Journal of Electron Spectroscopy and Related Phenomena 233 (2019) 83–89

Contents lists available at ScienceDirect

Journal of Electron Spectroscopy and Related Phenomena

journal homepage: www.elsevier.com/locate/elspec

Energy levels and transition probabilities of quartet Rydberg series in boron-like silicon

Yan Sun[a,*], Feng Hu[a], CuiCui Sang[b], Dong Dong Liu[a], Mao Fei Mei[a], Bing Cong Gou[c]

[a] School of Mathematic and Physical Science, Xuzhou University of Technology, Xuzhou 221111, China
[b] College of Science, Lanzhou University of Technology, Lanzhou 730050, China
[c] School of Physics, Beijing Institute of Technology, Beijing 100081, China

ARTICLE INFO

PACS:
31.15.Ar
31.10.+z
32.30.-r
32.70.Cs

Keywords:
Energy level
Line strength
Oscillator strength
Transition probability

ABSTRACT

Excited-state energy levels for quartet states $1s^22l2l'nl''$ (n = 2–7; $l=s, p$; $l'=s, p$; $l''=s, p, d, f$) pertaining to $^4L^{e,o}$ (m) ($L=S, P, D$; m = 1–5) Rydberg series in Si^{9+} ion are studied, using multi-configuration Rayleigh-Ritz variation method. Corrections for the relativistic effects, mass polarization effect, and quantum electrodynamics (QED) effect are taken into account to obtain the accurate relativistic energies. Transition probabilities, line strengths, oscillator strengths, and wavelengths of electric-dipole transitions between these quartet states are systematically calculated and compared with available reference data. These calculated data could be used for the identification of spectral lines arising from the coronal atmospheres and experimental study in future work.

1. Introduction

As the important element in the Universe, silicon is found in the process of stellar nucleosynthesis and supernova explosions [1]. Photoemission lines, in the extreme ultraviolet (EUV) and soft X-ray range, arising from various highly stripped silicon ions play a prominent role in the spectra of astrophysical plasmas from a wide variety of astronomical sources such as the solar corona [2,3], stellar atmospheres [4,5], and supernova remnants [6]. The information from these photoemission lines are often used to diagnose temperature, density [7] and element abundance [8] in astronomical object. During the past decades, considerable progress has been made in this field.

Early in 1970s, Träbert et al. [9–12] measured the lifetimes and identified the spectral lines from variously ionized silicon in the ranges of vacuum ultraviolet and x-ray, using the beam-foil technique. Later, Faenov [13] observed and measured the wavelengths of satellites due to 41 transitions in boron-like Si^{9+} ions in CO_2 laser-produced plasma. Recently, Bernhardt et al. [14] performed the measurement of electron-ion recombination coefficients of Si^{10+} and Si^{9+} ions, using the electron-ion merged-beams technique at the heavy-ion storage ring.

Theoretically, there are a few calculations of transition probabilities of boron-like Si^{9+} ion available in the literatures. In the work of the Opacity Project, energy levels, oscillator strengths and photo-ionization cross sections of multi-excited states in boron-like ions with Z = 6–16 were calculated in Ref. [15], by using the configuration interaction code CIV3 and *R*-matrix method. As part work of the IRON Project, radiative rates of the E1, E2 and M1 transitions of n = 2 complex in the boron isoelectronic sequence (Z = 8–26) were reported by Galavís et al. [16], using the computer program SUPERSTRUCTURE code. Merkelis et al. [17] calculated the energy spectra and electric dipole transitions in the boron isoelectronic sequence (Z = 8–26) between the levels of the n = 2 complex ($2s^22p$, $2s2p^2$ and $2p^3$), with the electron correlations are considered by the stationary second-order many-body perturbation theory (MBPT). For the n = 3 configurations in boron-like ions with Z = 6–30, Safronova et al. [18] calculated the corresponding energy levels and fine structure separations by relativistic many-body perturbation theory (RMBPT). Then, Vilkas et al. [19] studied the electric dipole transition probabilities for all levels up to n = 4 configurations of boron-like Si^{9+} ion, by the relativistic multireference many-body perturbation (MR-MP) theory. Calculations of energy levels and transition data of boron-like Si^{9+} ion were also performed by the relativistic coupled-cluster theory [20], multi-configuration Hartree-Fock (MCHF) method [21], multi-configuration Dirac-Fock (MCDF) method [22,23], 1/Z perturbation theory method [24], and AUTOSTRUCTURE code [5,25]. At present, the experimental and calculated transition coefficients for these configuration states in Si^{9+} ion are critically compiled.

* Corresponding author.
E-mail address: suenyangu@163.com (Y. Sun).

https://doi.org/10.1016/j.elspec.2019.04.007
Received 3 April 2019; Received in revised form 22 April 2019; Accepted 28 April 2019
Available online 29 April 2019
0368-2048/

These data can be available from the open NIST atomic spectra database [26] and CHIANTI atomic database [27]. However, most of earlier works mentioned above focused on the low-lying states with $n \le 4$. Studies on the $n \ge 5$ quartet series of Si^{9+} ion are still fragmentary. Besides, large discrepancies still exist between theoretical data and experimental results, and also between different theoretical methods. Given the increasing need for theoretical spectroscopic data in astrophysics, it is necessary to calculate more accurate atomic data such as the wavelengths and transition probabilities for the Rydberg quartet states of boron-like ions.

With this in mind, in this paper, we have performed systematical calculations of energy levels and radiative transitions of $1s^22l2l'nl''\ ^4L^{e,o}$ (m) ($L = S, P, D$; m = 1–5) Rydberg series in the boron-like Si^{9+} ion. The Rayleigh-Ritz variational results are reported for calculations done by considering the multi-configuration interaction with moderate-scale wave functions and the mass polarization and relativistic effects with the first-order perturbation theory. The QED effect corrections are also included. Further, the line strengths, oscillator strengths, transition probabilities, and transition wavelengths of the electric dipole transitions between centers of gravity of *LS* terms of these Rydberg series in Si^{9+} ion are calculated and discussed.

2. Theoretical method

For the boron-like Si^{9+} ion, the non-relativistic Hamiltonian in the *LS* coupling scheme is

$$H_0 = \sum_{i=1}^{5}\left[-\frac{1}{2}\nabla_i^2 - \frac{Z}{r_i}\right] + \sum_{\substack{i,j=1\\ i<j}}^{5}\frac{1}{r_{ij}}. \quad (1)$$

The wave function $\Psi_b(1, 2, 3, 4, 5)$ is constructed from *LS* coupled trial functions,

$$\Psi_b(1, 2, 3, 4, 5) = A\sum_i C_i\phi_{n(i)}(R)Y_{l(i)}^{LM}(\Omega)\chi_{SS_z}, \quad (2)$$

where A is the antisymmetrization operator, and C_i is a linear parameter. R represents, collectively, the radial parts of r_1, r_2, r_3, r_4, r_5 and Ω represents their angular part. $n(i)$ represents the combinations of the five principal quantum numbers in the ith configuration. $\phi_{n(i)}(R)$ is the set of radial functions,

$$\phi_{n(i)}(R) = \prod_{j=1}^{5} r_j^{n_j}\exp(-\alpha_j r_j). \quad (3)$$

A diff ;erent nonlinear parameter α_j is optimized for each electron. $l(i)$ is the orbital angular coupling function in ith configuration. L and M represent the total orbital angular momentum and its azimuthal component, respectively. $Y_{l(i)}^{LM}(\Omega)$ is the function of angular component and χ_{SS_z} is the function of spin wave function with the total spin S.

In this work, all the coupling modes of the orbital and spin angular momenta are considered. The configurations which have the same $Y_{l(i)}^{LM}(\Omega)$ and χ_{SS_z} are treated as a "partial wave". In each partial wave, the orbital angular coupling function $l(i)$ and spin wave function χ_{SS_z} are fixed, and the principal quantum numbers $n_j (j = 1 - 5)$ of electrons increase to expand configuration states functions (CSFs). If the maximum value of each n_j is given, the number of CSFs in each partial wave is restricted by

$$\sum_j n_j \le b. \quad (4)$$

Here b is an adjustable parameter. The set of CSFs in each partial wave expands until the convergence of the expectation value of the energies is obtained. Since all the principal quantum numbers of electrons are treated as "active", the valence-valence, core-valence, and core-core correlations could be considered in this work. For the low-lying quartet Rydberg series configuration states, the significant configuration interactions arise from the $n \le 5$ layers. While for the high-lying quartet Rydberg series configuration states, the configuration space expands to $n = 9$.

The variation of the non-relativistic energy E_b with respect to the linear parameters C_i leads to a secular equation whose eigenvalue is a function of the nonlinear parameters α_j. Then the E_b and wave functions are obtained via the Rayleigh-Ritz variation calculation by

$$\delta\left[\frac{\langle\Psi_b\ |H_0|\ \Psi_b\rangle}{\langle\Psi_b\ |\Psi_b\rangle}\right] = 0. \quad (5)$$

To saturate the wave functional space and further improve the non-relativistic energy E_b, the restricted variational method is used to calculate the restricted variational energy contributions ΔE_{RV}. Then the non-relativistic energies are obtained by $E_b + \Delta E_{RV}$. The detailed explanations of the restricted variational calculation were given in Refs. [28,29], and we will not repeat here.

Accurate transition wavelengths demand relativistic energy eigenvalues. Here, the correction $\Delta E_{mp} + \Delta E_{rel}$ for the mass polarization effect and the relativistic effects are also considered by using the first-order perturbation theory. The relativistic effects considered in this work are the correction to the kinetic energy, the Darwin term, the electron-electron contact term, and the orbit-orbit interaction term. To estimate QED effect for these states, the screened hydrogenic formula [30] is used. The detailed expressions for the correction ΔE_{QED} have been discussed in Ref. [31]. The total correction ΔE_{corr} is obtained by $\Delta E_{corr} = \Delta E_{mp} + \Delta E_{rel} + \Delta E_{QED}$. Then the centers of gravity of energy is obtained by

$$E_{total} = E_b + \Delta E_{RV} + \Delta E_{corr}. \quad (6)$$

For an electric dipole transition, the transition line strength S (in a.u.) is calculated by

$$S = |<\psi_f||\boldsymbol{D}||\psi_i>|^2, \quad (7)$$

where $\boldsymbol{D}$ is the electric dipole transition operator, ψ_f and ψ_i are the nonrelativistic wave functions of the final and initial configuration states, respectively. The relationship [32] between electric dipole transition probability A_{ik}, absorption oscillator strength f_{ki}, and line strength S is,

$$A_{ik} = \frac{6.6702\times10^{15}}{\lambda^2}\frac{g_k}{g_i}f_{ki} = \frac{2.0261\times10^{18}}{\lambda^3 g_i}S. \quad (8)$$

Here, λ (in Å) is the transition wavelength between the initial and final configuration states. The statistical weights g_i and g_k are obtained by

$$g_{i(k)} = (2L_{i(k)} + 1)(2S_{i(k)} + 1) \quad (9)$$

In this work, two expressions (the length form and velocity form) for the oscillator strength and transition probability are given. Ideally, these two forms of the same quantity should agree, so the discrepancies remaining between them can reflect the accuracy of theoretical calculation.

3. Results and discussion

In this work, moderate-scale multi-configuration wave functions are used to calculate the energies and wave functions of the quartet $^4L^{e,o}$ (m) ($L = S, P, D$; m = 1–5) Rydberg series in boron-like Si^{9+} ion. In the energy calculation of these quartet states, the radial and angular correlations are very important, especially for the $^4D^{e,o}$ states. For the even-parity complexes, the important orbital angular series $[l_1, l_2, l_3, l_4, l_5]$ are $[0, 0, 0, l, l]$, $[0, 0, 0, l, l+2]$, $[0, 0, 1, l, l+1]$, $[0, 0, 1, l, l+3]$, $[0, 0, 2, l, l]$, $[0, 1, 1, l, l]$, $[0, 1, 1, l, l+2]$, $[1, 1, 1, l, l+1]$, etc. For the odd-parity complexes, the important orbital angular series are $[0, 0, 0, l, l+1]$, $[0, 0, 1, l, l]$, $[0, 0, 1, l, l+2]$, $[0, 0, 2, l, l+1]$, $[0, 1, 1, l, l+1]$, $[0, 1, 2, l, l]$, $[1, 1, 1, l, l]$, $[1, 1, 1, l, l+2]$, etc. In both cases, the value of l is from 0 to 8, as the

Table 1
Centers of gravity of *LS* terms (a.u.) of quartet $^4L^{e,o}$ (m) ($L = S, P, D$; m = 1–5) Rydberg series in Si^{9+} ion. $E_b + \Delta E_{RV}$ is the nonrelativistic energy, ΔE_{corr} are corrections to the mass polarization, relativistic, and QED effects, and E_{total} is the total energy. E_S (Ry) is the energy level relative to the $2s^22p$ $^2P^o$ ground term.

Configuration states	$E_b + \Delta E_{RV}$	ΔE_{corr}	E_{total}	E_S(Ry)			
				Present	NIST	MR-MP	AS
$2s2p^2$ $^4P^e(1)$	−238.06445	−0.49280	−238.55725	1.4590	1.4599	1.4582	1.4599
$2s2p3p$ $^4P^e(2)$	−229.26089	−0.48695	−229.74784	19.0779		19.0761	19.0655
$2p^23s$ $^4P^e(3)$	−228.41526	−0.45910	−228.87436	20.8248		20.8150	20.8300
$2p^23d$ $^4P^e(4)$	−227.71286	−0.44819	−228.16105	22.2514	22.24	22.2437	22.3123
$2s2p4p$ $^4P^e(5)$	−226.57325	−0.48193	−227.05518	24.4632		24.3623	24.5152
$2s2p3s$ $^4P^o(1)$	−229.71401	−0.49528	−230.20929	18.1550	18.1553	18.1537	18.1470
$2s2p3d$ $^4P^o(2)$	−228.97960	−0.48254	−229.46214	19.6493	19.654	19.6543	19.6778
$2p^23p$ $^4P^o(3)$	−228.07832	−0.45381	−228.53213	21.5093		21.4912	21.5320
$2s2p4s$ $^4P^o(4)$	−226.73852	−0.48391	−227.22243	24.1287		24.1202	24.1610
$2s2p4d$ $^4P^o(5)$	−226.46405	−0.47932	−226.94337	24.6868		24.6896	24.7491
$2s2p3p$ $^4D^e(1)$	−229.38261	−0.48824	−229.87085	18.8318		18.8371	18.8358
$2p^23d$ $^4D^e(2)$	−227.76768	−0.44664	−228.21432	22.1449		22.0664	
$2s2p4p$ $^4D^e(3)$	−226.61094	−0.48586	−227.09680	24.3799		24.4393	24.4329
$2s2p4f$ $^4D^e(4)$	−226.39004	−0.48240	−226.87244	24.8287		24.8153	24.8865
$2s2p5p$ $^4D^e(5)$	−225.37523	−0.47962	−225.85485	26.8638			
$2s2p3d$ $^4D^o(1)$	−229.01712	−0.48423	−229.50135	19.5708	19.570	19.5708	19.6069
$2p^23p$ $^4D^o(2)$	−228.11632	−0.45398	−228.57030	21.4329		21.4116	21.4468
$2s2p4d$ $^4D^o(3)$	−226.47517	−0.48311	−226.95828	24.6570		24.6582	24.7103
$2p^24p$ $^4D^o(4)$	−225.37636	−0.45064	−225.82700	26.9195		26.9114	
$2s2p5d$ $^4D^o(5)$	−225.31052	−0.48062	−225.79114	26.9913			
$2s2p3p$ $^4S^e(1)$	−229.31907	−0.48525	−229.80432	18.9649		18.9613	18.9609
$2s2p4p$ $^4S^e(2)$	−226.59175	−0.48503	−227.07678	24.4200		24.4233	24.4791
$2s2p5p$ $^4S^e(3)$	−225.37095	−0.48280	−225.85375	26.8660			
$2s2p6p$ $^4S^e(4)$	−224.71671	−0.48010	−225.19681	28.1799			
$2s2p7p$ $^4S^e(5)$	−224.30469	−0.48083	−224.78552	29.0025			
$2p^3$ $^4S^o(1)$	−236.52523	−0.45974	−236.98497	4.6036	4.5989	4.5976	4.5985
$2p^23p$ $^4S^o(2)$	−227.95663	−0.45318	−228.40981	21.7539		21.7393	21.8557
$2p^24p$ $^4S^o(3)$	−225.33190	−0.45063	−225.78253	27.0085		27.0088	
$2p^25p$ $^4S^o(4)$	−224.12740	−0.44964	−224.57704	29.4195			
$2p^26p$ $^4S^o(5)$	−223.47932	−0.44831	−223.92763	30.7183			

NIST: Ref. [26]. MR-MP: multireference many-body perturbation, Ref. [19].
AS: AUTOSTRUCTURE, Refs. [25,27].

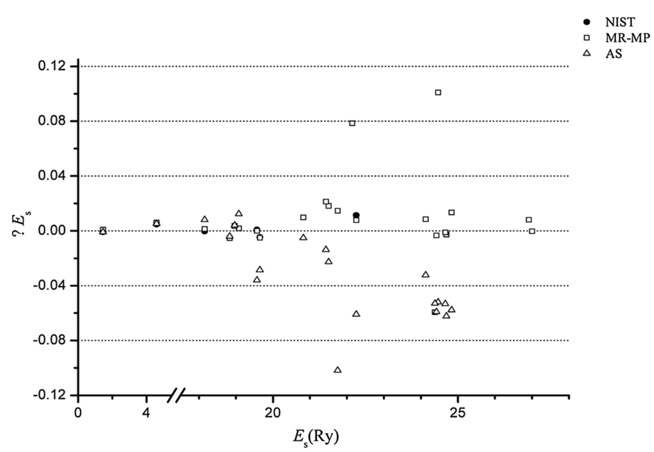

Fig. 1. Deviations of the calculated term center-of-gravity values from the reference results of the $^4L^{e,o}$ ($L = S, P, D$) Rydberg series in boron-like Si^{9+} ion.

energy contribution from the set with $l > 8$ is less than 10^{-5} a.u. and can be neglected. For an orbital angular momentum set $[l_1, l_2, l_3, l_4, l_5]$, all kinds of coupling modes which are possible to obtain the desired total orbital angular momentum L are considered. For the quartet state, four spin angular coupling modes are possible, namely,

$$\chi^1 = \{[(\frac{1}{2}, \frac{1}{2})0, \frac{1}{2}]\frac{1}{2}, \frac{1}{2}\}1, \frac{1}{2}; \chi^2 = \{[(\frac{1}{2}, \frac{1}{2})1, \frac{1}{2}]\frac{1}{2}, \frac{1}{2}\}1, \frac{1}{2}$$

$$\chi^3 = \{[(\frac{1}{2}, \frac{1}{2})1, \frac{1}{2}]\frac{3}{2}, \frac{1}{2}\}1, \frac{1}{2}; \chi^4 = \{[(\frac{1}{2}, \frac{1}{2})1, \frac{1}{2}]\frac{3}{2}, \frac{1}{2}\}2, \frac{1}{2}$$

The partial waves consist of all combinations of orbital-spin angular momentum coupling modes. For accurate calculation, all coupling modes are considered here, except for the partial waves with the energy contributions less than 10^{-5} a.u. To reach a high accuracy, more than 110 partial waves and 4000 configuration terms are used to construct high-quality wave functions in this work.

In Table 1, we list the calculated non-relativistic energies $E_b + \Delta E_{RV}$, corrections ΔE_{corr} to the mass polarization effect, relativistic effects and QED effect, and total energy levels E_{total} of the quartet Rydberg series $^4L^{e,o}$ (m) ($L = S, P, D$; m = 1–5) in Si^{9+} ion. The configurations of these Rydberg series are listed and numbered according to the values of energy levels. For the convenience of comparison, the relative energy levels E_S (with respect to the ground term $2s^22p$ $^2P^o$) are also given in Table 1, together with 6 NIST experimental values [26], the AS calculations [25] from the CHIANTI database [27], and theoretical data from MR-MP theory calculations [19]. In our calculations, the contribution from the finite nuclear size correction is not considered. According to the analysis and calculation in Refs. [33–35], the finite nuclear size effect to the total energy levels is estimated to be about 0.00005 a.u.. Combining the finite nuclear size effect and the omitted high-l configuration interactions, the estimation of uncertainty of calculated energy level is about 0.00015 a.u. Fig. 1 gives the deviations of the calculated term center-of-gravity values from the reference results for these Rydberg series in Si^{9+} ion. As can be seen from Fig. 1, our calculated values show good agreement with the values from NIST, due to large amount of electron correlation effects considered in this work. The root mean square (RMS) deviation between our results and NIST values is about 0.0054 Ry. Overall, our results show better agreement with the MR-MP theoretical data [19] than those from AS calculations [25], due to limited correlations between valence electrons are considered in the AS calculations. The RMS deviations between our calculations and those from MR-MP and AS are 0.0299 Ry and 0.0412 Ry, respectively. Large differences still exist between the theoretical data for these Rydberg

Table 2
Line strengths in length form S_P (a.u.), oscillator strengths f_{ki} (in the length f_l and velocity f_v), transition probabilities A_{ik} (in the length A_l and velocity A_v) (s^{-1}), and vacuum wavelengths λ (Å) for electric dipole transitions between the $^4L^{e,o}$ ($L = S, P, D$) Rydberg series terms in Si^{9+} ion. Only the transitions with $S_P > 3 \times 10^{-5}$ (a.u.) are listed. aEb means $a \times 10^b$.

Upper level	Lower level	S_P		A_{ik}			f_{ki}			λ(Å)	
		Present	Others	A_l	A_v	Others	f_l	f_v	Others	Present	Others
$2p^26p\ ^4S^o$	$2s2p^2\ ^4P^e$	2.27E-03		3.80E+10	3.79E+10		1.84E-03	1.84E-03		31.15	
$2p^25p\ ^4S^o$	$2s2p^2\ ^4P^e$	4.39E-03		6.42E+10	6.40E+10		3.41E-03	3.41E-03		32.59	
$2p^24p\ ^4S^o$	$2s2p^2\ ^4P^e$	1.09E-02		1.21E+11	1.21E+11		7.72E-03	7.67E-03		35.67	
$2s2p5d\ ^4D^o$	$2s2p^2\ ^4P^e$	6.60E-02		1.47E+11	1.46E+11		4.68E-02	4.64E-02		35.69	
$2p^24p\ ^4D^o$	$2s2p^2\ ^4P^e$	4.45E-02		9.83E+10	9.73E+10		3.15E-02	3.12E-02		35.79	
$2s2p4d\ ^4P^o$	$2s2p^2\ ^4P^e$	1.04E-01		2.90E+11	2.88E+11	2.52E+11[a]	6.70E-02	6.64E-02	5.79E-02[a]	39.23	39.13[a]
$2s2p4d\ ^4D^o$	$2s2p^2\ ^4P^e$	3.02E-01		5.05E+11	5.01E+11	4.21E+11[a]	1.95E-01	1.93E-01	1.61E-01[a]	39.28	
$2s2p4s\ ^4P^o$	$2s2p^2\ ^4P^e$	1.99E-02		5.18E+10	5.18E+10	3.82E+10[a]	1.25E-02	1.26E-02	9.22E-03[a]	40.20	40.14[a]
$2p^23p\ ^4S^o$	$2s2p^2\ ^4P^e$	4.41E-02		2.47E+11	2.44E+11	2.33E+11[a]	2.49E-02	2.46E-02	2.33E-02[a]	44.90	**44.93**[a]
$2p^23p\ ^4P^o$	$2s2p^2\ ^4P^e$	9.85E-02		1.77E+11	1.79E+11	1.77E+11[a]	5.49E-02	5.53E-02	5.48E-02[a]	45.45	45.40[a]
$2p^23p\ ^4D^o$	$2s2p^2\ ^4P^e$	1.24E-01		1.33E+11	1.33E+11	1.28E+11[a]	6.89E-02	6.91E-02	6.27E-02[a]	45.62	
$2s2p4p\ ^4P^e$	$2p^3\ ^4S^o$	6.64E-04		1.16E+09	1.12E+09	1.01E+09[a]	1.10E-03	1.06E-03	9.55E-04[a]	45.89	45.76[a]
$2s2p3d\ ^4P^o$	$2s2p^2\ ^4P^e$	5.99E-01		8.04E+11	8.02E+11	7.89E+11[a]	3.02E-01	3.02E-01	2.96E-01[a]	50.10	**50.07**[a]
$2s2p3d\ ^4D^o$	$2s2p^2\ ^4P^e$	1.89E+00	1.80E+00[b]	1.50E+12	1.50E+12	1.43E+12[a] 1.43E+12[b]	9.51E-01	9.49E-01	8.99E-01[a] 9.05E-01[b]	50.31	50.319[b]
$2p^23d\ ^4P^e$	$2p^3\ ^4S^o$	1.14E+00		1.40E+12	1.40E+12		1.68E+00	1.68E+00		51.64	
$2s2p3s\ ^4P^o$	$2s2p^2\ ^4P^e$	1.87E-01	1.89E-01[b] 1.89E-01[c]	1.94E+11	1.94E+11	1.87E+11[a] 1.96E+11[b]	8.65E-02	8.67E-02	8.36E-02[a] 8.75E-02[b] 8.75E-02[c]	54.58	**54.58**[a] 54.582[b] 54.56[c]
$2p^23s\ ^4P^e$	$2p^3\ ^4S^o$	1.08E-01		1.02E+11	1.02E+11	1.00E+11[a]	1.45E-01	1.45E-01	1.42E-01[a]	56.18	56.14[a]
$2s2p3p\ ^4P^e$	$2p^3\ ^4S^o$	9.89E-04		6.69E+08	6.50E+08	7.77E+08[a]	1.19E-03	1.16E-03	1.38E-03[a]	62.96	62.99[a]
$2s2p7p\ ^4S^e$	$2s2p3s\ ^4P^o$	7.81E-03		6.68E+09	6.22E+09		2.35E-03	2.19E-03		84.01	
$2s2p6p\ ^4S^e$	$2s2p3s\ ^4P^o$	1.41E-02		9.51E+09	9.85E+09		3.93E-03	4.07E-03		90.90	
$2p^26p\ ^4S^o$	$2p^23s\ ^4P^e$	1.51E-02		9.80E+09	9.83E+09		4.15E-03	4.17E-03		92.11	
$2s2p7p\ ^4S^e$	$2s2p3d\ ^4P^o$	2.65E-03		1.45E+09	1.59E+09		6.89E-04	7.54E-04		97.43	
$2s2p5p\ ^4S^e$	$2s2p3s\ ^4P^o$	3.59E-02		1.59E+10	1.51E+10		8.68E-03	8.25E-03		104.61	
$2s2p5p\ ^4D^e$	$2s2p3s\ ^4P^o$	2.04E-01		1.81E+10	1.74E+10		4.94E-02	4.77E-02		104.64	
$2p^25p\ ^4S^o$	$2p^23s\ ^4P^e$	3.72E-02		1.58E+10	1.53E+10		8.88E-03	8.61E-03		106.03	
$2s2p6p\ ^4S^e$	$2s2p3d\ ^4P^o$	5.33E-03		2.21E+09	2.22E+09		1.26E-03	1.26E-03		106.82	
$2p^26p\ ^4S^o$	$2p^23d\ ^4P^e$	3.10E-03		1.26E+09	1.19E+09		7.29E-04	6.88E-04		107.63	
$2s2p5d\ ^4D^o$	$2s2p3p\ ^4D^e$	1.24E-01		9.04E+09	9.06E+09		1.69E-02	1.69E-02		111.69	
$2s2p5d\ ^4D^o$	$2s2p3p\ ^4P^e$	3.96E-01		2.63E+10	2.55E+10		8.71E-02	8.45E-02		115.16	
$2s2p7p\ ^4S^e$	$2p^23p\ ^4P^o$	3.88E-05		1.09E+07	1.17E+07		8.07E-06	8.65E-06		121.61	
$2s2p5p\ ^4D^e$	$2s2p3d\ ^4D^o$	4.56E-03		2.37E+08	2.58E+08		5.54E-04	6.03E-04		124.95	
$2s2p5p\ ^4S^e$	$2s2p3d\ ^4P^o$	1.67E-02		4.19E+09	4.48E+09		3.34E-03	3.57E-03		126.27	
$2s2p5p\ ^4D^e$	$2s2p3d\ ^4P^o$	2.43E-02		1.22E+09	1.16E+09		4.87E-03	4.62E-03		126.31	
$2p^25p\ ^4S^o$	$2p^23d\ ^4P^e$	9.27E-03		2.28E+09	2.21E+09		1.85E-03	1.78E-03		127.13	
$2s2p4f\ ^4D^e$	$2s2p3s\ ^4P^o$	2.34E-02		9.31E+08	9.62E+08	5.57E+08[a]	4.34E-03	4.48E-03	2.54E-03[a]	136.55	
$2s2p6p\ ^4S^e$	$2p^23p\ ^4P^o$	5.09E-05		1.01E+07	1.90E+07		9.42E-06	1.78E-05		136.61	
$2s2p4p\ ^4P^e$	$2s2p3s\ ^4P^o$	4.51E-01		2.52E+10	2.64E+10	2.57E+10[a]	7.90E-02	8.25E-02	7.93E-02[a]	144.46	143.10[a]
$2s2p4p\ ^4S^e$	$2s2p3s\ ^4P^o$	1.53E-01		2.52E+10	2.54E+10	1.52E+10[a]	2.66E-02	2.69E-02	1.56E-02[a]	145.46	143.91[a]
$2s2p4p\ ^4D^e$	$2s2p3s\ ^4P^o$	8.88E-01		2.87E+10	2.88E+10	2.96E+10[a]	1.54E-01	1.54E-01	1.56E-01[a]	146.39	
$2p^24p\ ^4S^o$	$2p^23s\ ^4P^e$	1.56E-01		2.47E+10	2.52E+10		2.68E-02	2.73E-02		147.37	
$2s2p5d\ ^4D^o$	$2p^23s\ ^4P^e$	9.97E-02		3.13E+09	3.13E+09		1.71E-02	1.71E-02		147.78	
$2p^24p\ ^4D^o$	$2p^23s\ ^4P^e$	9.68E-01		2.93E+10	3.00E+10		1.64E-01	1.68E-01		149.52	
$2s2p4d\ ^4P^o$	$2s2p3p\ ^4D^e$	2.91E-02		1.30E+09	1.29E+09		2.84E-03	2.81E-03	1.43E-02[a]	155.64	
$2s2p4d\ ^4D^o$	$2s2p3p\ ^4D^e$	6.59E-01		1.74E+10	1.76E+10		6.40E-02	6.45E-02		156.44	155.15[a]
$2s2p4d\ ^4P^o$	$2s2p3p\ ^4S^e$	9.04E-01		3.78E+10	3.81E+10	2.40E+10[a]	4.31E-01	4.35E-01	2.67E-01[a]	159.26	157.43[a]
$2s2p4d\ ^4P^o$	$2s2p3p\ ^4P^e$	8.11E-01		3.20E+10	3.12E+10	3.24E+10[a]	1.26E-01	1.23E-01	1.25E-01[a]	162.47	160.33[a]
$2s2p4d\ ^4D^o$	$2s2p3p\ ^4P^e$	2.31E+00		5.36E+10	5.35E+10		3.57E-01	3.57E-01		163.34	
$2s2p5p\ ^4D^e$	$2p^23p\ ^4D^o$	1.92E-02		4.10E+08	4.05E+08		1.73E-03	1.71E-03		167.80	
$2s2p5p\ ^4S^e$	$2p^23p\ ^4P^o$	2.81E-04		2.89E+07	3.47E+07		4.18E-05	5.01E-05		170.12	
$2s2p5p\ ^4D^e$	$2p^23p\ ^4P^o$	4.45E-02		9.15E+08	8.39E+08		6.63E-03	6.07E-03		170.19	
$2s2p4s\ ^4P^o$	$2s2p3p\ ^4D^e$	4.97E-01		1.82E+10	1.92E+10	1.55E+10[a]	4.39E-02	5.12E-02	4.08E-02[a]	172.04	
$2s2p4f\ ^4D^e$	$2s2p3d\ ^4D^o$	1.26E+00		2.46E+10	2.44E+10		1.11E-01	1.10E-01		173.32	172.60[a]
$2s2p4f\ ^4D^e$	$2s2p3d\ ^4P^o$	6.78E+00		1.26E+11	1.25E+11	1.03E+11[a]	9.76E-01	9.69E-01	7.85E-01[a]	175.94	
$2s2p4s\ ^4P^o$	$2s2p3p\ ^4S^e$	9.66E-02		2.97E+09	2.99E+09	2.98E+09[a]	4.16E-02	4.19E-02	4.13E-02[a]	176.48	175.24[a]
$2s2p4s\ ^4P^o$	$2s2p3p\ ^4P^e$	5.78E-01		1.66E+10	1.67E+10	1.78E+10[a]	8.11E-02	8.14E-02	8.51E-02	180.42	178.84[a]
$2s2p7p\ ^4S^e$	$2s2p4s\ ^4P^o$	2.64E-02		2.04E+09	2.43E+09		3.57E-03	4.24E-03		186.98	
$2s2p5d\ ^4D^o$	$2p^23d\ ^4D^e$	5.00E-03		7.61E+07	7.65E+07		4.04E-04	4.06E-04		188.03	
$2s2p4p\ ^4P^e$	$2s2p3d\ ^4P^o$	2.88E-02		7.20E+08	8.05E+08	1.68E+09[a]	3.85E-03	4.31E-03	8.95E-03[a]	189.30	188.39[a]
$2s2p4p\ ^4D^e$	$2s2p3d\ ^4D^o$	7.63E-02		1.14E+09	1.10E+09	1.30E+09[a]	6.11E-03	5.90E-03	6.94E-03[a]	189.49	188.82[a]
$2p^24p\ ^4D^o$	$2p^23d\ ^4D^e$	1.05E-01		1.52E+09	1.68E+09		8.32E-03	9.19E-03		190.86	
$2s2p4p\ ^4S^e$	$2s2p3d\ ^4P^o$	1.54E-01		1.12E+10	1.07E+10	5.62E+09[a]	2.04E-02	1.94E-02	1.01E-02[a]	191.02	189.80[a]
$2p^24p\ ^4S^o$	$2p^23d\ ^4P^e$	6.72E-02		4.84E+09	4.98E+09		8.88E-03	9.14E-03		191.57	
$2s2p4p\ ^4D^e$	$2s2p3d\ ^4P^o$	7.73E-02		1.10E+09	9.91E+08	6.33E+08[a]	1.02E-02	9.19E-03	5.80E-03[a]	192.63	
$2s2p7p\ ^4S^e$	$2s2p4d\ ^4P^o$	2.34E-02		1.26E+09	1.15E+09		2.81E-03	2.56E-03		211.16	
$2p^23d\ ^4P^e$	$2s2p3s\ ^4P^o$	4.20E-03		6.44E+07	7.33E+07		4.78E-04	5.43E-04		222.46	
$2s2p6p\ ^4S^e$	$2s2p4s\ ^4P^o$	6.58E-02		2.93E+09	2.71E+09		7.41E-03	6.86E-03		224.94	
$2p^23d\ ^4D^e$	$2s2p3s\ ^4P^o$	1.11E-02		9.46E+07	9.71E+07	5.81E+07[a]	1.23E-03	1.26E-03	7.65E-04[a]	228.40	

(*continued on next page*)

Table 2 (*continued*)

Upper level	Lower level	S_P		A_{ik}			f_{ki}			λ(Å)	
		Present	Others	A_l	A_v	Others	f_l	f_v	Others	Present	Others
$2s2p4d\ ^4P^o$	$2p^23s\ ^4P^e$	1.30E-03		1.67E+07	2.55E+07		1.39E-04	2.13E-04		235.96	
$2s2p4d\ ^4D^o$	$2p^23s\ ^4P^e$	4.98E-03		3.75E+07	4.35E+07	3.93E+07[a]	5.30E-04	6.15E-04	5.40E-04[a]	237.80	
$2s2p6p\ ^4S^e$	$2s2p4d\ ^4P^o$	7.27E-02		2.08E+09	1.87E+09		7.06E-03	6.35E-03		260.88	
$2s2p4f\ ^4D^e$	$2p^23p\ ^4D^o$	3.70E-02		1.94E+08	1.93E+08	1.94E+08[a]	2.09E-03	2.08E-03	2.04E-03[a]	268.36	264.94[a]
$2s2p4f\ ^4D^e$	$2p^23p\ ^4P^o$	1.02E-01		5.00E+08	5.10E+08	3.33E+08[a]	9.41E-03	9.59E-03	6.11E-03[a]	274.53	
$2s2p4s\ ^4P^o$	$2p^23s\ ^4P^e$	3.12E-03		2.51E+07	2.28E+07	4.01E+07[a]	2.86E-04	2.60E-04	4.51E-04[a]	275.82	273.58[a]
$2p^3\ ^4S^o$	$2s2p^2\ ^4P^e$	7.57E-01	7.63E-01[b] 7.53E-01[c]	1.58E+10	1.56E+10	1.56E+10[a] 1.58E+10[b]	6.61E-02	6.57E-02	6.65E-02[b] 6.60E-02[c]	289.80	**290.36**[a] 290.30[b] 289.65[c]
$2s2p4p\ ^4P^e$	$2p^23p\ ^4P^o$	1.33E-02		7.65E+07	8.64E+07	1.11E+08[a]	1.09E-03	1.23E-03	1.56E-03[a]	308.50	305.47[a]
$2s2p4p\ ^4D^e$	$2p^23p\ ^4D^o$	1.73E-03		5.93E+06	4.24E+06		8.50E-05	6.08E-05		309.22	305.16[a]
$2s2p4p\ ^4S^e$	$2p^23p\ ^4P^o$	6.35E-03		1.05E+08	9.64E+07	9.63E+07[a]	5.13E-04	4.72E-04	4.57E-04[a]	313.08	309.21[a]
$2s2p4p\ ^4D^e$	$2p^23p\ ^4P^o$	1.06E-02		3.37E+07	3.60E+07	7.19E+07[a]	8.48E-04	9.06E-04	1.77E-03[a]	317.45	
$2s2p5p\ ^4S^e$	$2s2p4s\ ^4P^o$	4.66E-01		6.40E+09	5.13E+08		3.54E-02	2.84E-02		332.91	
$2s2p5p\ ^4D^e$	$2s2p4s\ ^4P^o$	2.64E+00		7.23E+09	6.82E+09		2.01E-01	1.89E-01		333.17	
$2s2p4p\ ^4P^e$	$2p^23p\ ^4S^o$	2.35E-03		1.04E+07	7.52E+06	1.10E+07[a]	5.29E-04	3.83E-04	5.81E-04[a]	336.36	342.66[a]
$2p^23p\ ^4P^o$	$2s2p3p\ ^4D^e$	7.46E-01		3.20E+09	2.95E+09	3.07E+09[a]	3.33E-02	3.07E-02	3.15E-02[a]	340.36	
$2p^23p\ ^4S^o$	$2s2p3p\ ^4P^e$	5.09E-01		6.53E+09	6.37E+09	8.25E+09[a]	3.79E-02	3.69E-02	4.38E-02[a]	340.53	326.60[a]
$2p^23s\ ^4P^e$	$2s2p3s\ ^4P^o$	1.40E+00		5.92E+09	5.79E+09	6.25E+09[a]	1.04E-01	1.01E-01	1.08E-01[a]	341.32	339.64[a]
$2s2p5d\ ^4D^o$	$2s2p4p\ ^4D^e$	5.57E-01		1.33E+09	1.39E+09		2.43E-02	2.54E-02		348.97	
$2p^23d\ ^4P^e$	$2s2p3d\ ^4P^o$	4.25E-01		1.67E+09	1.68E+09		3.07E-02	3.09E-02		350.20	
$2p^23p\ ^4D^o$	$2s2p3p\ ^4D^e$	1.98E+00		4.69E+09	4.55E+09	4.62E+09[a]	8.60E-02	8.36E-02	8.12E-02[a]	350.35	349.02[a]
$2p^23d\ ^4D^e$	$2s2p3d\ ^4D^o$	8.82E-02		2.01E+08	1.88E+08	2.78E+08[a]	3.78E-03	3.54E-03	5.45E-03[a]	354.03	362.76[a]
$2p^24p\ ^4S^o$	$2s2p4p\ ^4P^e$	4.91E-01		5.41E+09	5.18E+09		3.47E-02	3.47E-02		358.03	
$2p^23p\ ^4P^o$	$2s2p3p\ ^4S^e$	4.89E-01		1.80E+09	1.77E+09	1.84E+09[a]	1.04E-01	1.02E-01	1.04E-01[a]	358.16	354.44[a]
$2s2p4d\ ^4P^o$	$2p^23d\ ^4D^e$	3.04E-02		1.11E+08	1.22E+08	2.10E+08[a]	1.29E-03	1.41E-03	2.26E-03[a]	358.51	
$2p^24p\ ^4D^o$	$2s2p4p\ ^4D^e$	1.97E+00		4.31E+09	4.01E+09		8.32E-02	7.73E-02		358.83	
$2s2p4d\ ^4D^o$	$2p^23d\ ^4D^e$	6.47E-03		1.37E+07	1.48E+07		2.71E-04	2.92E-04		362.76	
$2p^23d\ ^4D^e$	$2s2p3d\ ^4P^o$	4.07E-01		8.47E+08	7.87E+08	7.05E+08[a]	2.82E-02	2.62E-02	2.45E-02[a]	365.15	
$2s2p4d\ ^4P^o$	$2p^23d\ ^4P^e$	9.46E-03		3.05E+07	2.98E+07		6.40E-04	6.25E-04		374.19	
$2p^23p\ ^4P^o$	$2s2p3p\ ^4P^e$	5.03E-01		1.61E+09	1.48E+09	1.73E+09[a]	3.40E-02	3.11E-02	3.55E-02[a]	374.80	369.46[a]
$2p^23p\ ^4D^o$	$2s2p3p\ ^4P^e$	7.38E-01		1.29E+09	1.23E+09	1.31E+09[a]	4.83E-02	4.61E-02	4.80E-02[a]	386.94	
$2s2p5p\ ^4D^e$	$2s2p4d\ ^4D^o$	5.11E-01		7.36E+08	6.22E+08		1.88E-02	1.59E-02		412.93	
$2s2p5p\ ^4S^e$	$2s2p4d\ ^4P^o$	4.66E-01		3.23E+09	3.75E+09		2.82E-02	3.28E-02		418.17	
$2s2p5p\ ^4D^e$	$2s2p4d\ ^4P^o$	1.41E-03		1.94E+06	1.86E+06		8.51E-05	8.14E-05		418.59	
$2s2p5d\ ^4D^o$	$2s2p4f\ ^4D^e$	4.96E-02		6.71E+07	7.00E+07		1.79E-03	1.86E-03		421.38	
$2p^24p\ ^4D^o$	$2s2p4f\ ^4D^e$	2.28E-03		2.80E+06	2.00E+06		7.96E-05	5.71E-05		435.84	
$2s2p4s\ ^4P^o$	$2p^23d\ ^4D^e$	1.25E-02		2.17E+07	2.09E+07	2.56E+07[a]	4.13E-04	4.04E-04	4.59E-04[a]	459.37	
$2s2p4s\ ^4P^o$	$2p^23d\ ^4P^e$	4.44E-03		6.55E+06	7.39E+06	1.40E+07[a]	2.31E-04	2.61E-04	5.08E-04[a]	485.44	493.08[a]
$2p^23s\ ^4P^e$	$2s2p3d\ ^4D^o$	5.04E-02		2.22E+07	3.84E+07	2.42E+07[a]	1.05E-03	1.82E-03	1.21E-03[a]	726.71	
$2p^23s\ ^4P^e$	$2s2p3d\ ^4P^o$	1.62E-02		5.89E+06	8.26E+06	5.77E+06[a]	5.30E-04	7.44E-04	5.34E-04[a]	775.19	791.38[a]
$2p^23p\ ^4S^o$	$2p^23s\ ^4P^e$	1.98E+00		1.06E+09	1.03E+09	1.41E+09[a]	5.10E-02	4.95E-02	5.57E-02[a]	980.83	888.85[a]
$2s2p3p\ ^4P^e$	$2s2p3s\ ^4P^o$	5.12E+00		8.97E+08	9.05E+08	8.63E+08[a]	1.31E-01	1.32E-01	1.28E-01[a]	987.41	992.26[a]
$2s2p3d\ ^4P^o$	$2s2p3p\ ^4D^e$	5.32E-01		6.48E+07	5.67E+07		7.25E-03	6.34E-03		1114.83	
$2s2p3p\ ^4S^e$	$2s2p3s\ ^4P^o$	1.59E+00		5.66E+08	5.59E+08	5.21E+08[a]	3.58E-02	3.54E-02	3.30E-02[a]	1125.13	1120.61[a]
$2p^23d\ ^4P^e$	$2p^23p\ ^4P^o$	2.56E+00		2.34E+08	2.07E+08		5.28E-02	4.68E-02		1227.88	
$2s2p3d\ ^4D^o$	$2s2p3p\ ^4D^e$	1.49E+00		8.05E+07	7.53E+07		1.84E-02	1.72E-02		1233.13	
$2p^23d\ ^4D^e$	$2p^23p\ ^4D^o$	2.25E+00		1.08E+08	9.32E+07	8.28E+07[a]	2.66E-02	2.29E-02	2.29E-02[a]	1279.97	1356.40[a]
$2s2p4f\ ^4D^e$	$2s2p4s\ ^4P^o$	8.07E-02		3.71E+06	4.35E+06		1.57E-03	1.84E-03		1301.87	
$2p^23p\ ^4P^o$	$2p^23s\ ^4P^e$	5.49E+00		3.93E+08	3.75E+08	4.16E+08[a]	1.04E-01	9.95E-02	1.06E-01[a]	1331.39	1298.51[a]
$2s2p3d\ ^4P^o$	$2s2p3p\ ^4S^e$	2.80E+00		2.00E+08	1.96E+08	1.80E+08[a]	1.60E-01	1.56E-01	1.31E-01[a]	1331.59	1271.02[a]
$2s2p3p\ ^4D^e$	$2s2p3s\ ^4P^o$	8.09E+00		3.36E+08	4.53E+08		1.52E-01	2.05E-01		1346.30	
$2p^23d\ ^4D^e$	$2p^23p$ 4Po	6.34E+00		2.18E+08	1.57E+08	1.56E+08[a]	1.12E-01	8.08E-02	9.35E-02[a]	1433.69	
$2p^23p\ ^4D^o$	$2p^23s\ ^4P^e$	9.16E+00		2.76E+08	2.66E+08	2.91E+08[a]	1.55E-01	1.49E-01	1.58E-01[a]	1498.53	
$2s2p3d\ ^4P^o$	$2s2p3p\ ^4P^e$	1.31E+00		5.47E+07	5.57E+07		2.09E-02	2.12E-02		1594.83	
$2p^23d\ ^4P^e$	$2p^23p\ ^4S^o$	2.32E+00		6.38E+07	6.62E+07		9.63E-02	9.99E-02		1831.65	
$2s2p3d\ ^4D^o$	$2s2p3p\ ^4P^e$	5.07E+00		8.18E+07	8.09E+07		6.94E-02	6.86E-02		1848.52	
$2s2p4p\ ^4P^e$	$2s2p4s\ ^4P^o$	1.97E+01		1.64E+08	1.69E+08	1.77E+08[a]	1.83E-01	1.88E-01	1.85E-01[a]	2724.32	2576.65[a]
$2s2p4d\ ^4P^o$	$2s2p4p\ ^4D^e$	5.66E-01		3.65E+06	5.60E+06		2.90E-03	4.44E-03		2969.71	
$2s2p4p\ ^4S^e$	$2s2p4s\ ^4P^o$	5.40E+00		8.94E+07	9.27E+07	7.49E+07[a]	4.37E-02	4.53E-02	2.74E-02[a]	3128.34	2879.18[a]
$2s2p4d\ ^4D^o$	$2s2p4p\ ^4D^e$	7.97E+00		2.27E+07	2.26E+07		3.68E-02	3.66E-02		3289.36	3303.26[a]
$2s2p4d\ ^4P^o$	$2s2p4p\ ^4S$	1.09E+01		4.62E+07	5.33E+07	2.60E+07[a]	2.42E-01	2.79E-01	1.34E-01[a]	3415.35	3374.80[a]
$2s2p4p\ ^4D^e$	$2s2p4s\ ^4P^o$	3.01E+01		6.38E+07	6.77E+07	8.37E+07[a]	2.10E-01	2.23E-01	2.30E-01[a]	3626.86	
$2s2p4d\ ^4P^o$	$2s2p4p\ ^4P^e$	8.43E+00		2.10E+07	1.66E+07		5.23E-02	4.15E-02		4075.15	
$2s2p4f\ ^4D^e$	$2s2p4d\ ^4D^o$	3.45E+00		2.34E+06	1.97E+06		9.86E-03	8.31E-03		5308.04	
$2s2p4f\ ^4D^e$	$2s2p4d\ ^4P^o$	1.87E+01		7.14E+06	6.06E+06		7.36E-02	6.25E-02		6423.83	
$2s2p5d\ ^4D^o$	$2s2p5p\ ^4D^e$	1.93E+01		5.35E+06	5.62E+06		4.10E-02	4.31E-02		7151.82	

a. CHIANT-Ref. [27]. b. NIST-Ref. [26]. c. MCHF-Ref. [21].
The data in bold font in the last column indicates experimental data from CHIANTI [27].

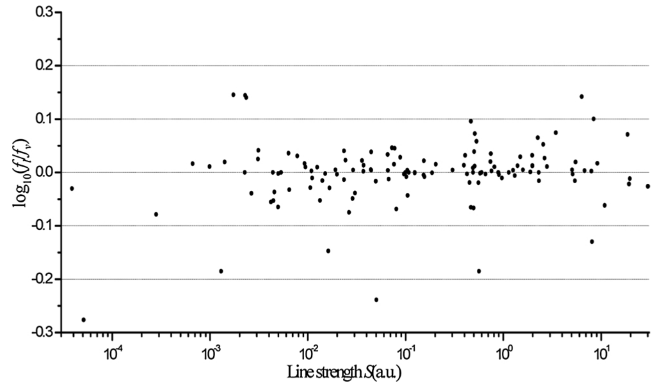

Fig. 2. Comparisons of the length form of oscillator strengths with the velocity form of oscillator strengths calculated in this work.

states especially for the high-lying states with the relative energy $E_S > 21$ Ry. However, no more experimental data are available for these states, we could not give a comparison. More accurate experimental data are needed to identify it.

Using the calculated wave functions and energy levels, we further systematically study the electric-dipole transitions between these quartet $^4L^{e,o}$ ($L = S$, P, D) Rydberg series in Si^{9+} ion. In this work, the oscillator strengths and transition probabilities are calculated in the frame of non-relativistic wave functions. In order to eliminate the largest part of uncertainty related to inaccuracy of the non-relativistic energy levels, the oscillator strengths and transition probabilities are rescaled to wavelengths from the relativistic levels. In Table 2, we list the calculated line strengths S_P, oscillator strengths f_{ki}, transition probabilities A_{ik}, and transition wavelengths for the electric-dipole transitions between these $^4L^{e,o}$ Rydberg series terms in Si^{9+} ion, except the weak transitions with line strength $S_P < 3 \times 10^{-5}$ (a.u.) All the oscillator strengths and transition probabilities are given in two forms: the length and velocity gauges. The consistency between the results from these two gauges is usually used as an indicator of the overall accuracy the calculated wave functions. In order to estimate the uncertainty of the calculated oscillator strengths, in Fig. 2, we compare the calculated oscillator strengths in the form of f_l with the velocity form f_v, along with line strengths S_P in Si^{9+} ion. It can be deduced from Fig. 2 that the deviations between the results in two gauges in 100 transition are less than 10%, in 20 transition are less than 20%, and in 12 transitions are more than 20%. Fig. 3 presents the comparison of the calculated oscillator strengths f_l with the referenced oscillator strength values from CHIANTI [27] database calculated with the AS method. Fig. 3 shows that large discrepancies exist between our calculated oscillator strengths and those from the AS theoretical method, especially in the range of $S_P < 0.1$ (a.u.). Unfortunately, limited theoretical or experimental data are available, so we could not make further overall comparison. Table 2 also lists the transition parameters from the NIST [26] and MCHF theoretical data [21] for three transitions: $2p^3$ $^4S^o$—$2s2p^2$ $^4P^e$, $2s2p3s$ $^4P^o$—$2s2p^2$ $^4P^e$, and $2s2p3d$ $^4D^o$—$2s2p^2$ $^4P^e$. Good agreements are obtained when comparing the calculated results in this work and the referenced data for these three transitions. For example, for the $2p^3$ $^4S^o$—$2s2p^2$ $^4P^e$ transition, our calculated line strength $S_P = 0.757$ and oscillator strength $f_l = 0.0662$, agree well with the NIST experimental results $S = 0.763$, $f = 0.0665$, and are also in accord with the MCHF theoretical data $S = 0.753$, $f = 0.0660$. As the transition probabilities are proportional to oscillator strengths, the deviations A_l from A_v values should have the similar magnitude as the deviations of oscillator strengths discussed above.

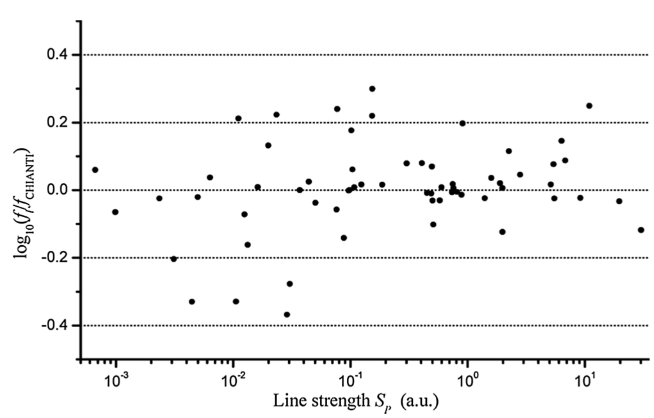

Fig. 3. Comparisons of the length form of oscillator strengths f_l with the referenced data from CHIANTI [27] database.

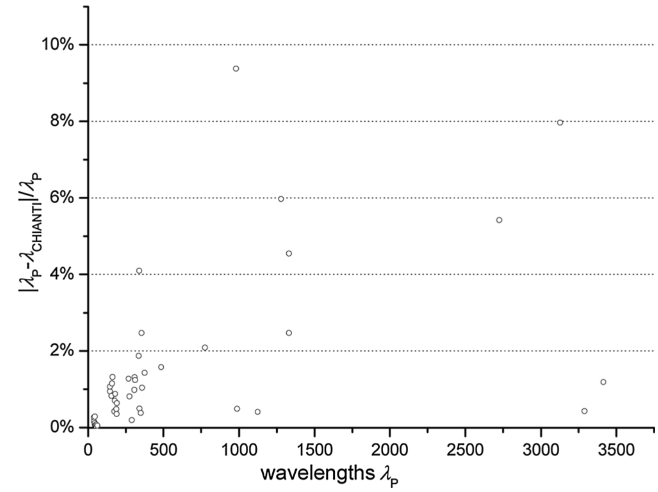

Fig. 4. Comparisons of the calculated transition wavelengths with the referenced data from CHIANTI [27] database.

In the last two columns of Table 2, we tabulate the calculated transition wavelengths, together with the NIST experimental results [26] wherever available, the data from CHIANTI database [27], and the theoretical data from the MCHF calculations [21]. Our calculated data are in good agreement with the NIST experimental results and the MCHF theoretical data. For example, the present calculated wavelength value 54.58 Å for the transition of $2s2p3s$ $^4P^o$—$2s2p^2$ $^4P^e$, agree well with the NIST experimental value 54.582 Å [26] and MCHF theoretical data 54.56 Å [21]. Fig. 4 gives the comparisons of the present transition wavelengths with the data from the CHIANTI database [27]. Here, 4 transition wavelengths from CHIANTI are experimental values, which are labelled with bold font in the last column of Table 2. The remaining referenced wavelengths from CHIANTI are calculated by the AS method [25]. As can be seen from Fig. 4, our calculated wavelengths generally agree well with the CHIANTI data in the range of $\lambda \leq 500$ Å, but do not fit well in the range of $\lambda > 500$ Å. It is because the wavelength of long wave is more sensitive to the deviation of energy levels. The largest deviation is for the transition $2p^2 3p$ $^4S^o$—$2p^2 3s$ $^4P^e$, our calculated wavelength 980.83 Å is about 9.4% larger than the AS theoretical value 888.85 Å [25]. The large deviation is resulting from the disagreement in the energy level calculation of the $2p^2 3p$ $^4S^o$ state. Most of these transition wavelengths are in the range of EUV or soft X-ray, which are particular importance in astrophysical applications. The present calculated data would provide theoretical support for identification of astrophysical spectral lines and other applications.

4. Conclusions

The radiative transition processes of the quartet $^4L^{e,o}$ ($L = S$, P, D) Rydberg series in boron-like Si^{9+} ion are systematically studied, by employing the multi-configuration Rayleigh-Ritz variation method. The energy levels and the radiative transition parameters are calculated in detail. The calculated results have been compared with available accurate experimental data from NIST and CHIANTI database and other theoretical data. Good agreements are found between the data obtained in this work and the experimental results. The good agreements between the oscillator strengths from the length and velocity gauges indicates that the calculated values should be generally accurate. However, there are still large discrepancies for some high-lying Rydberg states, for which further studies are needed. The present results will provide reliable theoretical data for further experiments.

Acknowledgments

The authors are grateful to K. T. Chung for providing the computational codes. This work was supported by the National Natural Science Foundation of China, under grant nos. 11604284 and 51506184, the Natural Science Foundation of the Jiangsu Higher Education Institutions of China, under grant no. 17KJB140025, Special Foundation for Theoretical Physics Research Program of China under grant no. 11547145, Yan Sun is supported by the QinLan project of Jiangsu Province of China. Dong Dong Liu is supported by the Nature Science Foundation of Xuzhou University of Technology of China under grant no. XKY2018122. CHIANTI is a collaborative project involving George Mason University, the University of Michigan (USA) and the University of Cambridge (UK).

References

[1] D. Bernhardt, A. Becker, C. Brandau, et al., Absolute rate coefficients for photorecombination of beryllium-like and boron-like silicon ions, J. Phys. B: Atom. Mol. Phys. 49 (2016) 074004.
[2] A. Antunes, F. Nagase, N.E. White, ASCA observations of the coronal X-ray emission of Algol, Astrophys. J. 436 (1994) L83–L86.
[3] H.M. Antia, Sarbani Basu, The discrepancy between solar abundances and helioseismology, Astrophys. J. Lett. 620 (2005) L129–L132.
[4] M. Güdel, Y. Nazé, X-ray spectroscopy of stars, Astron. Astrophys. Rev. 17 (2009) 309–408.
[5] G.Y. Liang, A.D. Whiteford, N.R. Badnell, R-matrix electron-impact excitation data for B-like Si and its application in cool stars, A&A 499 (2009) 943–954.
[6] A. Decourchelle, J.L. Sauvageot, M. Audard, et al., XMM-Newton observation of the Tycho supernova remnant, A&A 365 (2001) L218.
[7] J.E. Vernazza, H.E. Mason, Density sensitivity of the solar EUV emission from boron-like ions, Astrophys. J. 226 (1978) 720–728.
[8] A. Martin, G. Nicolas, S. Jacques, The solar chemical composition, Nucl. Phys. A 777 (2006) 1–4.
[9] E. Träbert, B.C. Fawcett, Identification of satellite lines in the X-ray spectrum of foil-excited silicon, J. Phys. B: Atom. Mol. Phys. 12 (1979) L441–L447.
[10] E. Träbert, P.H. Heckmann, H.V. Buttlar, Beam-foil lifetimes of highly ionized silicon, Z. Physik A 281 (1977) 333–339.
[11] E. Trabert, P.H. Heckmann, W. Schlagheckt, H.V. Buttlar, Beam-foil lifetime studies of highly ionized silicon, Phys. Scr. 21 (1980) 27–34.
[12] E. Träbert, G. Schneider, P.H. Heckmann, Beam-foil measurements of branching ratios and transition probabilities in boron-like Si X and P XI, Phys. Scr. 27 (1983) 407–412.
[13] A.Ya. Faenov, Precise measurements and theoretical calculations of He-like ion resonance line satellites radiated from Be-, B-, C-, N-, O-, and F-like ions, Phys. Scr. 49 (1994) 41–50.
[14] D. Bernhardt, A. Becker, C. Brandau, et al., Absolute rate coefficients for photorecombination of beryllium-like and boron-like silicon ions, J. Phys. B: At. Mol. Opt. Phys. 49 (2016) 074004.
[15] J.A. Fernley, A. Hibbert, A.E. Kingston, M.J. Seaton, Atomic data for opacity calculations: XXIV. The boron-like sequence, J. Phys. B: At. Mol. Opt. Phys. 32 (1999) 5507–5522.
[16] M.E. Galavís, C. Mendoza, C.J. Zeippen, Atomic data from the IRON Project XXIX. Radiative rates for transitions within the n = 2 complex in ions of the boron isoelectronic sequence, Astron. Astrophys. Suppl. Ser. 131 (1998) 499–522.
[17] G. Merkelis, M.J. Vilkas, G. Gaigalas, R. Kisielius, MBPT calculation of energy spectra and E1 transition probabilities for boron isoelectronic sequence, Phys. Scr. 51 (1995) 233–251.
[18] U.I. Safronova, W.R. Johnson, M.S. Safronova, Relativistic many-body calculations of energies of n = 3 states for the boron isoelectronic sequence, Z = 6-30, At. Data Nucl. Data 69 (1998) 183–215.
[19] M.J. Vilkas, Y. Ishikawa, E. Träbert, Relativistic many-body perturbation calculations of boron-like silicon, Si X, Phys. Scr. 72 (2005) 181–199.
[20] H.S. Nataraj, B.K. Sahoo, B.P. Das, R.K. Chaudhuri, D. Mukherjee, Theoretical studies of the atomic transitions in boron-like ions: Mg VIII, Si X and S XII, J. Phys. B: At. Mol. Opt. Phys. 40 (2007) 3153–3162.
[21] C.F. Fischer, G. Tachiev, Breit-Pauli energy levels, lifetimes, and transition probabilities for the beryllium-like to neon-like sequences, At. Data Nucl. Data 87 (2004) 1–184.
[22] K.T. Cheng, Y.K. Kim, J.P. Desclaux, Electric dipole, quadrupole, and magnetic dipole transition probabilities of ions isoelectronic of the first-row atoms, Li through F, At. Data Nucl. Data 24 (1979) 111–189.
[23] H.L. Zhang, D.H. Sampson, Relativistic distorted-wave collision strengths and oscillator strengths for all possible n = 2-n = 3 transitions in B-like ions, At. Data Nucl. Data 58 (1994) 255–305.
[24] U.I. Safronova, A.S. Shlyaptseva, Inner-shell excitation energy and autoionization rates for Li-, Be-, B-like ions with Z = 6-54, Phys. Scr. 54 (1996) 254–270.
[25] G.Y. Liang, N.R. Badnell, G. Zhao, R-matrix electron-impact excitation data for the B-like isoelectronic sequence, A&A 547 (2012) A87.
[26] A. Kramida, Reader J. Yu Ralchenko, NIST ASD Team. NIST Atomic Spectra Database (Ver. 5.5.6), [Online]. Available: [2018, 12 24] (2018) https://physics.nist.gov/asd.
[27] G. Del Zanna, K.P. Dere, P.R. Young, E. Landi, H.E. Mason, CHIANTI – an atomic database for emission lines. Version 8, A&A 582 (2015) A56.
[28] Y. Sun, F. Chen, L. Zhuo, B.C. Gou, Energies, fine structures, and radiative lifetimes for the multi-excited quartet states of B-like oxygen, Int. J. Quantum. Chem. 112 (2012) 1114–1121.
[29] Y. Sun, C.C. Sang, F. Hu, et al., Rydberg series for quartet states of Li-like sulfur ion, J. Quant. Spectrosc. Radiat. Transf. 187 (2017) 30–37.
[30] G.W.F. Drake, Quantum electrodynamic effects in few-electron atomic systems, Adv. Mol. Phys. 18 (1982) 399–460.
[31] B. Lin, H.G. Berry, T. Shibata, A.E. Livingston, H.P. Garnir, T. Bastin, J. Désequelles, I. Savukov, $1s2s2p^2 3s\ {}^6P - 1s2p^3 3s\ {}^6S^o$ transitions in O IV, Phys. Rev. A 67 (2003) 062507.
[32] W.C. Martin, W.L. Wiese, Atomic, moleCular, and Optical Physics Handbook (Version 2.2), Available: (2019) https://www.nist.gov/pml/atomic-spectroscopy-compendium-basic-ideas-notation-data-and-formulas.
[33] K.T. Chung, X.W. Zhu, Z.W. Wang, Ionization potential for ground states of berylliumlike systems, Phys. Rev. A 47 (1993) 1740–1751.
[34] Z.B. Chen, H.W. Hu, K. Ma, X.B. Liu, X.L. Guo, S. Li, B.H. Zhu, L. Huang, K. Wang, Influence of dense plasma on the energy levels and transition properties in highly charged ions, Phys. Plasmas 25 (2018) 032108.
[35] Z.B. Chen, C.C. Sang, K. Wang, Theoretical determination of energies, wavelengths, and transition probabilities for EUV and SXR spectral lines in Rb XXXIV, Sr XXXV, Zr XXXVII, and Nb XXXVIII, J. Quant. Spectrosc. Radiat. Transf. 225 (2019) 76–83.