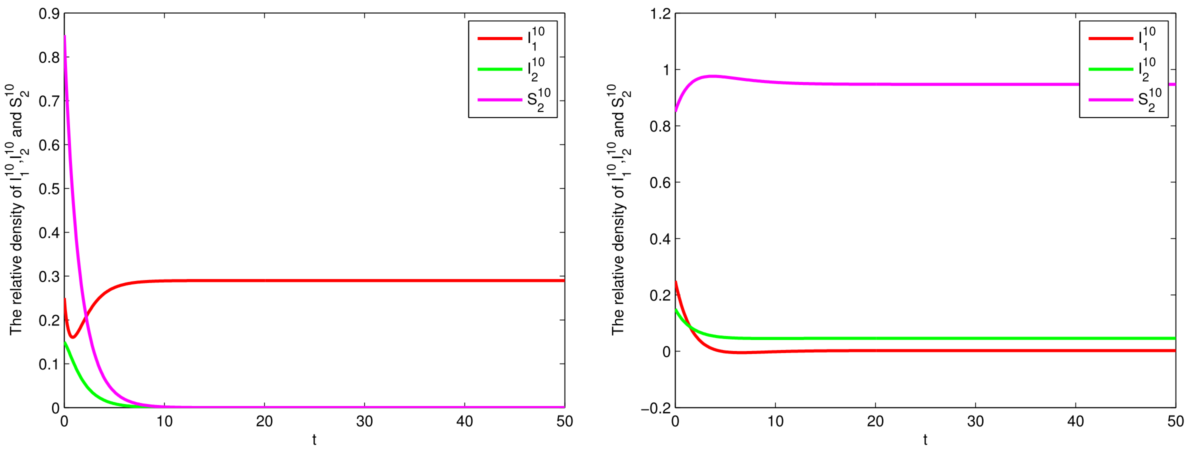


Fig. 2. The time series and orbits of system (3.2) with $k = 10$ when $R_0 < 1$.



(a) (b)

Fig. 3. The time series and orbits of system (3.2) with $k = 10$.

0.4, $\gamma_1 = \gamma_2 = 0.8$, $\sigma_{21} = 0.6$, $k = 10$, $N = 1000\,000$. By Fig. 3(b), we know that the unique alcohol-present-in-community 2-only-equilibrium $\hat{E}_2\left(\hat{S}_2^k, \vec{0}, \hat{I}_2^k\right)$ is global asymptotically stable.

To verify the existence and stability of alcohol present equilibrium, Let $\alpha_2 = 0.2$, $\sigma_{21} = 0.6$, $\alpha_1 = 0.4$, $\gamma_1 = 1$, $\gamma_2 = 0.8$, $\sigma_{12} = 0.8$, $k = 10$, $N = 1000\,000$, by Fig. 4, we know that the alcohol present equilibrium of the system (3.2) is globally asymptotically stable when $\alpha_1 \neq 0$ and $\alpha_2 \neq 0$.

In order to study the effect of transmission rates σ_{12} and σ_{21} on the system (3.2), we assume that $\sigma_{12} = 0.8$ and $\sigma_{21} = 0.2$, that is to say, the transmission rate at which people leave from community 1 to community 2 is greater than that of from community 2 to community 1, and let $\alpha_2 = 0.4$, $\alpha_1 = 0.2$, $\gamma_1 = 0.8$, $\gamma_2 = 0.6$, $k = 10$, $N = 1000\,000$, we get Fig. 5(a). On the contrary, let $\sigma_{21} = 0.8$ and $\sigma_{12} = 0.2$, we get Fig. 5(b). From Fig. 5(a) and (b), we know that the transmission rate of people moving to another community is bigger, the risk of alcoholism in another community is greater.

In Fig. 6, the densities of alcoholics with different degree are presented. Let $\sigma_{12} = 0.6$, $\alpha_1 = 0.2$, $\alpha_2 = 0.4$, $\gamma_1 = \gamma_2 = 0.8$, $\sigma_{21} = 0.6$, $N = 1000\,000$. It is easy to know that the larger degree leads to larger value of the alcoholism level.

4. Conclusions and discussions

A new alcoholism model on scale-free networks with community structure and voluntary drinking is introduced. Local and global stability of the alcohol free equilibrium of our model with two communities when $\alpha_1^k = \alpha_1$, $\alpha_2^k = \alpha_2$, $\gamma_1^k = \gamma_1$, $\sigma_{12}^k = \sigma_{12}$, $\sigma_{21}^k = \sigma_{21}$ are investigated. Furthermore, we study the existence of all the equilibria of our model for some special cases, numerical simulations are also conducted to explain and extend our analytic results.

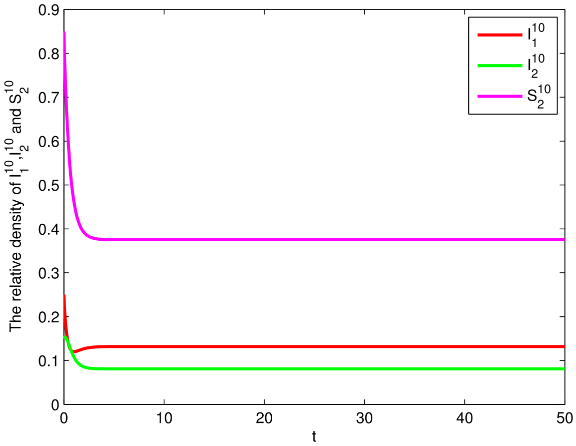
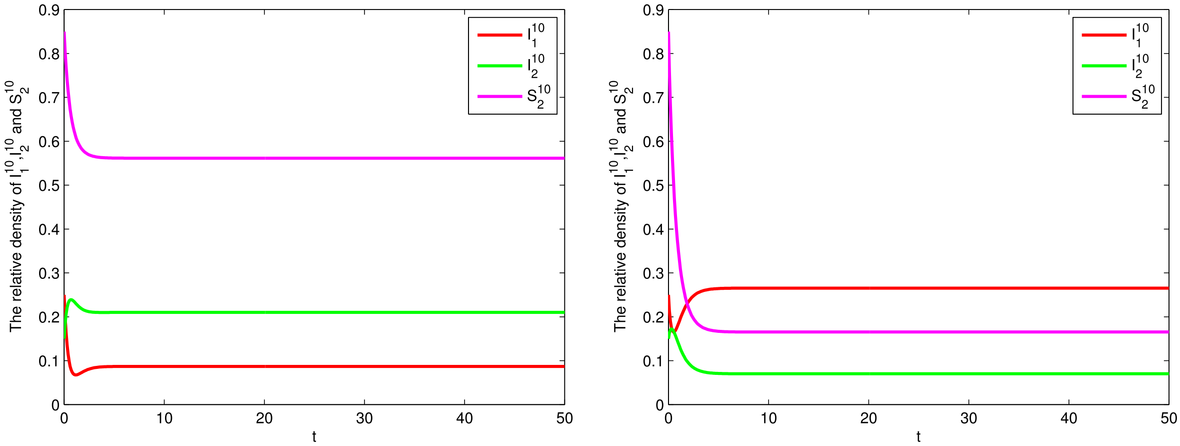
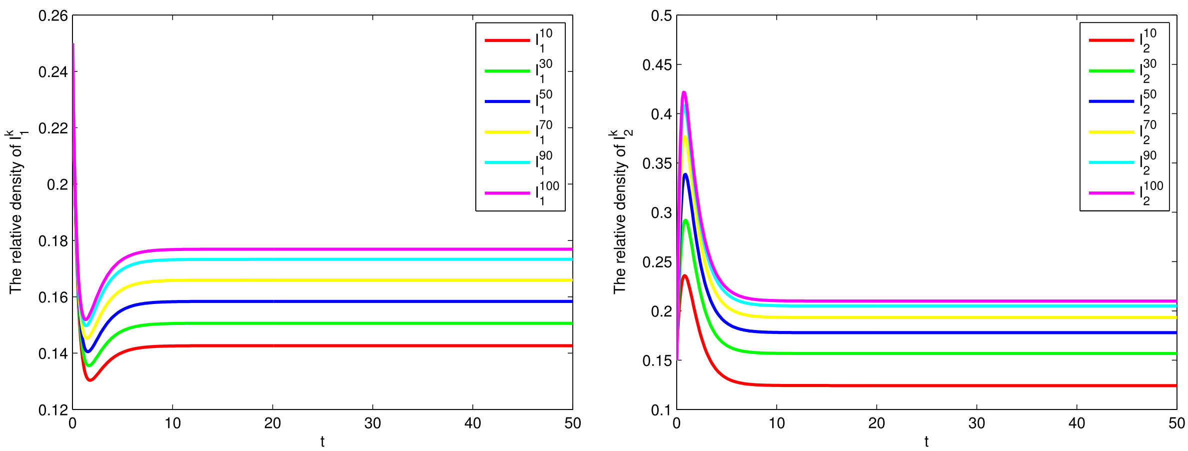


Fig. 4. The time series and orbits of system (3.2) with $k = 10$.



(a) (b)

Fig. 5. The time series and orbits of system (3.2) with different parameter values σ_{21} and σ_{12}.



(a) (b)

Fig. 6. The time series and orbits of system (3.2) with $k = 10, 30, 50, 70, 90, 100$.

Since some people drink for their own reasons, we know that when $\alpha_2 \neq 0$ and $\sigma_{21} \neq 0$, system (3.2) does not have the alcohol free equilibrium. When $\alpha_1 \neq 0$ and $\sigma_{12} \neq 0$, we also know that system (3.5) does not have the alcohol free equilibrium. These results show that alcoholism as a social epidemic disease, is different from common epidemic diseases. Furthermore, from our numerical simulations, we know that the transmission rate of people moving to another community is bigger. The risk of alcoholism in another community is greater. These mean that the effect of transmission rates σ_{12} and σ_{21} on the system (3.2) is very important.

It is very interesting to study our alcoholism model with n communities for more general cases. We leave these work in the future.

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