IET Control Theory & Applications

Research Article

IET Journals
The Institution of Engineering and Technology

Distributed iterative learning coordination control for leader–follower uncertain non-linear multi-agent systems with input saturation

ISSN 1751-8644
Received on 4th November 2018
Revised 9th May 2019
Accepted on 10th June 2019
E-First on 11th July 2019
doi: 10.1049/iet-cta.2018.6268
www.ietdl.org

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Abstract: In this study, the fully distributed adaptive iterative learning coordination control of the uncertain non-linear leader–follower multi-agent systems with input saturation is studied. Under the alignment initial condition and Lyapunov theory, a novel adaptive distributed control protocol with a fully saturated parameter learning law is designed. Despite the existence of input saturation, the global perfect consensus tracking can be realised over a finite time interval. Besides, the consensus tracking problem is extended to the formation control problem as well. Ultimately, the validity of theoretical analysis of this study is shown by two examples.

1 Introduction

In the past few decades, the distributed coordination control for multi-agent systems (MASs) has earned considerable concerns from multidisciplinary researchers on account of its broad use in transport systems, distributed wireless communication networks, service robots, spacecraft formation flying [1] and so on. Based on different collaborative tasks, the coordination control is divided mainly into two kinds of controls, one of which is the consensus control [2] that the states of agents arrive at a common state through their interactions, the other is the formation control [3] which is about manoeuvring many floating single agents to operate and keep a predesigned form. Up to now, as a basic problem of MASs, the consensus problem has been extensively researched by scholars from different perspectives, such as the switching [4, 5] or fixed [6, 7] topology, the known [8, 9] or unknown [10, 11] directions, with [12–14] or without [15, 16] a leader and so on. While the consensus problem with a leader is named as the consensus tracking.

It should be noted that the above literature on MASs are executed relying on the implicit assumption, i.e. all agents can run without any limitations, which is apparently impossible in practical applications. Since most physical actuators in the actual control systems may suffer amplitude saturation due to hardware restrictions, which is called input saturation. It means that the amplitude of input is finite over a bounded domain and cannot be arbitrarily large. The performance of systems would be caused by instability or destroyed by this sort of saturation non-linearities [17]. Therefore, it is significant and necessary to analyse the system with input saturation. Many works with respect to input saturation have been reported in [18–25]. Lin and Lin [18] solved the control problem of an individual linear system with actuator saturation using the low gain feedback technique. Later, the authors [19, 20] applied this method to achieve the semi-global consensus, while the global consensus was obtained in [21–23]. Different from [18–23], the author in [24, 25] utilised the distributed optimal control scheme to make the tracking errors uniformly ultimately bounded.

However, all the aforementioned studies realised the asymptotic consensus on the infinite time interval. If the arbitrary high accuracy is required for the coordination control of MASs over a finite time interval, these works cannot resolve. As is well known, iterative learning control (ILC) is a sole and valid control technique realising repetitive control task on a limited temporal interval for accurate tracking requirements. Interested readers please refer to survey papers [26, 27] for an excellent review of the progresses on ILC, and the reference therein. ILC for an individual non-linear system [28, 29], there are also some papers on input saturation. For instance, an ILC method for a kind of non-linear uncertain systems with input saturation was designed in [30], and the non-linearly parametric systems with input saturation were investigated in [31–33]. Moreover, as an important branch of ILC, optimal ILC has been widely investigated in [34–36]. The authors of [34, 35] proposed some model predictive control (MPC)-based ILC schemes by incorporating input saturation and output constraint into the optimisation problem, and Chi *et al.* [36] considered a constrained data-driven optimal ILC for a class of non-linear systems with input saturation and output constraint directly. Recently, ILC has been utilised to deal with the coordination problem of MASs [37–41], where the authors of [37–39] solved the formation control problem via ILC but the authors of [40, 41] handled the consensus tracking problem using ILC and distributed optimal ILC. In addition, the adaptive ILC for MASs without input saturation has been reported in [42–47]. The authors in [42, 43] addressed the consensus problems of the first- and high-order non-linear MASs with constraints, respectively. Chen and Li [44] investigated the perfect consensus problem for second-order linearly parameterised MASs with imprecise communication topology structure and Bu et al. [45] proposed a distributed model free adaptive ILC method for a class of unknown non-linear MASs to perform consensus tracking. An adaptive ILC of consensus problem for the leader–follower non-linear MASs was presented in [46] and the consensus tracking in L_T^2-sence was obtained, and Li and Li [47] introduced the fully distributed adaptive ILC to tackle the coordination control of MASs.

By the above observations, although numerous papers went into MASs with input saturation or ILC, the coordination control for MASs with input saturation has not been addressed under the framework of ILC. Therefore, it is highly desirable for us to develop a new ILC protocol for MASs with input saturation. This inspires us to put forward the fully distributed coordination control for the uncertain non-linear leader–follower MASs with input saturation utilising an adaptive ILC algorithm.

In this paper, we focus mainly on the distributed coordination control for the uncertain non-linear leader–follower MASs with input saturation using adaptive ILC. Under the alignment condition, a novel adaptive distributed control protocol, which is independent of any global information is presented, and a fully saturated parameter learning law for coupling gains is designed. Meanwhile, even though there exists input saturation in the system dynamics of each follower agent, the global perfect consensus tracking can be achieved on $[0, T]$. Furthermore, we extend the consensus results to the formation control problem. Finally, two examples testify the efficiency of theoretical analysis in this paper. In conclusion, the main contributions of the paper can be summarised as follows: (i) unlike the literatures on MASs with input saturation, such as [19–25], in this paper, the global perfect consensus tracking can be gained on $[0, T]$; (ii) literatures [30–33] investigated the convergence problem of an individual system with input saturation under the identical initial condition, whereas we take into account the coordination control problem of MASs with input saturation under the alignment condition, which is more practical; and (iii) [44, 47] settled the fully distributed coordination control for MASs without input saturation. To be applied more highly to the real world, we have tackled the coordination problem of the ILC-based MASs with input saturation, at the same time, the fully distributed coordination control protocols are also proposed. It is the first time to solve the distributed coordination problem of the uncertain non-linear MASs with input saturation using adaptive ILC under the alignment condition.

The remainder of the paper is divided into the following sections. The preliminaries are given in Section 2. Section 3 is the problem formulation. In Section 4, the distributed consensus algorithm for the uncertain MASs with input saturation is presented. Section 5 is the extension of the consensus results to the formation control and Section 6 enumerates two illustrative examples. At last, the conclusion is provided in Section 7.

2 Preliminaries

An undirected graph is denoted as $\bar{G} = (\bar{V}, \bar{E}, \bar{A})$, where $\bar{V} = \{\bar{v}_1, \ldots, \bar{v}_N\}$ is the set of vertices and $\bar{E} \subseteq \bar{V} \times \bar{V}$ is the set of edges. $\bar{A} = [a_{ij}] \in R^{N \times N}$ is the weighted adjacency matrix of the graph $\bar{G}$. If there is an edge between agents v_i and v_j, i.e. $(v_j, v_i) \in \bar{E}$, then $a_{ij} = a_{ji} = 1$, otherwise $a_{ij} = a_{ji} = 0$. Moreover, it is assumed that $a_{ii} = 0$. The set of all neighbours of the ith agent is $N_i = \{v_j: (v_j, v_i) \in \bar{E}\}$. The Laplacian matrix of $\bar{G}$ is $L = D - \bar{A}$, where $D = \mathrm{diag}\{d_1, \ldots, d_N\}$ with $d_i = \sum_{j=1}^{N} a_{ij}$. A path is a sequence of connected edges in the graph. For the undirected graph $\bar{G}$, the adjacency matrix $\bar{A}$ is symmetric and the graph $\bar{G}$ is connected if there is a path between any two vertices.

In what follows, we mainly concern G' associated with the system consisting of N follower agents whose topology graph is denoted by $\bar{G}$ and one leader (labelled as 0). b_i denotes the access of the ith agent to the leader, i.e. $b_i = 1$ if agent v_i has direct access to the full information of the leader, otherwise, $b_i = 0$. It is obvious that $H = L + B$ is a symmetric matrix associated with G', where L is the Laplacian matrix of $\bar{G}$ and $B = \mathrm{diag}\{b_1, \ldots, b_n\}$.

Lemma 1: If graph G' is connected, then the symmetric matrix H associated with G' is positive definite [4].

Definition 1: For a scalar $u(t)$, a saturation function $\mathrm{sat}(u(t), u^*)$ is defined as

$$\mathrm{sat}(u(t), u^*) \overset{\Delta}{=} \begin{cases} u(t) & |u(t)| \le u^* \\ \mathrm{sign}(u(t))u^* & \text{else,} \end{cases} \tag{1}$$

where $u^* > 0$.

To facilitate the subsequent analysis, three properties in regard to saturation function will be given below.

Property 1: For a given $u_0(t)$, satisfying $\max_{t \in [0, T]} |u_0(t)| \le u^*$, then

$$[u_0(t) - \mathrm{sat}(u(t), u^*)]^2 \le [u_0(t) - u(t)]^2 \ \ [30]. \tag{2}$$

Property 2: For $h = \mathrm{sat}(\mu, b) + s$, where $b > 0$, then the following inequality is established [30]. i.e.,

$$|\mathrm{sat}(h, b) - h| \le |s|. \tag{3}$$

Property 3: For $m, r \in R$, if m satisfies $|m| < r^*$, then

$$[m - \mathrm{sat}(r, r^*)][r - \mathrm{sat}(r, r^*)] \le 0 \ \ [48]. \tag{4}$$

3 Problem formulation

In view of a set of N identical follower agents with a leader in the repetitive environment, at the kth iteration, each follower agent is governed by

$$\dot{x}_i^k(t) = \eta(x_i^k(t), t) + \mathrm{sat}(u_i^k(t), u^*), \tag{5}$$

where $i = 1, 2, \ldots, N$; $x_i^k(t) \in R$ and $u_i^k(t) \in R$ are the state and input of the ith agent; $\eta(x_i^k(t), t)$ is an unknown time-varying global Lipschitz continuously differentiable function in $x_i^k(t)$ and piecewise continuous in t; and $\mathrm{sat}(u_i^k(t), u^*)$ is a saturation function defined in Definition 1.

The leader whose state is denoted as $x_0(t) \in R$ satisfies the following dynamic

$$\dot{x}_0(t) = \eta(x_0(t), t) + u_0(t), \tag{6}$$

where $u_0(t) \in R$ is the leader's input.

Remark 1: Literatures [12–14, 44–47] on the leader–follower MASs, there is a tacit assumption that the input of the leader $u_0(t) = 0$. This assumption might be rigorous in different circumstances. Actually, to refrain from dangerous barriers or get an ideal consensus, non-zero control can be exerted on the leader. Also, this part aims to deal with the ordinary form of the leader–follower consensus, i.e. the input of the leader is non-zero. From now on, we will give the assumption on this.

Assumption 1: The control input of the leader is finite, i.e. $|u_0(t)| \le u^*$.

Define the consensus error for the MASs (5) and (6) as

$$\delta_i^k(t) = x_i^k(t) - x_0(t). \tag{7}$$

The ultimate target of the paper is to find a series of appropriate control protocols $\{u_i^k(t), 0 \le t \le T, i = 1, 2, \ldots, N;\ k \in Z^+\}$ such that $x_i^k(t) \to x_0(t)$ when $k \to \infty$, i.e. $\lim_{k \to \infty} \delta_i^k(t) = 0$, $i = 1, 2, \ldots, N$. In other words, each follower agent can perfectly track the leader in the iteration domain, i.e. the global perfect consensus tracking can be achieved.

Furthermore, the agent only knows the information from its neighbours. Hence, we should define the distributed error for the ith agent as

$$e_i^k(t) = \sum_{j=1}^{N} a_{ij}(x_j^k(t) - x_i^k(t)) + b_i(x_0(t) - x_i^k(t)). \tag{8}$$

According to the definition of (8), the compact form of the error can be expressed as

$$e^k(t) = -(L + B)[x^k(t) - 1_N x_0(t)] = -H\delta^k(t), \tag{9}$$

where

$$\begin{aligned} 1_N &= [1, 1, \dots, 1]^{\mathrm{T}} \in R^N, \\ e^k(t) &= [e_1^k(t), e_2^k(t), \dots, e_N^k(t)]^{\mathrm{T}}, \\ x^k(t) &= [x_1^k(t), x_2^k(t), \dots, x_N^k(t)]^{\mathrm{T}}, \\ \delta^k(t) &= [\delta_1^k(t), \delta_2^k(t), \dots, \delta_N^k(t)]^{\mathrm{T}}. \end{aligned} \tag{10}$$

Remark 2: It can be seen that the consensus tracking problem of this paper is the typical tracking issue when $N = 1$. It is non-trivial from an individual system to MASs. On the one hand, the state of the leader can be only acquired by some follower agents. Therefore, the error (7) is unable to directly design the control protocol. We define the error (8) for the consensus control purpose. On the other hand, the communication topology among all agents plays a crucial role in consensus analyse. While selecting the Lyapunov–Krasovskii functional, the topology structure should be considered.

Assumption 2: The alignment initial condition is satisfied, i.e. $x_i^k(0) = x_i^{k-1}(T)$ and $x_0(0) = x_0(T)$.

Remark 3: Though the authors of [30–33] investigated the individual uncertain non-linear system with input saturation, only the identical initial condition (i.i.c) was utilised. Nevertheless, the i.i.c cannot be applied to the MASs. Since each follower agent cannot have direct access to the state of the leader, it is hard to make the i.i.c hold in the MASs. Moreover, the alignment condition only requires that the initial state of each iteration for the follower agents equals to the final state of the previous iteration, regardless of the leader. As far as the leader is spatially closed. Therefore, it is less restrictive than the i.i.c to some extent in the MASs. Meanwhile, many existing works including [42–47] without input saturation have utilised the alignment initial condition to the MASs. Consequently, it is of great value to assume this condition in the MASs with input saturation.

Assumption 3: $\eta(x', t)$ is an unknown function which satisfies

$$|\eta(x_1', t) - \eta(x_2', t)| \le l|x_1' - x_2'|, \tag{11}$$

for all x_1' and x_2' in R, where $l > 0$.

In what follows, the variable t will be omitted when no confusion would arise.

4 Distributed consensus scheme via adaptive ILC for uncertain MASs with input saturation

During the kth iteration, the error dynamic of the ith agent can be calculated as

$$\dot{\delta}_i^k = \eta(x_i^k, t) + \tilde{u}_i^k - \eta(x_0, t) - u_0, \tag{12}$$

where $\tilde{u}_i^k = \mathrm{sat}(u_i^k, u^*)$. Then, the vector form is

$$\dot{\delta}^k = \eta(x^k) + \tilde{u}^k - 1_N\eta(x_0) - 1_N u_0, \tag{13}$$

where

$$\begin{aligned} \eta(x^k) &= [\eta(x_1^k, t), \eta(x_2^k, t), \dots, \eta(x_N^k, t)]^{\mathrm{T}}, \\ \tilde{u}^k &= [\tilde{u}_1^k, \tilde{u}_2^k, \dots, \tilde{u}_N^k]^{\mathrm{T}}. \end{aligned} \tag{14}$$

For the sake of working out the consensus issue of MASs (5) and (6), the distributed learning-based protocol is

$$u_i^k(t) = \hat{\phi}_i^k(t)e_i^k + \mathrm{sat}(u_i^{k-1}, u^*), u_i^{-1}(t) = 0, \quad \forall t \in [0, T], \tag{15}$$

with a fully saturated difference adaptive learning law for adjusting the coupling control gain

$$\hat{\phi}_i^k(t) = \mathrm{sat}(\hat{\phi}_i^{k-1}(t) + q_i(e_i^k)^2, \phi^*), \hat{\phi}_i^{-1}(t) = 0, \tag{16}$$

where $\hat{\phi}_i^k(t)$ is the time-varying control gain among neighbouring agents for the ith agent with $\hat{\phi}_i^0(0) > 0$; $q_i > 0$ is a designed constant and ϕ^* is the saturation bound of $\hat{\phi}_i^k(t)$, which can be chosen by the designer.

Remark 4: It should be noted that the control protocol (15) is a novel distributed adaptive ILC protocol, which includes the adaptive term $\hat{\phi}_i^k(t)e_i^k$ and the saturation term $\mathrm{sat}(u_i^{k-1}, u^*)$. The adaptive term has the time-varying control gain $\hat{\phi}_i^k(t)$, which can make the control protocol fully distributed; and $\mathrm{sat}(u_i^{k-1}, u^*)$ is used to obtain the perfect consensus tracking over $[0, T]$. At the same time, it can be seen from the saturated difference adaptive law (16), when $\hat{\phi}_i^k(t)$ reaches the upper bound ϕ^*, the control protocol (15) becomes the closed-loop P-type ILC law in [30]. Furthermore, it is obvious that $\hat{\phi}_i^k(t)$ is bounded. Then, $\hat{\phi}_i^0(0) > 0$ can guarantee $\hat{\phi}_i^k(t) > 0$.

The control protocol (15) is represented as the compact form

$$u^k = \tilde{u}^{k-1} + \hat{\Phi}^k e^k = \tilde{u}^{k-1} - \hat{\Phi}^k H\delta^k, \tag{17}$$

where

$$\begin{aligned} u^k &= [u_1^k, u_2^k, \dots, u_N^k]^{\mathrm{T}} \in R^N, \\ \tilde{u}^{k-1} &= [\mathrm{sat}(u_1^{k-1}), \mathrm{sat}(u_2^{k-1}), \dots, \mathrm{sat}(u_N^{k-1})]^{\mathrm{T}} \in R^N, \\ \hat{\Phi}^k &= \mathrm{diag}\{\hat{\phi}_1^k(t), \hat{\phi}_2^k(t), \dots, \hat{\phi}_N^k(t)\} \in R^{N \times N}. \end{aligned} \tag{18}$$

Theorem 1: Consider the connected graph G' of the MASs (5) and (6), Assumptions 1–3 hold, then N follower agents expressed by (5) under the protocol (15) and the learning-based updating law (16) for the coupling control gains can perfectly track the leader (6) in the iteration domain on $[0, T]$, i.e. the global perfect consensus tracking can be achieved, and the variables involved in the closed system are all bounded.

Proof: First of all, a Lyapunov function is constructed as

$$\begin{aligned} V^k(t) &= \frac{1}{2}(e^k)^{\mathrm{T}}H^{-1}e^k \\ &= \frac{1}{2}(\delta^k)^{\mathrm{T}}H\delta^k. \end{aligned} \tag{19}$$

Taking the derivative of (19) and recalling the error dynamic (13) and the protocol (17) yield

$$\begin{aligned} \dot{V}^k(t) &= (\delta^k)^{\mathrm{T}}H\dot{\delta}^k \\ &= (\delta^k)^{\mathrm{T}}H(\eta(x^k) + \tilde{u}^k - 1_N\eta(x_0) - 1_N u_0) \\ &= (\delta^k)^{\mathrm{T}}H(\eta(x^k) - 1_N\eta(x_0) + \tilde{u}^k - u^k + u^k - 1_N u_0) \\ &= (\delta^k)^{\mathrm{T}}H(\eta(x^k) - 1_N\eta(x_0) + \tilde{u}^k - u^k + \tilde{u}^{k-1} \\ &\quad - \hat{\Phi}^k(t)H\delta^k - 1_N u_0). \end{aligned} \tag{20}$$

From Assumption 3, we know

$$
\begin{aligned}
&(\delta^k)^{\mathrm{T}} H[\eta(x^k) - 1_N \eta(x_0)] \\
&\quad = -\sum_{i=1}^{N} e_i^k [\eta(x_i^k) - \eta(x_0)] \\
&\quad \le \sum_{i=1}^{N} |e_i^k| [|\eta(x_i^k) - \eta(x_0)|] \\
&\quad \le l \sum_{i=1}^{N} |e_i^k| |\delta_i^k| \\
&\quad = l |e^k|^{\mathrm{T}} |\delta^k| \\
&\quad = l |-H\delta^k|^{\mathrm{T}} |\delta^k| \\
&\quad \le l \| H\delta^k \| \| \delta^k \| \\
&\quad \le l \lambda_{\max}(H) (\delta^k)^{\mathrm{T}} \delta^k .
\end{aligned} \tag{21}
$$

Thus, (20) becomes

$$
\begin{aligned}
\dot{V}^k(t) \le{}& l\lambda_{\max}(H)(\delta^k)^{\mathrm{T}}\delta^k - (\delta^k)^{\mathrm{T}} H \hat{\Phi}^k(t) H \delta^k \\
&+ (\delta^k)^{\mathrm{T}} H(\tilde{u}^k - u^k + \tilde{u}^{k-1} - 1_N u_0) .
\end{aligned} \tag{22}
$$

Select a Lyapunov–Krasovskii functional as

$$
\begin{aligned}
E^k(t) ={}& V^k + \sum_{i=1}^{N} \frac{1}{2q_i} \int_0^t (\tilde{\phi}_i^k(\tau))^2 \mathrm{d}\tau \\
&+ \sum_{i=1}^{N} \int_0^t \frac{1}{2\hat{\phi}_i^k(\tau)} (\delta u_i^k)^2 \mathrm{d}\tau,
\end{aligned} \tag{23}
$$

where $\tilde{\phi}_i^k(t) = \phi - \hat{\phi}_i^k(t)$ with $\phi > 0$ and $\delta u_i^k = u_i^k - u_0$.

Now, the difference of $E^k(t)$ can be written as

$$
\begin{aligned}
\Delta E^k ={}& E^k - E^{k-1} \\
={}& V^k + \sum_{i=1}^{N} \frac{1}{2q_i} \int_0^t [(\tilde{\phi}_i^k(\tau))^2 - (\tilde{\phi}_i^{k-1}(\tau))^2] \mathrm{d}\tau \\
&+ \sum_{i=1}^{N} \int_0^t \frac{1}{2\hat{\phi}_i^k} (\delta u_i^k)^2 \mathrm{d}\tau \\
&- \sum_{i=1}^{N} \int_0^t \frac{1}{2\hat{\phi}_i^{k-1}} (\delta u_i^{k-1})^2 \mathrm{d}\tau - V^{k-1} \\
={}& \int_0^t \dot{V}^k d\tau + \sum_{i=1}^{N} \frac{1}{2q_i} \int_0^t [(\tilde{\phi}_i^k)^2 - (\tilde{\phi}_i^{k-1})^2] \mathrm{d}\tau \\
&+ \sum_{i=1}^{N} \int_0^t \frac{1}{2\hat{\phi}_i^k} (\delta u_i^k)^2 \mathrm{d}\tau + V^k(0) \\
&- \sum_{i=1}^{N} \int_0^t \frac{1}{2\hat{\phi}_i^{k-1}} (\delta u_i^{k-1})^2 \mathrm{d}\tau - V^{k-1} .
\end{aligned} \tag{24}
$$

Noting that $\hat{\phi}_i^k(t) \ge \hat{\phi}_i^{k-1}(t)$, (24) becomes

$$
\begin{aligned}
\Delta E^k(t) \le{}& \int_0^t \dot{V}^k \mathrm{d}\tau + \sum_{i=1}^{N} \frac{1}{2q_i} \int_0^t [(\tilde{\phi}_i^k)^2 - (\tilde{\phi}_i^{k-1})^2] \mathrm{d}\tau \\
&+ \sum_{i=1}^{N} \int_0^t \frac{1}{2\hat{\phi}_i^k} [(\delta u_i^k)^2 - (\delta u_i^{k-1})^2] \mathrm{d}\tau \\
&+ V^k(0) - V^{k-1}(t) .
\end{aligned} \tag{25}
$$

With the help of the relation $(f - g)^2 - (f - y)^2 = (y - g)[2(f - g) + (g - y)]$ and (16), it can be obtained that

$$
\begin{aligned}
&\sum_{i=1}^{N} \frac{1}{2q_i} \int_0^t [(\tilde{\phi}_i^k)^2 - (\tilde{\phi}_i^{k-1})^2] \mathrm{d}\tau \\
&= \sum_{i=1}^{N} \frac{1}{2q_i} \int_0^t [(\phi - \hat{\phi}_i^k)^2 - (\phi - \hat{\phi}_i^{k-1})^2] \mathrm{d}\tau \\
&= \sum_{i=1}^{N} \frac{1}{2q_i} \int_0^t (\hat{\phi}_i^{k-1} - \hat{\phi}_i^k)[2(\phi - \hat{\phi}_i^k) + (\hat{\phi}_i^k - \hat{\phi}_i^{k-1})] \mathrm{d}\tau \\
&= \sum_{i=1}^{N} \frac{1}{q_i} \int_0^t \tilde{\phi}_i^k (\hat{\phi}_i^{k-1} - \hat{\phi}_i^k)) \mathrm{d}\tau \\
&\quad - \sum_{i=1}^{N} \frac{1}{2q_i} \int_0^t (\hat{\phi}_i^k - \hat{\phi}_i^{k-1})^2 \mathrm{d}\tau \\
&\le \sum_{i=1}^{N} \frac{1}{q_i} \int_0^t \tilde{\phi}_i^k (\hat{\phi}_i^{k-1} - \hat{\phi}_i^k) \mathrm{d}\tau \\
&\quad = \sum_{i=1}^{N} \frac{1}{q_i} \int_0^t \tilde{\phi}_i^k (\hat{\phi}_i^{k-1} + q_i (e_i^k)^2 - q_i (e_i^k)^2 - \hat{\phi}_i^k) \mathrm{d}\tau \\
&\quad = \sum_{i=1}^{N} \frac{1}{q_i} \int_0^t \tilde{\phi}_i^k [\hat{\phi}_i^{k-1} + q_i (e_i^k)^2 - \hat{\phi}_i^k] \mathrm{d}\tau \\
&\qquad - \sum_{i=1}^{N} \int_0^t \tilde{\phi}_i^k (e_i^k)^2 \mathrm{d}\tau .
\end{aligned} \tag{26}
$$

Based on Property 3 and (16), we have

$$
\begin{aligned}
&\tilde{\phi}_i^k [\hat{\phi}_i^{k-1} + q_i (e_i^k)^2 - \hat{\phi}_i^k] \\
&= (\phi - \hat{\phi}_i^k)[\hat{\phi}_i^{k-1} + q_i (e_i^k)^2 - \hat{\phi}_i^k] \\
&= [\phi - \mathrm{sat}(\hat{\phi}_i^{k-1} + q_i (e_i^k)^2, \phi^*)][\hat{\phi}_i^{k-1} + q_i (e_i^k)^2 - \hat{\phi}_i^k] \le 0.
\end{aligned} \tag{27}
$$

Accordingly

$$
\begin{aligned}
&\sum_{i=1}^{N} \frac{1}{2q_i} \int_0^t [(\tilde{\phi}_i^k)^2 - (\tilde{\phi}_i^{k-1})^2] \mathrm{d}\tau \\
&\le -\sum_{i=1}^{N} \int_0^t \tilde{\phi}_i^k (e_i^k)^2 \mathrm{d}\tau \\
&= -\phi \sum_{i=1}^{N} \int_0^t (e_i^k)^2 \mathrm{d}\tau + \sum_{i=1}^{N} \int_0^t \hat{\phi}_i^k (e_i^k)^2 \mathrm{d}\tau \\
&= -\phi \int_0^t (\delta^k)^{\mathrm{T}} H^2 \delta^k \mathrm{d}\tau + \int_0^t (\delta^k)^{\mathrm{T}} H \hat{\Phi}^k H \delta^k \mathrm{d}\tau .
\end{aligned} \tag{28}
$$

From Property 1 and the control protocol (15), the third term on the RHS of (25) is reexpressed as

$$
\begin{aligned}
&\sum_{i=1}^{N}\int_0^t \frac{1}{2\hat{\phi}_i^k}[(\delta u_i^k)^2-(\delta u_i^{k-1})^2]\,\mathrm{d}\tau \\
&= \sum_{i=1}^{N}\int_0^t \frac{1}{2\hat{\phi}_i^k}[(u_i^k-u_0)^2-(u_i^{k-1}-u_0)^2]\,\mathrm{d}\tau \\
&\le \sum_{i=1}^{N}\int_0^t \frac{1}{2\hat{\phi}_i^k}[(u_i^k-u_0)^2-(\tilde{u}_i^{k-1}-u_0)^2]\,\mathrm{d}\tau \\
&= \sum_{i=1}^{N}\int_0^t \frac{1}{2\hat{\phi}_i^k}(\tilde{u}_i^{k-1}-u_i^k)[2(u_0-u_i^k)+(u_i^k-\tilde{u}_i^{k-1})]\,\mathrm{d}\tau \\
&= \sum_{i=1}^{N}\int_0^t \frac{1}{2\hat{\phi}_i^k}(-\hat{\phi}_i^k e_i^k)[2(u_0-u_i^k)]\,\mathrm{d}\tau \\
&\quad -\sum_{i=1}^{N}\int_0^t \frac{1}{2\hat{\phi}_i^k}(u_i^k-\tilde{u}_i^{k-1})^2\,\mathrm{d}\tau \\
&= \sum_{i=1}^{N}\int_0^t e_i^k(u_i^k-u_0)\,\mathrm{d}\tau-\sum_{i=1}^{N}\int_0^t \frac{\hat{\phi}_i^k}{2}(e_i^k)^2\,\mathrm{d}\tau \\
&= \int_0^t (\delta^k)^{\mathrm{T}}H(1_N u_0-u^k)\,\mathrm{d}\tau-\frac{1}{2}\int_0^t (\delta^k)^{\mathrm{T}}H\hat{\Phi}^k H\delta^k\,\mathrm{d}\tau .
\end{aligned}
\tag{29}
$$

Substituting (22), (28) and (29) into (25) results

$$
\begin{aligned}
\Delta E^k(t) \le{}& l\lambda_{\max}(H)\int_0^t (\delta^k)^{\mathrm{T}}\delta^k\,\mathrm{d}\tau-\frac{1}{2}\int_0^t (\delta^k)^{\mathrm{T}}H\hat{\Phi}^k(\tau)H\delta^k\,\mathrm{d}\tau \\
&+\int_0^t (\delta^k)^{\mathrm{T}}H(\tilde{u}^k-2u^k+\tilde{u}^{k-1})\,\mathrm{d}\tau \\
&-\phi\int_0^t (\delta^k)^{\mathrm{T}}H^2\delta^k\,\mathrm{d}\tau+V^k(0)-V^{k-1}(t) .
\end{aligned}
\tag{30}
$$

From Property 2 and the control protocol (15), it follows that $\left|u_i^k-\tilde{u}_i^k\right| \le \hat{\phi}_i^k(t)\left|e_i^k\right|$, which leads to the following inequality:

$$
\begin{aligned}
&\int_0^t (\delta^k)^{\mathrm{T}}H(\tilde{u}^k-2u^k+\tilde{u}^{k-1})\,\mathrm{d}\tau \\
&= -\sum_{i=1}^{N}\int_0^t e_i^k(\tilde{u}_i^k-u_i^k+\tilde{u}_i^{k-1}-u_i^k)\,\mathrm{d}\tau \\
&\le \sum_{i=1}^{N}\int_0^t \left|e_i^k\right|(\left|\tilde{u}_i^k-u_i^k\right|+\left|\tilde{u}_i^{k-1}-u_i^k\right|)\,\mathrm{d}\tau \\
&\le \sum_{i=1}^{N}\int_0^t \left|e_i^k\right|(\hat{\phi}_i^k\left|e_i^k\right|+\hat{\phi}_i^k\left|e_i^k\right|)\,\mathrm{d}\tau \\
&= 2\sum_{i=1}^{N}\int_0^t \hat{\phi}_i^k(e_i^k)^2\,\mathrm{d}\tau \\
&= 2\int_0^t (\delta^k)^{\mathrm{T}}H\hat{\Phi}^k H\delta^k\,\mathrm{d}\tau .
\end{aligned}
\tag{31}
$$

Taking (31) into (30), it is obvious that

$$
\begin{aligned}
\Delta E^k(t) \le{}& l\lambda_{\max}(H)\int_0^t (\delta^k)^{\mathrm{T}}\delta^k\,\mathrm{d}\tau+\frac{3}{2}\int_0^t (\delta^k)^{\mathrm{T}}H\hat{\Phi}^k(\tau)H\delta^k\,\mathrm{d}\tau \\
&-\phi\int_0^t (\delta^k)^{\mathrm{T}}H^2\delta^k\,\mathrm{d}\tau+V^k(0)-V^{k-1}(t) .
\end{aligned}
\tag{32}
$$

For the positive definite matrix H, (32) becomes

$$
\begin{aligned}
\Delta E^k \le{}& V^k(0)-V^{k-1}(t) \\
&-[\phi\lambda_{\min}^2(H)-l\lambda_{\max}(H)-\frac{3}{2}\phi^*\lambda_{\max}^2(H)]\int_0^t (\delta^k)^{\mathrm{T}}\delta^k d\tau \\
\le{}& -c\int_0^t (\delta^k)^{\mathrm{T}}\delta^k\,\mathrm{d}\tau+V^k(0)-V^{k-1}(t),
\end{aligned}
\tag{33}
$$

where $c=\phi\lambda_{\min}^2(H)-l\lambda_{\max}(H)-\frac{3}{2}\phi^*\lambda_{\max}^2(H)$ with ϕ being the large enough constant, such that $c>0$; $\lambda_{\min}(H)$ and $\lambda_{\max}(H)$ are the minimum and maximum eigenvalues of H. Therefore, we have

$$
\Delta E^k(t) \le -c\int_0^t (\delta^k)^{\mathrm{T}}\delta^k\,\mathrm{d}\tau+V^k(0)-V^{k-1}(t) . \tag{34}
$$

Let $t=T$, according to Assumption 2

$$
\Delta E^k(T) \le -c\int_0^T (\delta^k)^{\mathrm{T}}\delta^k\,\mathrm{d}\tau . \tag{35}
$$

That is

$$
E^k(T) \le E^{k-1}(T) . \tag{36}
$$

From (34) and (36), we can obtain (see (37)) . Hence, $E^k(t)$ is finite for any iteration if $E^0(T)$ is limited.

Next, the boundedness of $E^0(t)$ will be shown.

$$
E^0(t)=V^0+\sum_{i=1}^{N}\frac{1}{q_i}\int_0^t (\tilde{\phi}_i^0)^2\,\mathrm{d}\tau+\sum_{i=1}^{N}\int_0^t \frac{1}{2\hat{\phi}_i^0}(\delta u_i^0)^2\,\mathrm{d}\tau, \tag{38}
$$

and

$$
\begin{aligned}
\dot{E}^0(t) &= \dot{V}^0(t) + \sum_{i=1}^{N}\frac{1}{q_i}(\tilde{\phi}_i^0(t))^2 + \sum_{i=1}^{N}\frac{1}{2\hat{\phi}_i^0}(\delta u_i^0)^2 \\
&\le l\lambda_{\max}(H)(\delta^0)^{\mathrm{T}}\delta^0 - (\delta^0)^{\mathrm{T}}H\hat{\Phi}^0(t)H\delta^0 + (\delta^0)^{\mathrm{T}}H(\tilde{u}^0 - u^0 \\
&\quad + \tilde{u}^{-1} - 1_N u_0) + \sum_{i=1}^{N}\frac{1}{2\hat{\phi}_i^0(\tau)}[(\delta u_i^0)^2 - (\delta u_i^{-1})^2] \\
&\quad + \sum_{i=1}^{N}\frac{1}{2q_i}(\tilde{\phi}_i^{-1})^2 + \sum_{i=1}^{N}\frac{1}{2q_i}[(\tilde{\phi}_i^0)^2 - (\tilde{\phi}_i^{-1})^2] \\
&\quad + \sum_{i=1}^{N}\frac{1}{2\hat{\phi}_i^0}(\delta u_i^{-1})^2 \\
&\le l\lambda_{\max}(H)(\delta^0)^{\mathrm{T}}\delta^0 - (\delta^0)^{\mathrm{T}}H\hat{\Phi}^0(t)H\delta^0 + (\delta^0)^{\mathrm{T}}H(\tilde{u}^0 - u^0 \\
&\quad + \tilde{u}^{-1} - 1_N u_0) - \phi(\delta^0)^{\mathrm{T}}H^2\delta^0 + (\delta^0)^{\mathrm{T}}H\hat{\Phi}^0(t)H \\
&\quad + (\delta^0)^{\mathrm{T}}H(1_N u_0 - u^0) + \sum_{i=1}^{N}\frac{1}{2q_i}\phi^2 \\
&\quad - \frac{1}{2}(\delta^0)^{\mathrm{T}}H\hat{\Phi}^0 H + \sum_{i=1}^{N}\frac{1}{2\hat{\phi}_i^0}u_0^2 \\
&= l\lambda_{\max}(H)(\delta^0)^{\mathrm{T}}\delta^0 - \frac{1}{2}(\delta^0)^{\mathrm{T}}H\hat{\Phi}^0 H\delta^0 + (\delta^0)^{\mathrm{T}}H(\tilde{u}^0 - 2u^0 \\
&\quad + \tilde{u}^{-1}) - \phi(\delta^0)^{\mathrm{T}}H^2\delta^0 + \sum_{i=1}^{N}\frac{1}{2q_i}\phi^2 + \sum_{i=1}^{N}\frac{1}{2\hat{\phi}_i^0}u_0^2 \\
&\le l\lambda_{\max}(H)(\delta^0)^{\mathrm{T}}\delta^0 + 2(\delta^0)^{\mathrm{T}}H\hat{\Phi}^0 H\delta^0 + \phi^2\sum_{i=1}^{N}\frac{1}{2q_i} \\
&\quad + \sum_{i=1}^{N}\frac{1}{2\hat{\phi}_i^0}u_0^2 - \phi(\delta^0)^{\mathrm{T}}H^2\delta^0 - \frac{1}{2}(\delta^0)^{\mathrm{T}}H\hat{\Phi}^0 H\delta^0 \\
&= l\lambda_{\max}(H)(\delta^0)^{\mathrm{T}}\delta^0 + \frac{3}{2}(\delta^0)^{\mathrm{T}}H\hat{\Phi}^0 H\delta^0 + \phi^2\sum_{i=1}^{N}\frac{1}{2q_i} \\
&\quad + \sum_{i=1}^{N}\frac{1}{2\hat{\phi}_i^0}u_0^2 - \phi(\delta^0)^{\mathrm{T}}H^2\delta^0 \\
&\le -\left[\phi\lambda_{\min}^2(H) - l\lambda_{\max}(H) - \frac{3}{2}\phi^*\lambda_{\max}^2(H)\right](\delta^0)^{\mathrm{T}}\delta^0 \\
&\quad + \phi^2\sum_{i=1}^{N}\frac{1}{2q_i} + \sum_{i=1}^{N}\frac{1}{2\hat{\phi}_i^0}u_0^2 \\
&\le -c(\delta^0)^{\mathrm{T}}\delta^0 + \sum_{i=1}^{N}\frac{1}{2q_i}\phi^2 + \sum_{i=1}^{N}\frac{1}{2\hat{\phi}_i^0}u_0^2 .
\end{aligned} \tag{39}
$$

Since $u_0(t)$ is continuous on $[0, T]$. Therefore, we can denote K as

$$K = \max_{\substack{t \in [0,T] \\ 1 \le i \le N}}\left[\sum_{i=1}^{N}\frac{1}{2q_i}\phi^2 + \sum_{i=1}^{N}\frac{1}{2\hat{\phi}_i^0}u_0^2\right] < \infty . \tag{40}$$

Consequently, it follows that

$$E^0(t) \le \left|E^0(0)\right| + \left|\int_0^t \dot{E}^0\,\mathrm{d}\tau\right| \le \frac{1}{2}(\delta^0)^{\mathrm{T}}H\delta^0 + TK < \infty . \tag{41}$$

The finiteness of $E^0(t)$ implies that $E^0(T)$ is finite. In the meanwhile, the uniformly boundedness of $E^k(t)$ is ensured, for all $k \in Z^+$ on $[0, T]$. Moreover, from the definition of $E^k(t)$, the uniformly boundedness of $\delta^k(t)$ can be acquired on $[0, T]$. The adaptive learning law (16) renders the boundedness of $\hat{\phi}_i^k(t)$. From (15), it can be concluded that $u_i^k(t)$ is uniformly bounded. Thus, the boundedness of all arguments involved in the system are guaranteed.

Finally, let us prove the learning consensus property. Applying (35) repeatedly, it can be derived that

$$
\begin{aligned}
E^k(T) &= E^0(T) + \sum_{j=1}^{k}\Delta E^j(T) \\
&\le E^0(T) - c\sum_{j=1}^{k}\int_0^T (\delta^j)^{\mathrm{T}}\delta^j\,\mathrm{d}\tau .
\end{aligned} \tag{42}
$$

According to (42) and the positiveness of $E^k(T)$, we can attain that

$$c\sum_{j=1}^{k}\int_0^T (\delta^j)^{\mathrm{T}}\delta^j\,\mathrm{d}\tau \le E^0(T) . \tag{43}$$

On account of the finiteness of $E^0(T)$, we have that the series $\sum_{j=1}^{k}\int_0^T (\delta^j)^{\mathrm{T}}\delta^j\,\mathrm{d}\tau$ is convergent. Then, $\lim_{k\to\infty}\int_0^T (\delta^k)^{\mathrm{T}}\delta^k\,\mathrm{d}\tau = 0$.

$$
\begin{aligned}
E^k(t) &= \Delta E^k(t) + E^{k-1}(t) \\
&\le -c\int_0^t (\delta^k)^{\mathrm{T}}\delta^k\,\mathrm{d}\tau + V^k(0) - V^{k-1}(t) \\
&\quad + V^{k-1}(t) + \sum_{i=1}^{N}\frac{1}{2q_i}\int_0^t (\tilde{\phi}_i^{k-1}(\tau))^2\,\mathrm{d}\tau \\
&\quad + \sum_{i=1}^{N}\int_0^t \frac{1}{2\hat{\phi}_i^{k-1}(\tau)}(\delta u_i^{k-1})^2\,\mathrm{d}\tau \\
&= -c\int_0^t (\delta^k)^{\mathrm{T}}\delta^k\,\mathrm{d}\tau + V^k(0) + \sum_{i=1}^{N}\frac{1}{2q_i}\int_0^t (\tilde{\phi}_i^{k-1}(\tau))^2\,\mathrm{d}\tau \\
&\quad + \sum_{i=1}^{N}\int_0^T \frac{1}{2\hat{\phi}_i^{k-1}(\tau)}(\delta u_i^{k-1})^2\,\mathrm{d}\tau \\
&= -c\int_0^t (\delta^k)^{\mathrm{T}}\delta^k\,\mathrm{d}\tau + E^{k-1}(T) \\
&\le E^{k-1}(T) \\
&\quad \vdots \\
&\le E^0(T)
\end{aligned} \tag{37}
$$

Recalling the error dynamic (13), it is known that $\dot{\delta}^k$ is uniformly finite over $[0, T]$. Above all, from Barbalat-like lemma [48], $\lim_{k \to \infty} \delta^k(t) = 0$, i.e. $\lim_{k \to \infty} \delta_i^k(t) = 0$. Namely, each follower agent can perfectly track the leader over $[0, T]$, the global perfect consensus tracking is derived.

Remark 5: In the dynamics of each follower agent (5), no input dynamics and external disturbances are taken into account. Actually, if there exist external disturbances, we can refer to [44, 47] to handle these problems. Besides, the algorithms presented in the paper can be extended to a class of uncertain non-linear MASs having the following form:

$$\begin{aligned} \text{Followers:} \quad & \dot{\boldsymbol{x}}_i^k = \boldsymbol{\eta}(\boldsymbol{x}_i^k, t) + \boldsymbol{B}\text{sat}(\boldsymbol{u}_i^k, \boldsymbol{u}^*), \quad i = 1, 2, \ldots, N; \\ \text{Leader:} \quad & \dot{\boldsymbol{x}}_0(t) = \boldsymbol{\eta}(\boldsymbol{x}_0, t) + \boldsymbol{B}\boldsymbol{u}_0, \end{aligned} \tag{44}$$

where $\boldsymbol{x}_i^k \in R^n$, $\boldsymbol{\eta}(\boldsymbol{x}_i^k, t) \in R^n$ is an unknown time-varying global Lipschitz continuously differentiable vector-valued function in $\boldsymbol{x}_i^k$ and piecewise continuous in t; $\boldsymbol{B} \in R^{n \times m}$ is a known column full rank matrix; the vector-valued function $\text{sat}(\boldsymbol{u}(t), \boldsymbol{u}^*) = [\text{sat}(u_1(t), u_1^*), \text{sat}(u_2(t), u_2^*), \ldots, \text{sat}(u_m(t), u_m^*)] \in R^m$; $\boldsymbol{x}_0 \in R^n$ and $\boldsymbol{u}_0 \in R^m$. Then, the consensus analysis is similar to the proof of Theorem 1. For the general systems, if the uncertain non-linear function in (5) can be parameterised and $\boldsymbol{B}$ is square and unknown, we may refer to [49] for further details.

5 Extension to formation control for the uncertain non-linear MASs with input saturation

The objective of the section is to generalise the previous consensus results to the formation problem of the uncertain non-linear MASs with input saturation. Specifically, the necessary and sufficient condition for the MASs (5) and (6) to realise the formation control is that all follower agents form a desired formation at a certain distance from the leader, and the formation control can be transformed into the consensus control through simple transformation. Since we have solved the consensus problem, it is not difficult to deal with the formation control problem.

At present, the position error of the ith agent at the kth iteration is defined as

$$\bar{x}_i^k = x_i^k - \Delta_i, \tag{45}$$

where Δ_i indicates the expected distance of the ith agent from the leader.

The purpose of formation control is to seek a sequence of protocols $u_i^k(t)$ such that each follower agent keeps a desired distant from the leader in the iteration domain over $[0, T]$.

From (45), we define the formation divergence error of the ith agent as

$$\delta_i^k(t) = \bar{x}_i^k(t) - x_0(t)\,. \tag{46}$$

Accordingly, the formation control can be re-expressed as the consensus control, i.e. $\lim_{k \to \infty} \delta_i^k = 0$.

Meanwhile, the local neighbourhood formation error of the ith agent is

$$e_i^k = \sum_{j=1}^{N} a_{ij}(\bar{x}_j^k - \bar{x}_i^k) + b_i(x_0 - \bar{x}_i^k)\,. \tag{47}$$

Assumption 4: The alignment initial condition is satisfied, i.e. $\bar{x}_i^k(0) = \bar{x}_i^{k-1}(T), i = 1, 2, \ldots, N$; and the leader is spatially closed, i.e. $x_0(0) = x_0(T)$. Thereby, $\delta_i^k(0) = \delta_i^{k-1}(T)$.

Theorem 2: Consider the connected graph G' of MASs (5) and (6), Assumptions 1, 3 and 4 hold, then N follower agents represented by (5) under the protocol (15) and the learning-based updating law (16) for coupling control gains with the error (47) guarantee that the desired formation is formed in the iteration domain, and all the variables involved in the closed system are bounded.

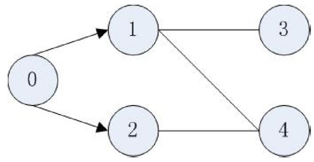

Fig. 1 *Topology graph (vertex 0 expresses the leader)*

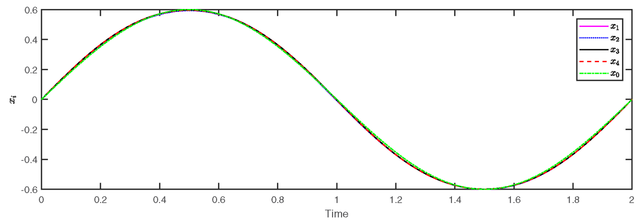

Fig. 2 *States of all agents in case 1*

6 Simulations

There are two illustrative examples to validate the efficiency of the protocols designed in this paper. The first example is a numerical simulation, which includes two cases, i.e. the consensus and formation control cases for the leader–follower MASs with four follower agents and one leader. Besides, the multi-vehicle systems composed of four follower vehicles and one leader vehicle is stated in Example 2, which can be considered as the MASs. The communication topology graph between agents both in Examples 1 and 2 is demonstrated in Fig. 1. It is obvious that the information of the leader can be received by the first and second agents.

$$L = \begin{pmatrix} 2 & 0 & -1 & -1 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ -1 & -1 & 0 & 2 \end{pmatrix}, \quad B = \text{diag}\{1, 1, 0, 0\}\,. \tag{48}$$

Example 1: Consider the MASs (5) and (6) with $\eta(x_i^k, t) = 2\cos(x_i^k)\cos(\pi t)$, $t \in [0, T]$, where $T = 2$, $k = 20$, $i = 1, 2, 3, 4$; and the state of the leader is $x_0 = 0.6\sin(\pi t)$. By computing, $u_0 = 0.6\pi\cos(\pi t) - 2\cos(0.6\sin \pi t)\cos(\pi t)$ and $\max_{t \in [0,2]}|u_0(t)| = 0.1150$. Thus, we choose $u^* = 2$ in the simulation. Furthermore, $\phi^* = 5$.

Case 1: Consensus control for the uncertain non-linear MASs. Choose $x_1(0) = x_2(0) = x_3(0) = x_4(0) = 3$, and the initial values of the parameters are $\hat{\phi}_1(0) = 0.8, \hat{\phi}_2(0) = 0.5, \hat{\phi}_3(0) = 0.6, \hat{\phi}_4(0) = 1$. Applying the protocol (15) and the adaptive learning law (16), select $q_1 = 1, q_2 = 0.8, q_3 = 1, q_4 = 1$. The simulation results for 20 iterations are displayed in Figs. 2 and 3.

Fig. 2 draws the states of all agents at the 20*th* iteration. The consensus errors $\delta_i^k(t)$, designed distributed control gains $\hat{\phi}_i^k(t)$ and control inputs $u_i^k(t), i = 1, 2, 3, 4$ are depicted in Fig. 3. From Figs. 2 and 3, we can learn that each follower agent tracks the leader perfectly on $[0, 2]$, in other words, the global perfect consensus tracking can be realised even the existence of input saturation. Simultaneously, the signals involved are finite along the whole iteration axis. Finally, the above results are not only consistent with Theorem 1 but also verify the effectiveness of the designed protocols further.

Case 2. Formation control for the uncertain non-linear MASs. Here, let us consider the formation control for the MASs (5) and (6). The initial state of each follower agent is the same as in case 1, and the initial values of parameters are $\hat{\phi}_1(0) = 2, \hat{\phi}_2(0) = 3,$

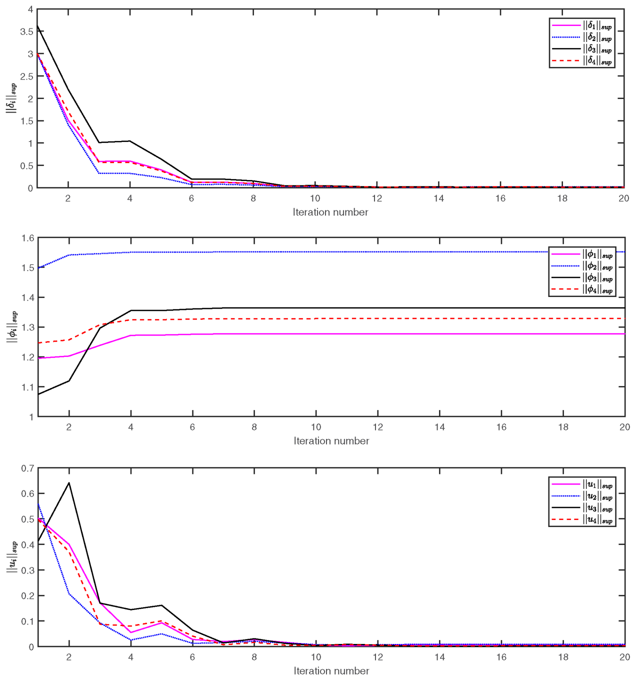

Fig. 3 *Response curves of agents in case 1*

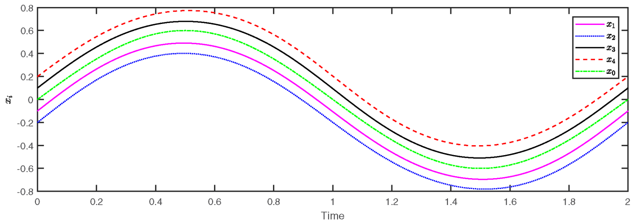

Fig. 4 *States of all agents in case 2*

$\hat{\phi}_3(0) = 2, \hat{\phi}_4(0) = 2$. Applying the protocol (15) and adaptive learning law (16) with the error (47), we select $q_1 = q_2 = q_3 = q_4 = 1$, and the desired relative distances are $\Delta_1 = -0.1, \Delta_2 = -0.2, \Delta_3 = 0.1, \Delta_4 = 0.2$. The simulation results for 20 cycles are demonstrated in Figs. 4 and 5.

It can be learned that four follower agents form the desired formation in Figs. 4 and 5 describe that the formation errors asymptotically approach to zero in the iteration domain and designed distributed control gains, as well as protocols, are all finite. The validity of this part is obvious.

Example 2: In the real world, a set of vehicles need to attain a common configuration to achieve some kind of coordination task, such as loading a workpiece [50]. The same velocity is required for all the vehicles to accomplish this task. Hence, in this part, the velocity consensus problem of four follower vehicles and one leader vehicle will be shown in the iteration domain.

The dynamics of all follower vehicles are expressed as $\dot{v}_i^k = 1/m_i[\mathrm{sat}(u_i^k) - \eta(v_i^k, t)]$ where v_i^k is the velocity, m_i is the mass, u_i^k is the input, $\eta(v_i^k, t) = k_i \tanh(v_i^k)$ can be considered as the sum of resistance, and $v_0(t) = 0.01\sin(\pi t)$. By computing, $u_0(t) = 0.01\pi\cos(\pi t) - \tanh(0.01\sin(\pi t))$ and $\max_{t \in [0,2]}|u_0(t)| = 0.033$. Therefore, we choose $u^* = 2$ in the simulation. The initial states are $v_1(0) = -1, v_2(0) = -0.6, v_3(0) = -1.2, v_4(0) = -2$, and the initial values of the parameters are $\hat{\phi}_1(0) = \hat{\phi}_2(0) = \hat{\phi}_3(0) = \hat{\phi}_4(0) = 1$. $T = 2$, $k = 20$, $\phi^* = 5$, $1/m_i = 1$, $i = 1, 2, 3, 4$; $q_1 = q_2 = q_3 = q_4 = 2; k_1 = 0.5, k_2 = 0.2, k_3 = 0.4, k_4 = 0.3$.

Utilising the distributed protocol (15) and learning law (16) to the multi-vehicle systems, the results for 20 loops displaced in Fig. 6, which show that the algorithms proposed in the paper are also

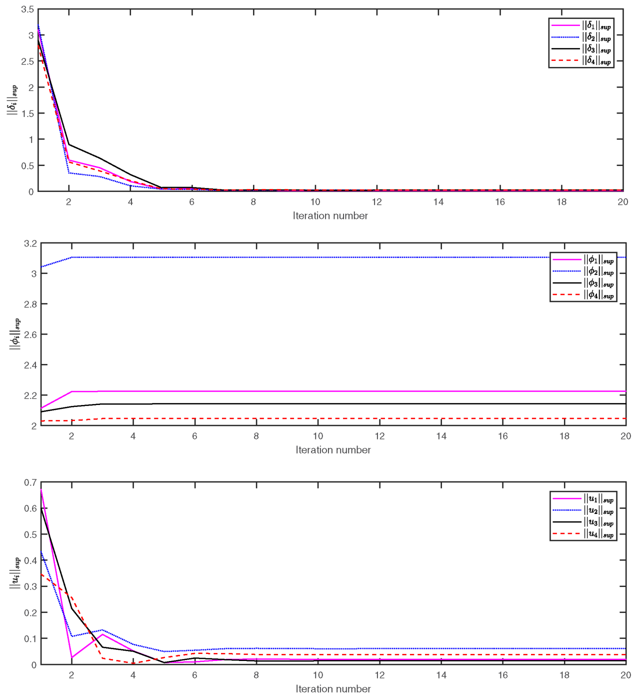

Fig. 5 *Response curves of agents in case 2*

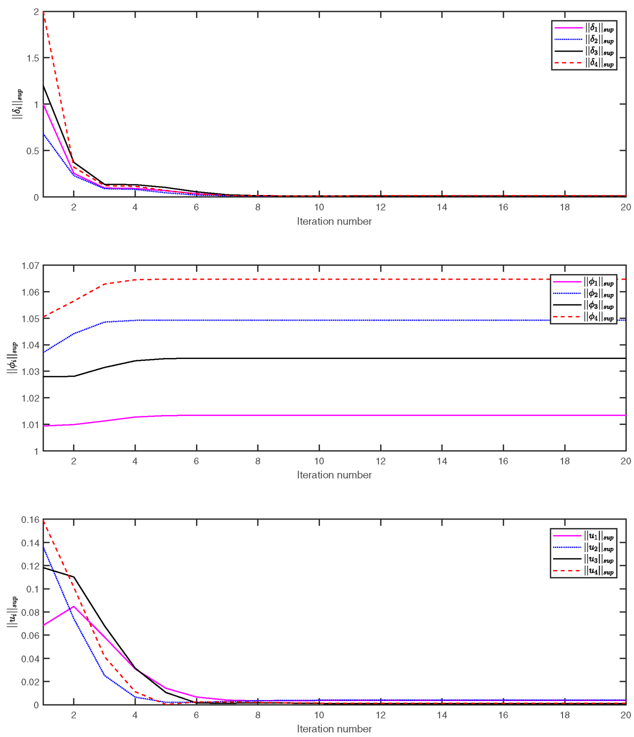

Fig. 6 *Responses curves of four follower vehicles in Example 2*

valid in the multi-vehicle systems, that is to say, all the agents can achieve the velocity consensus and other arguments involved are bounded.

7 Conclusion

The distributed coordination control protocols for the uncertain non-linear MASs with input saturation using adaptive ILC are designed. By Lyapunov theory and under the alignment initial condition, the proposed protocols with fully saturated learning laws can guarantee the global perfect consensus tracking in spite of the existence of the input saturation. Also, we have extended the consensus results to the formation control problem of the MASs. The algorithms raised in this paper are verified by two illustrative examples.

8 Acknowledgment

This work was supported by the National Natural Science Foundations of China under grant nos. 61573013 and 61603286.

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