

Noise and delay sustained chimera state in small world neuronal network

TANG Jun^{1*}, ZHANG Juan¹, MA Jun² & LUO JinMing³

¹ School of Physics, China University of Mining and Technology, Xuzhou 221116, China;

² Department of Physics, Lanzhou University of Technology, Lanzhou 730050, China;

³ School of Mathematics, China University of Mining and Technology, Xuzhou 221116, China

Received December 7, 2017; accepted July 24, 2018; published online October 19, 2018

Chimera state in neuronal network means the coexistence of synchronized and desynchronized firing patterns. It attracts much attention recently due to its possible relevance to the phenomenon of unihemispheric sleep in mammals. In this paper, we search for chimera state in a noisy small-world neuronal network, in which the neurons are delayed coupled. We found both transient and permanent chimera state when time delay is close to a critical value. The chimera state occurs due to the competition between the synchronized and desynchronized patterns in the neuronal network. On the other hand, intermediate intensity of noise facilitates the occurrence of delay-sustained chimera states. Comparison between noise and delay shows that time delay is the key factor determining the chimera state, whereas noise is a subordinate one.

delay, noise, chimera state, neuronal network

Citation: Tang J, Zhang J, Ma J, et al. Noise and delay sustained chimera state in small world neuronal network. *Sci China Tech Sci*, 2019, 62: 1134–1140, <https://doi.org/10.1007/s11431-017-9282-x>

1 Introduction

Recently, chimera state has attracted much attention in the field of nonlinear dynamics. Chimera states arise in ensembles of identical oscillators when the oscillators exhibit radically different dynamics, within which one group of oscillators exhibit synchronized oscillations whereas the others exhibit desynchronized behavior. Chimera state was first reported by Kuramoto and his colleagues [1] in a ring of simple oscillators. After the first finding, chimera state has been extensively studied in various of computational models, such as simple phase oscillators [2–11], pendulum-like model [12], Stuart-Landau oscillators [13], Josephson junction model [14], chaotic oscillators [15–17], and FitzHugh-Nagumo (FHN) type models [18–21], etc. On the other hand, the experimental verification of chimera states was demon-

strated in mechanical [22], optical [23], chemical [24], and electronic systems [25].

Chimera behavior is of particular importance in neuron system, given that synchronized firing of neurons plays key role in pathological states such as seizures [26], Parkinson's disease [27]. Due to the potential application of chimera states in the phenomenon of unihemispheric sleep, which has been reported in birds, dolphins [28] and human beings [29], many theoretical work have been performed based on numerical simulation [2–21] and bifurcation analysis [30]. Omelchenko et al. [19] have studied the robustness of chimera states in systems of nonlocally coupled FitzHugh-Nagumo (FHN) neurons. They find different multichimera states arising in a transition from classical chimera states, depending on the coupling strength.

Glaze et al. [31] demonstrate chimera-like behaviors in a Hodgkin-Huxley-type model of thermally sensitive neurons. They identified the regions of parameter space for which

*Corresponding author (email: tjuns1979@126.com)

chimera behavior occurs for different coupling schemes. Schmidt et al. [18] discuss chimera pattern in a two-dimensional networks of coupled neuron oscillators. The network models support hybrid states composed of coherent and incoherent regions, and a number of chimera patterns including spots, grids, rings, and stripes are identified.

Inhomogeneity in the dynamics of oscillators and coupling topology were widely assumed to be the necessary ingredients for chimera states. Recent studies show that chimera states can be found also when the elements of the system are nonidentical [32] or when the topology is not regular [33,34] or even global [35]. Omelchenko et al. [36] demonstrate that chimera states are robust with respect to inhomogeneity both in the dynamics of oscillators and coupling topology.

In reality, the dynamics of a neuronal network often involves time delay due to the finite signal propagation time in biological networks [37]. Recently, neuronal networks with time delay have received considerable attention [38–47]. Fan and Wang [46] reported that the interplay between delay and coupling strength in a Hindmarsh-Rose neuronal system can not only enhance or destroy the synchronizations but also can induce the regular transitions of bursting firing patterns. Our former work has studied delay induced synchronization transitions in a small-world (SW) neuronal network [47]. We found that the neuronal network may transit from a desynchronized pattern to a synchronized one when the time delay increase across a critical value, and the synchronized pattern depends on the noise intensity sensitively. Noise effect is also a hot topic in the study of neural network [48–53]. It has been confirmed that noise could determine the synchronization or coherence through the mechanism of stochastic resonance or coherence resonance in neuronal systems [48–51]. Our former work found that noise could change the firing patterns of coupled neurons during neuronal information transmission, and noise facilitates the occurrence of episodic spikes of neurons [52]. Based on a sleep-wake-cycling neuronal model, Jin et al. [53] found noise could trigger transitions between sleep and awake states.

Motivated by the above consideration and based on the similar models used in refs. [39,47], we attempt to find chimera states with the value of time delay close to the critical value, for which the synchronization transition occurs. We focus on how the noise and time delay sustained the chimera state cooperatively, and how long the transient chimera state can exist for. The remainder of this paper is organized as follows. In sect. 2, a noisy SW neuronal network and corresponding equations are introduced. In sect. 3, how the time delay determines the occurrence of chimera state is studied, and the influence of noise on the delay-sustained chimera state is investigated. The paper ends with conclusions in sect. 4.

2 Model and simulation

The FitzHugh-Nagumo (FHN)-type equations are employed to describe the electric activities of neurons since this neuron model combines computational efficiency and the controllability of excitability. The neuron coupling in the cortex and other brain regions is mainly local, with relatively sparse long distance projections, which suggests a SW topological structure rather than regular one. A SW network is implemented as in ref. [54]. At first, a regular network consisting of N neurons is constructed, in which every neuron connects to its k nearest neighbors. Then each link in the regular network is selected and is removed and reconnected to another randomly chosen neuron with probability p . The rewiring parameter p , k , and N are important factors determining the network topology. Following refs. [39,47], we set $N=100$, $p=0.2$, and $k=4$ throughout this paper. The equations of the model are as follows:

$$\begin{aligned} \varepsilon \frac{du_i(t)}{dt} &= u_i(t) - \frac{u_i^3(t)}{3} - v_i(t) + \sum_j g_{ij} [u_j(t-\tau) - u_i(t)], \\ \frac{dv_i(t)}{dt} &= u_i(t) + a + \xi_i(t), \end{aligned} \quad (1)$$

where $u_i(t)$ represents the membrane potential of the i th neuron, and $v_i(t)$ is the corresponding recovery variable. The parameter g_{ij} is the coupling parameter between the i and j th neuron. If the two neurons are coupled to each other, $g_{ij}=0.03$, or else $g_{ij}=0$. τ denotes the information transmission time delay. We set time scale separation $\varepsilon=0.01$. We set the values of g_{ij} and ε following refs. [39,47,54], which guarantee the weak noise could not induce spontaneous firing in single neuron and the firing in one neuron could be transferred to the adjacent one successfully. This FHN-type model possesses two Hopf bifurcation points with bifurcation parameter $a=1, -1$. If $|a|>1$, the system has one stable fixed point that corresponds to its quiescent state, while for $|a|<1$, a limit cycle represents the spontaneously and periodically firing of the neuron. The quiescent neuron ($|a|$ slightly larger than 1) is excitable because it fires a spike when perturbed away from the fixed point by external stimuli [55]. The value of a is fixed at 1.1 throughout this paper, at which the neuron model has a single stable fixed point and is excitable.

We suppose each neuron is subjected to an additive noise $\xi_i(t)$. The statistical properties of the Gaussian white noise are given by

$$\langle \xi_i(t) \rangle \geq 0, \quad \langle \xi_i(t) \xi_j(t') \rangle = D \delta_{ij} \delta(t-t'), \quad (2)$$

where D is the corresponding noise intensity, δ is the Kronecker symbol, and δ_{ij} denotes that the noise is spatial uncorrelated. We know that noise is able to induce coherent or incoherent firing of excitable neurons, thus, noise is a critical factors determining the synchronized firing patterns of neuronal networks.

Random selected initial values are allocated to all neurons. We use a forward Euler integration scheme with a time step 10^{-3} time unit. Simulations verify further time step reduction does not significantly improve accuracy. The numerical algorithm presented by Sancho et al. [56] will be used to simulate the noise.

3 Results and discussion

Chimera state in neuronal network means the coexistence of synchronized and desynchronized groups. Our former work had reported the SW neuronal network exhibits synchronized firing pattern for special values of time delay, whereas desynchronized pattern for others [47]. It is reasonable for us to expect the occurrence of Chimera state for delay value close to the critical value, which is about $\tau_c=2.60$. For examples, the firing patterns for 3 typical τ values close to τ_c are plotted in Figure 1. We can see that all neurons in the network start to fire after about 40 time units in average. Obviously, the coexistence of synchronized and desynchronized groups indicates the occurrence of chimera state. Although the synchronized (desynchronized) group in Figure 1(a) (Figure 1(c)) disappears rapidly, and the whole network approaches toward a completely desynchronized (synchronized) state finally, the transient chimera state occurs explicitly. On the other hand, the persistent existence of synchronized and desynchronized groups in Figure 1(b) indicates a persistent chimera state, even a permanent one.

In fact, the chimera state occurs due to the competition between the synchronized and desynchronized groups in the neuronal network. Comparing the figures in Figure 1, the

proportion of synchronized group increases with time delay τ . To illustrate this change, the average value of u is plotted against time in Figure 2, in which the value of τ is same as that in Figure 1. Obviously, the more the neurons are in the synchronized group, the more average u oscillates coherently.

To our knowledge, chimera state are always reported in regular network. Omelchenko et al. [36] found chimera state in a regular network with few additional random links. Herein this paper, the SW neuronal network without delay could not sustain chimera state, but special time delay in the coupling is able to support chimera state.

In ref. [46], we identified the parameter regions for synchronization based on the final firing pattern, and transient dynamics was ignored. As mentioned above, the transient chimera state may imply in the transient dynamics of the network. Thus, we anticipate chimera state could be found in extended region of time delay. To identify the parameter region of the chimera state, the time period t_m , in which the chimera state exists, are recorded for different value of time delay (see Figure 3). It should be pointed out that the largest simulated time in the paper is 6000 time units, thus peak values of $t_m \approx 6000$ correspond to permanent chimera states. The permanent-like chimera state only occurs in the region $\tau \in (2.60, 2.70)$ time units. When $\tau < 2.45$ or $\tau > 2.90$, the system reaches to the completely desynchronized or synchronized state rapidly ($t_m \approx 0$), i.e., the chimera state could not be found. For other values of τ , although t_m is small, the transient chimera state occurs before the system reaches the final desynchronized or synchronized patterns. We conclude suitable value of time delay allows the occurrence of chimera state, and the chimera state could be found for parameter

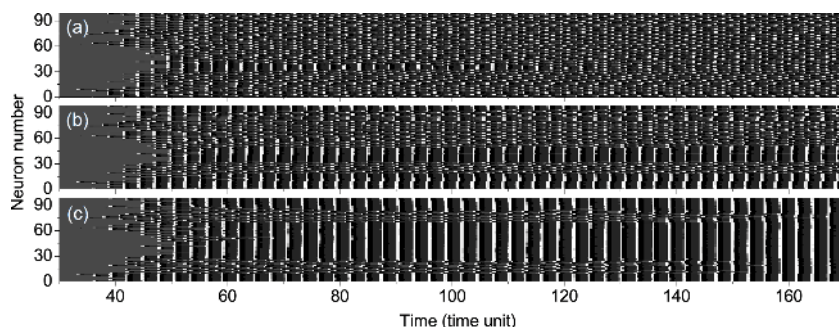


Figure 1 Space-time plots of u for different τ . (a) $\tau=2.57$; (b) $\tau=2.65$; (c) $\tau=2.80$. Noise intensity $D=0.0003$. The black-to-white gray scale represents the lowest value -2.5 to the highest value 2 . This gray scale will be used in all space-time plots throughout this paper.

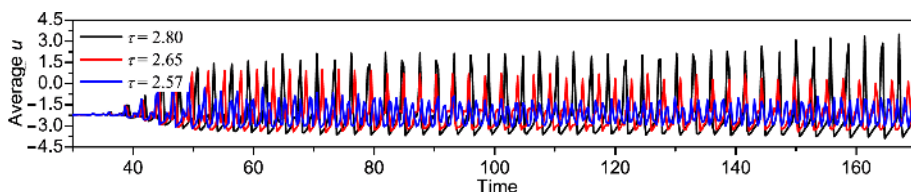


Figure 2 Time-evolution of u averaging over the whole network. All values of parameters are same as that in Figure 1.

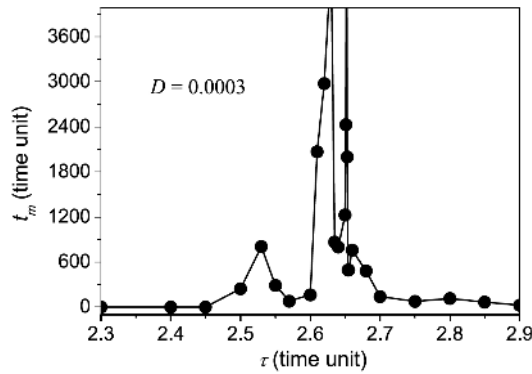


Figure 3 The time period t_m in which the chimera state exists, as a function of time delay τ . Noise intensity $D=0.0003$.

region $\tau \in (2.45, 2.90)$ approximately.

It should be noted that the obvious fluctuation in t_m may be due to the randomness originated from the noise term in eq. (1) and the random rewiring when the SW network is structured.

All above results are obtained for noise intensity $D=0.0003$. Given that the synchronized pattern in neuronal network depends on the noises sensitively, we investigate the chimera state for varies of noise intensities. t_m is plotted against time delay τ in Figure 4 for different noise intensities. The numerical simulation shows that the chimera state could be found explicitly in broad region of noise intensity. Figure 4 tells us that the parameter region for the permanent chimera state is shifted due to the change of noise intensity, but the direction of shift is not certain. On the other hand, the parameter region for the permanent chimera state is broadened significantly by the enhancement of noise intensities. It could be concluded that noise aids time delay in sustaining

chimera states.

Certainly, too weak or strong noise may prevent us from obtaining reasonable synchronized pattern in the neuronal network. In our network model, too weak noise ($D < 0.0002$) could not make the neurons fire, i.e., all neurons are in the quiescent state. For examples, the firing pattern for $D=0.00005$ are plotted in Figure 5(a), no any spike is found. On the other hand, numerical simulation shows that too strong noise ($D > 0.005$) makes the synchronized groups sporadic, which we could not define as chimera state or not (see Figure 5(b) as an example). In the mathematical modelling of biological system, extreme strong noise may drive the system deviating from its stable behaviors, which make the modelling results lost its biological meaning. Thus, we will ignore the firing patterns shown in Figure 5.

To illustrate the influence of noise more clearly, we investigated the firing pattern for varying noise with fixed time delay. As an example, the firing patterns for different noise with $\tau=2.40$ are plotted in Figure 6. We can see that chimera state could not be found for weak or strong noise (see Figure 6(a) and (c)). The transient chimera state occurs for intermediate intensity of noise.

We also study the dependence of chimera state on noise for other values of time delay. As shown in Figure 7, t_m reaches a peak value for the intermediate intensity of noise whatever the time delay τ is. This kind of noise-sustained chimera state is very similar to the noise-induced coherent behaviors in many nonlinear system, which are called coherence or stochastic resonance. On the other hand, we should note the difference in the vertical axis of all subfigures in Figure 7. In Figure 7(a), (e) and (f), the peak values of t_m indicate the transient chimera states, whereas, in Figure 7(b)–(d), the peak values represent the permanent chimera states. The

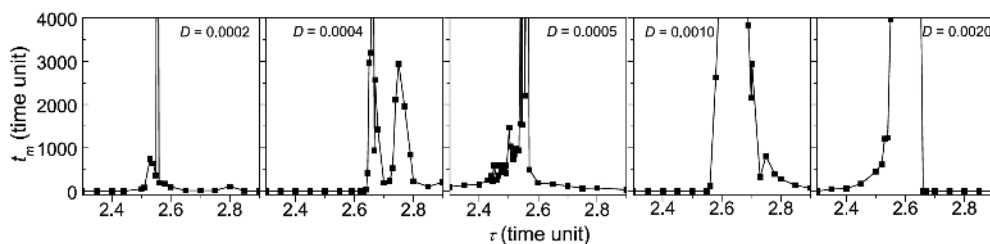


Figure 4 The time period t_m in which the chimera state exists, as a function of time delay τ for different noise intensity.

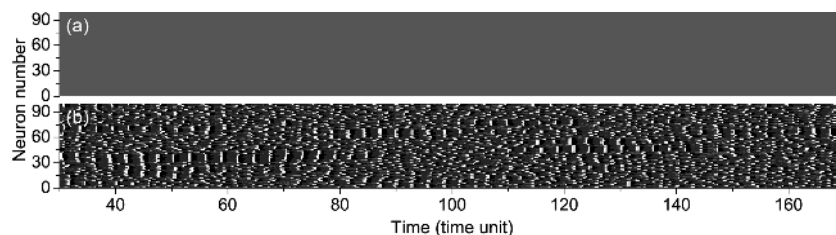


Figure 5 Space-time plots of u for different noise intensity. (a) $D=0.00005$; (b) $D=0.006$. Time delay $\tau=2.80$.

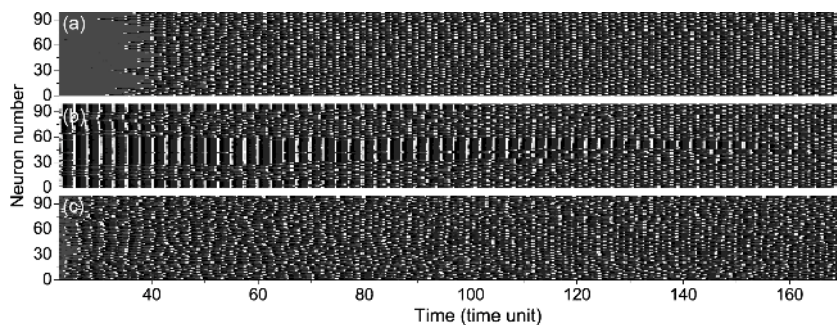


Figure 6 Space-time plots of u for different noise intensity. (a) $D=0.0004$; (b) $D=0.0008$; (c) $D=0.0015$. Time delay $\tau=2.40$.

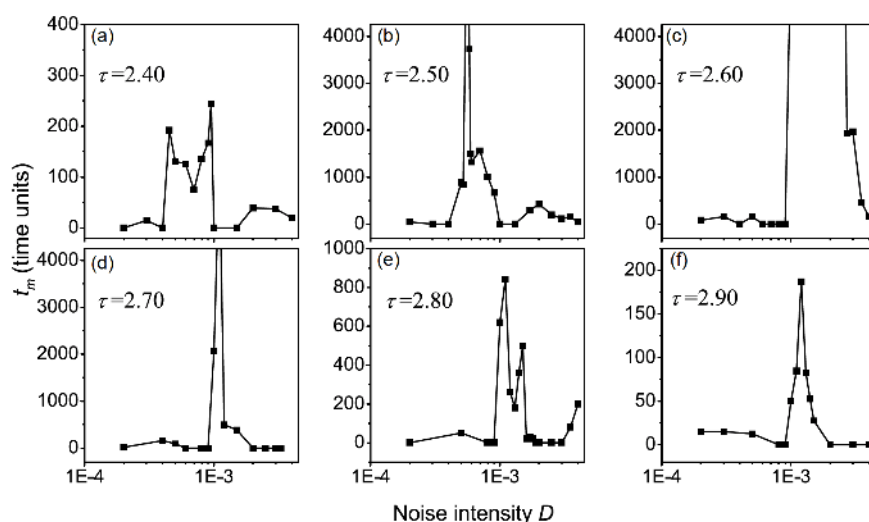


Figure 7 The time period t_m in which the chimera state exists, as a function of noise intensity D for different time delay τ . (a) $\tau=2.4$; (b) $\tau=2.5$; (c) $\tau=2.6$; (d) $\tau=2.7$; (e) $\tau=2.8$; (f) $\tau=2.9$.

huge difference in the peak values tells us that the time delay τ is the key factor determining the occurrence of chimera state, although intermediate amount of noise facilitates the enhancement of t_m .

4 Conclusions

In summary, we investigate chimera state in a noisy delayed-coupled SW neuronal network, in which FHN equations are employed to model the single neuron dynamics. Our former work reported that the neuronal network may transit from a desynchronized pattern to a synchronized one when the time delay increase across a critical value, and the synchronized pattern depends on the noise intensity sensitively [47]. In this paper, we find transient or permanent chimera states with the value of time delay close to the critical value τ_c for which the synchronization transition occurs. The chimera state occurs due to the competition between the synchronized and desynchronized patterns in the neuronal network, which is not similar to that has been reported in many theoretical works. We focus on the effect of time delay on the occurrence of

chimera state. In other study about chimera state, time delay is ignored mostly [10,13,14,17,20, 21,31,36]. The time period, during which the chimera state exists, is recorded to illuminate the transient and permanent chimera states. The transient or permanent chimera state could be found for broad parameter region $\tau \in (2.45, 2.90)$. On the other hand, intermediate amount of noise facilitates the occurrence of chimera states, but time delay is the deterministic factor.

In the present study, the chimera state is originated from the coexistence of synchronized and desynchronized. In fact, it has been widely reported that there exist many other synchronization states in neuronal network, such as, complete synchronization, antiphase synchronization, phase synchronization etc. The coexistence of other synchronization states will be an interesting problem [57–61], which motivates our future work.

It should be pointed out that there are a large number of network realizations with the same values of all parameters. Our results are obtained from only one network realization. To test the generalization of the results, we perform parts of the study on several other network realizations with the same parameter values. The test shows that although the figures

are not totally same, all the results about occurrence of chimera state are similar for different network realizations. That implies that the results in this paper are general and independent on the network realization.

This work was supported by the Fundamental Research Funds for the Central Universities of China (Grant No. 2015XKMS080(JT)).

- 1 Kuramoto Y, Battogtokh D. Coexistence of coherence and incoherence in nonlocally coupled phase oscillators. *Nonlinear Phenom Complex Syst*, 2002, 5: 380–385
- 2 Zhang X, Bi H, Guan S, et al. Model bridging chimera state and explosive synchronization. *Phys Rev E*, 2016, 94: 012204
- 3 Singha J, Gupte N. Spatial splay states and splay chimera states in coupled map lattices. *Phys Rev E*, 2016, 94: 052204
- 4 Maksimenko V A, Makarov V V, Bera B K, et al. Excitation and suppression of chimera states by multiplexing. *Phys Rev E*, 2016, 94: 052205
- 5 Abrams D M, Strogatz S H. Chimera states for coupled oscillators. *Phys Rev Lett*, 2004, 93: 174102
- 6 Abrams D M, Mirollo R, Strogatz S H, et al. Solvable model for Chimera states of coupled oscillators. *Phys Rev Lett*, 2008, 101: 084103
- 7 Laing C R. The dynamics of chimera states in heterogeneous Kuramoto networks. *Phys D-Nonlinear Phenomena*, 2009, 238: 1569–1588
- 8 Martens E A, Laing C R, Strogatz S H. Solvable model of spiral wave chimeras. *Phys Rev Lett*, 2010, 104: 044101
- 9 Omel'chenko O E, Wolfrum M, Yanchuk S, et al. Stationary patterns of coherence and incoherence in two-dimensional arrays of non-locally-coupled phase oscillators. *Phys Rev E*, 2012, 85: 036210
- 10 Martens E A. Chimeras in a network of three oscillator populations with varying network topology. *Chaos*, 2010, 20: 043122
- 11 Wolfrum M, Omel'chenko O E, Yanchuk S, et al. Spectral properties of chimera states. *Chaos*, 2011, 21: 013112
- 12 Maistrenko Y, Brezetsky S, Jaros P, et al. Smallest chimera states. *Phys Rev E*, 2017, 95: 010203
- 13 Premalatha K, Chandrasekar V K, Senthilvelan M, et al. Chimeralike states in two distinct groups of identical populations of coupled Stuart-Landau oscillators. *Phys Rev E*, 2017, 95: 022208
- 14 Mishra A, Saha S, Hens C, et al. Coherent libration to coherent rotational dynamics via chimeralike states and clustering in a Josephson junction array. *Phys Rev E*, 2017, 95: 010201
- 15 Omel'chenko O E, Wolfrum M, Maistrenko Y L. Chimera states as chaotic spatiotemporal patterns. *Phys Rev E*, 2010, 81: 065201
- 16 Cano A V, Cosenza M G. Chimeras and clusters in networks of hyperbolic chaotic oscillators. *Phys Rev E*, 2017, 95: 030202
- 17 Ujjwal S R, Punetha N, Prasad A, et al. Emergence of chimeras through induced multistability. *Phys Rev E*, 2017, 95: 032203
- 18 Schmidt A, Kasimatis T, Hizanidis J, et al. Chimera patterns in two-dimensional networks of coupled neurons. *Phys Rev E*, 2017, 95: 032224
- 19 Omelchenko I, Omel'chenko O E, Hövel P, et al. When nonlocal coupling between oscillators becomes stronger: Patched synchrony or multichimera states. *Phys Rev Lett*, 2013, 110: 224101
- 20 Laing C R, Chow C C. Stationary bumps in networks of spiking neurons. *Neural Comput*, 2001, 13: 1473–1494
- 21 Sakaguchi H. Instability of synchronized motion in nonlocally coupled neural oscillators. *Phys Rev E*, 2006, 73: 031907
- 22 Martens E A, Thutupalli S, Fourrière A, et al. Chimera states in mechanical oscillator networks. *Proc Natl Acad Sci USA*, 2013, 110: 10563–10567
- 23 Hagerstrom A M, Murphy T E, Roy R, et al. Experimental observation of chimeras in coupled-map lattices. *Nat Phys*, 2012, 8: 658–661
- 24 Tinsley M R, Nkomo S, Showalter K. Chimera and phase-cluster states in populations of coupled chemical oscillators. *Nat Phys*, 2012, 8: 662–665
- 25 Larger L, Penkovsky B, Maistrenko Y. Virtual chimera states for delayed-feedback systems. *Phys Rev Lett*, 2013, 111: 054103
- 26 Isomura Y, Fujiwara-Tsakamoto Y, Takada M. A network mechanism underlying hippocampal seizure-like synchronous oscillations. *NeuroSci Res*, 2008, 61: 227–233
- 27 Tass P A, Qin L, Hauptmann C, et al. Coordinated reset has sustained aftereffects in Parkinsonian monkeys. *Ann Neurol*, 2012, 72: 816–820
- 28 Rattenborg N C, Amlaner C J, Lima S L. Behavioral, neurophysiological and evolutionary perspectives on unihemispheric sleep. *NeuroSci BioBehav Rev*, 2000, 24: 817–842
- 29 Rial R, González J, Gené L, et al. Asymmetric sleep in apneic human patients. *Am J Physiol-Regulatory Integrative Comp Physiol*, 2013, 304: R232–R237
- 30 Omel'chenko O E. Coherence-incoherence patterns in a ring of non-locally coupled phase oscillators. *Nonlinearity*, 2013, 26: 2469–2498
- 31 Glaze T A, Lewis S, Bahar S. Chimera states in a Hodgkin-Huxley model of thermally sensitive neurons. *Chaos*, 2016, 26: 083119
- 32 Laing C R. Chimeras in networks of planar oscillators. *Phys Rev E*, 2010, 81: 066221
- 33 Shanahan M. Metastable chimera states in community-structured oscillator networks. *Chaos*, 2010, 20: 013108
- 34 Zhu Y, Zheng Z, Yang J. Chimera states on complex networks. *Phys Rev E*, 2014, 89: 022914
- 35 Yeldesbay A, Pikovsky A, Rosenblum M. Chimeralike states in an ensemble of globally coupled oscillators. *Phys Rev Lett*, 2014, 112: 144103
- 36 Omelchenko I, Provata A, Hizanidis J, et al. Robustness of chimera states for coupled FitzHugh-Nagumo oscillators. *Phys Rev E*, 2015, 91: 022917
- 37 Kandel E R, Schwartz J H, Jessell T M. Principles of Neural Science. Amsterdam: Elsevier, 1991
- 38 Wang Q, Perc M, Duan Z, et al. Synchronization transitions on scale-free neuronal networks due to finite information transmission delays. *Phys Rev E*, 2009, 80: 026206
- 39 Yu W T, Tang J, Ma J, et al. Heterogeneous delay-induced asynchrony and resonance in a small-world neuronal network system. *EPL*, 2016, 114: 50006
- 40 Qian Y, Zhang Z. Effects of time delay and connection probability on self-sustained oscillations and synchronization transitions in excitable Erdős-Rényi random networks. *Commun Nonlinear Sci Numer Simul*, 2017, 47: 127–138
- 41 Sun X, Perc M, Kurths J. Effects of partial time delays on phase synchronization in Watts-Strogatz small-world neuronal networks. *Chaos*, 2017, 27: 053113
- 42 Zheng Y G, Bao L J. Effect of topological structure on synchronizability of network with connection delay. *Chaos Solitons Fractals*, 2017, 98: 145–151
- 43 Zhu J, Liu X. Locking induced by distance-dependent delay in neuronal networks. *Phys Rev E*, 2016, 94: 052405
- 44 Yang X L, Hu L P, Sun Z K. How time-delayed coupling influences differential feedback control of bursting synchronization in modular neuronal network. *Nonlinear Dyn*, 2016, 86: 1797–1806
- 45 Qian Y. Time delay and long-range connection induced synchronization transitions in Newman-Watts small-world neuronal networks. *PLoS ONE*, 2014, 9: e96415
- 46 Fan D G, Wang Q Y. Synchronization and bursting transition of the coupled Hindmarsh-Rose systems with asymmetrical time-delays. *Sci China Technol Sci*, 2017, 60: 1019–1031
- 47 Tang J, Ma J, Yi M, et al. Delay and diversity-induced synchronization transitions in a small-world neuronal network. *Phys Rev E*, 2011, 83: 046207
- 48 Yilmaz E, Baysal V, Perc M, et al. Enhancement of pacemaker induced stochastic resonance by an autapse in a scale-free neuronal network. *Sci China Technol Sci*, 2016, 59: 364–370
- 49 Chen Y L, Yu L C, Chen Y. Reliability of weak signals detection in

- neurons with noise. *Sci China Technol Sci*, 2016, 59: 411–417
- 50 Wang Z Q, Xu Y, Yang H. Lévy noise induced stochastic resonance in an FHN model. *Sci China Technol Sci*, 2016, 59: 371–375
- 51 Tang J, Jia Y, Yi M, et al. Multiplicative-noise-induced coherence resonance via two different mechanisms in bistable neural models. *Phys Rev E*, 2008, 77: 061905
- 52 Tang J, Liu T B, Ma J, et al. Effect of calcium channel noise in astrocytes on neuronal transmission. *Commun Nonlinear Sci Numer Simul*, 2016, 32: 262–272
- 53 Jin W, Lin Q, Wang A, et al. Computer simulation of noise effects of the neighborhood of stimulus threshold for a mathematical model of homeostatic regulation of sleep-wake cycles. *Complexity*, 2017, 2017: 1–7
- 54 Li Q, Gao Y. Spiking regularity in a noisy small-world neuronal network. *BioPhys Chem*, 2007, 130: 41–47
- 55 Zhou C, Kurths J, Hu B. Array-enhanced coherence resonance: Nontrivial effects of heterogeneity and spatial independence of noise. *Phys Rev Lett*, 2001, 87: 098101
- 56 Sancho J M, San Miguel M, Katz S L, et al. Analytical and numerical studies of multiplicative noise. *Phys Rev A*, 1982, 26: 1589–1609
- 57 Zhao Z, Gu H. Identifying time delay-induced multiple synchronous behaviours in inhibitory coupled bursting neurons with nonlinear dynamics of single neuron. *Procedia IUTAM*, 2017, 22: 160–167
- 58 Gu H, Zhao Z. Dynamics of time delay-induced multiple synchronous behaviors in inhibitory coupled neurons. *PLoS ONE*, 2015, 10: e0138593
- 59 Zhao Z, Gu H. The influence of single neuron dynamics and network topology on time delay-induced multiple synchronous behaviors in inhibitory coupled network. *Chaos Solitons Fractals*, 2015, 80: 96–108
- 60 Gu H, Pan B, Li Y. The dependence of synchronization transition processes of coupled neurons with coexisting spiking and bursting on the control parameter, initial value, and attraction domain. *Nonlinear Dyn*, 2015, 82: 1191–1210
- 61 Gu H G, Chen S G, Li Y Y. Complex transitions between spike, burst or chaos synchronization states in coupled neurons with coexisting bursting patterns. *Chin Phys B*, 2015, 24: 050505