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扁球薄壳在大挠度下的动力学行为

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摘要: 根据薄壳非线性动力学理论, 由扁球薄壳大挠度基本方程, 在周边固定夹紧的条件下, 用修正迭代法求出二次近似解析解, 把大挠度解作为扁球薄壳的初挠度处理, 推导出扁球薄壳在大挠度下的非线性动力学基本方程。利用扁球面壳的非线性动力学变分方程和协调方程, 在夹紧固定的边界条件下, 用 Galerkin 方法得到一个含二次、三次项非线性受迫振动微分方程, 通过求 Melnikov 函数, 给出可能发生混沌运动的条件, 通过数字仿真绘出平面相图, 证实混沌运动的存在。

关键词: 非线性; 大挠度; 修正迭代法; 混沌运动

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Dynamic behavior of flat spherical shallow shells under large deflection

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Abstract: On the basis of nonlinear dynamical theory and according to control equations of flat spherical shallow shells under large deflection, the secondary approximate analytic solution was obtained by using a modified iteration method in the condition of fixedly clamped perimeter. Then, taking the large deflection solution as initial deflection of the flat spherical shells, the nonlinear dynamic control equations of the latter were derived in the case of large deflection. Employing nonlinear dynamic variational equation and compatible equation with boundary condition of clamped fixing, a nonlinear differential equation of forced vibration with second and third order terms was obtained by using Galerkin approach. By means of finding the Melnikov function, a condition was given to the probable occurrence of chaotic motion. The existence of the latter was justified by the phase plane plotted with numerical simulation.

Key words: nonlinearity; large deflection; modified iteration method; chaotic motion

在板壳方面的非线性动力学特性的研究非常多, 特别是对其非线性动力稳定性、分岔与混沌的研究较多。但在动静载荷作用下板壳的非线性分岔和混沌研究的很少。而文[1]研究双曲面扁壳在动、静载荷作用下的稳定性问题, 发现了一些奇趣现象, 次谐响应, 倍分岔, 混沌行为。

1 扁球面壳混合边值问题

引入无量纲对原问题^[2]进行简化:

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$$\begin{aligned}\rho &= r/R, \quad s_0 = \frac{12(1-\mu^2)}{Eh^3}RN_r, \\ Q &= \frac{[12(1-\mu^2)]^{3/2}}{Eh^4}R^4q, \quad k = \frac{2H}{h}\sqrt{12(1-\mu^2)}, \\ \gamma_1 &= \frac{12(1-\mu^2)R^4\gamma}{Eh^3}, \quad c_1 = \frac{12(1-\mu^2)R^4c}{Eh^3}, \\ y &= \sqrt{12(1-\mu^2)}\frac{w}{h}\end{aligned}$$

1) 静态无量纲方程:

$$L_{12}(y_j) = Q + \frac{1}{\rho} \frac{\partial}{\partial \rho} \left[s_0 \left(K\rho + \frac{\partial y_j}{\partial \rho} \right) \right] \quad (1)$$

$$L_{22}(y_j) = - \left[\left(K\rho + \frac{1}{2} \frac{\partial y_j}{\partial \rho} \right) \frac{\partial y_j}{\partial \rho} \right] \quad (2)$$

$$\text{式中: } L_{12} = \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho}$$

$$L_{22} = \rho \frac{\partial}{\partial \rho} \frac{1}{\rho} \frac{\partial}{\partial \rho}$$

边界条件:

$$\text{当 } \rho=1, y_j = \frac{\partial y_i}{\partial \rho} = 0, \frac{\partial s_i}{\partial \rho} - k s_j = 0 \quad (3)$$

$$\text{当 } \rho=0, y_j = y_0, \frac{1}{\rho} \frac{\partial y_i}{\partial \rho}, s_j \text{ 有限} \quad (4)$$

2) 静动态无量纲方程:

$$L_{12}(y_d) = Q_d + \frac{1}{\rho} \frac{\partial}{\partial \rho} \left[s_0 \frac{\partial y_d}{\partial \rho} + s_d \left(k \rho + \frac{\partial y_i}{\partial \rho} + \frac{\partial y_d}{\partial \rho} \right) \right] - c_1 \frac{\partial y_d}{\partial t} - \gamma_1 \frac{\partial^2 y_d}{\partial t^2} \quad (5)$$

$$L_{22}(y_d) = - \left[k \rho + \frac{1}{2} \frac{\partial y_d}{\partial \rho} + \frac{\partial y_i}{\partial \rho} \right] \frac{\partial y_d}{\partial \rho} \quad (6)$$

边界条件:

$$\text{当 } \rho=1, y_d = \frac{\partial y_d}{\partial \rho} = 0, \frac{\partial s_d}{\partial \rho} - k s_d = 0 \quad (7)$$

$$\text{当 } \rho=0, y_d, \frac{1}{\rho} \frac{\partial y_d}{\partial \rho}, s_d \text{ 有限} \quad (8)$$

初始条件:

$$\text{当 } t=0, y_d(\rho, t) = y_d(\rho, 0), \frac{\partial y_d(\rho, t)}{\partial t} = \frac{\partial y_d(\rho, 0)}{\partial t}.$$

2 问题的求解

用修正迭代法^[3]求解静态方程

$$y^3 = \left[-\frac{1}{2160} \rho^{12} - \frac{1}{240} \rho^{10} + \frac{5}{288} \rho^8 - \frac{(9\mu-11)}{216(\mu-1)} \rho^6 + \frac{(277-197\mu)}{4320(\mu-1)} \rho^4 - \frac{(19\mu-29)}{1080(\mu-1)} \rho^2 \right] y^3_0 - \left[\frac{1}{960} \rho^{10} - \frac{5}{576} \rho^8 + \frac{(13\mu-17)}{432(\mu-1)} \rho^6 - \frac{(331\mu-491)}{8640(\mu-1)} \rho^4 + \frac{(217-137\mu)}{8640(\mu-1)} \rho^2 \right] y^2_0 k - \left[\frac{1}{2304} \rho^8 - \frac{1}{288} \rho^6 + \frac{13}{2304} \rho^4 - \frac{1}{384} \rho^2 \right] y_0 k^2 + (1-\rho^2)^2 y_0 \quad (9)$$

$$S_j = -\frac{1}{3} \beta_1 y_0^2 (\rho^7 - 4\rho^5 + 6\rho^3 - 6\rho) - \frac{1}{3} k \beta_1 y_0 (\rho^5 - 3\rho^3 + 5\rho) \quad (10)$$

$$y_d = (1-\rho^2)^2 f(t)$$

$$s = s_1 f(t) + s_2 f(t)^2$$

$$y = y_0 (1-\rho^2)^2 \quad (11)$$

用 y_3 代替 y_j , 由式(6)可得

$$L_2(y_1) = - \left[k \rho \frac{\partial y}{\partial \rho} + \frac{\partial y_3}{\partial \rho} \frac{\partial y}{\partial \rho} \right] \quad (12)$$

$$L_2(y_2) = - \frac{1}{2} \left(\frac{\partial y}{\partial \rho} \right) \quad (13)$$

边界条件均为

$$\rho=0, s_1, s_2 \text{ 有限};$$

$$\rho=1, \frac{\partial s_1}{\partial \rho} - k s_1 = 0, \frac{\partial s_2}{\partial \rho} - k s_2 = 0 \quad (14)$$

可解得 s_1, s_2

$$s_1 = \left[\begin{aligned} & \frac{\rho^{15}}{10080} - \frac{17\rho^{13}}{15120} + \frac{13\rho^{11}}{2160} - \\ & \frac{(7\mu-8)\rho^9}{360(\mu-1)} + \frac{(467\mu-607)\rho^7}{12960(\mu-1)} - \\ & \frac{(47\mu-67)\rho^5}{1296(\mu-1)} + \frac{(19\mu-29)\rho^3}{1080(\mu-1)} - \\ & \frac{(33\mu^2-74\mu+34)\rho}{11340(\mu-1)^2} \end{aligned} \right] y_0^4 - \frac{1}{3} \left[\rho^7 - 3\rho^5 + 6\rho^3 - \frac{(3\mu-5)\rho}{(\mu-1)} \right] y_0^2 + \left[\frac{1}{4032} \rho^{13} - \frac{23}{8640} \rho^{11} + \frac{(9\mu-11)\rho^9}{720(\mu-1)} - \frac{7(103\mu-143)\rho^7}{25920(\mu-1)} + \frac{(799\mu-1199)\rho^5}{25920(\mu-1)} - \frac{(137\mu-217)\rho^3}{8640(\mu-1)} + \frac{(501\mu^2-1148\mu+535)\rho}{181440(1-\mu)^2} \right] k y_0^3 + \frac{1}{34560} \left[4\rho^{11} - 42\rho^9 + 125\rho^7 - 160\rho^5 + 90\rho^3 - \frac{(17\mu-11)\rho}{(\mu-1)} \right] k^2 y_0^2 - \frac{1}{6} \left[\rho^5 - 3\rho^3 + \frac{(\mu-2)}{\mu-1} \rho \right] k y_0 \quad (15)$$

$$s_2 = \left[-\frac{1}{6} \rho^7 + \frac{2}{3} \rho^5 - \rho^3 + \frac{3\mu-5}{6(\mu-1)} \rho \right] y_0^2 \quad (16)$$

由式(5)可得其变分方程为

$$\int_0^{\frac{2\pi}{\omega}} \int_0^1 \left\{ L_1(y_d) - Q_d - \frac{1}{\rho} \frac{\partial}{\partial \rho} \left[s_0 \frac{\partial y_d}{\partial \rho} + s_d \left(\frac{\partial y_d}{\partial \rho} + \frac{\partial y_i}{\partial \rho} + k \rho \right) \right] + c_1 \frac{\partial y_d}{\partial t} + \gamma_1 \frac{\partial^2 y_d}{\partial t^2} \right\} \rho \delta y_d dt = 0$$

取 $\varphi_a = \cos(\Omega t)$, $\mu = \frac{1}{3}$, $c_1 = \frac{\beta_1 y_0^2}{\gamma_1}$, 并将等式两边同时除 $\frac{1}{10} \gamma_1$ 得

$$f_{(t)}'' + cf_{(t)}' + \omega^2 f_{(t)} - \beta f_{(t)}^2 + g f_{(t)}^3 = g \cos(\Omega t) \quad (17)$$

$$\omega^2 = \frac{8107571}{73556683200} \frac{\gamma_0^6}{\gamma_1} - \frac{24949471}{117690693120} \frac{\gamma_0^5 k}{\gamma_1} + \left\{ \frac{95}{2673} + \frac{263113k^2}{2179457280} \right\} \frac{\gamma_0^4}{\gamma_1} + \left\{ \frac{7199}{136080} - \frac{186563k^2}{10346434560} \right\} \frac{\gamma_0^3 k^3}{\gamma_1} + \left\{ \frac{100}{7} + \frac{167k^2}{7776} + \frac{2749k^4}{3448811520} \right\} \frac{\gamma_0^2}{\gamma_1} - \left\{ \frac{40}{3} + \frac{59k^2}{36288} \right\} \frac{\gamma_0 k}{\gamma_1} + \frac{19k^2}{9\gamma_1} + \frac{320}{3\gamma_1}$$

$$\beta = \frac{95}{3564} \frac{\gamma_0^4}{\gamma_1} - \frac{901k}{36288} \gamma_0^3 + \left\{ \frac{100}{7} + \frac{157}{72576} k^2 \right\} \frac{\gamma_0^2}{\gamma_1} - \frac{20}{3} \frac{\gamma_0 k}{\gamma_1}$$

$$\alpha = \frac{100\gamma_0^2}{21\gamma_1}, \quad c = \frac{c_1}{\gamma_1}, \quad g = \frac{5Q_d}{3y_0\gamma_1}$$

取 $\tau = \omega t$, $f_{(t)} = \frac{\omega}{\sqrt{\alpha}} \eta_{(\tau)}$, 则式(17)转化为

$$\eta_{(\tau)}'' + \frac{c}{\omega} \eta_{(\tau)}' + \eta_{(\tau)} - \frac{\beta}{\omega \sqrt{\alpha}} \eta_{(\tau)}^2 + \eta_{(\tau)}^3 = \frac{\sqrt{\alpha}}{\omega^3} g \cos\left(\frac{\Omega}{\omega}\tau\right) \quad (18)$$

再取 $\alpha' = \frac{c}{\omega}$, $\beta' = \frac{\beta}{\omega \sqrt{\alpha}}$, $g' = \frac{\sqrt{\alpha}}{\omega^3} g$, 则式(18)可简化为

$$\eta_{(\tau)}'' + \alpha' \eta_{(\tau)}' + \eta_{(\tau)} - \beta' \eta_{(\tau)}^2 + \eta_{(\tau)}^3 = g' \cos\left(\frac{\Omega}{\omega}\tau\right) \quad (19)$$

3 求 Melnikov 函数

令 $\frac{\Omega}{\omega} = \omega$, 式(19)的自由振动解(初速度为零)为^[4,5]

$$\eta_l(\tau) = \frac{2}{a - b \sin \tau} \quad (20)$$

$$d\eta_l(\tau) = \eta_k(\tau) = \frac{2b \cos \tau}{(a - b \sin \tau)^2}$$

$$M(\tau_0) = \int_{-\infty}^{+\infty} [-\alpha' \eta_k^2(\tau) + g' \eta_k(\tau) \cos \omega(\tau + \tau_0)] d\tau$$

$$1) \text{ 当 } \omega=1 \text{ 时, } g'\left(\frac{2}{3}\beta' - \sqrt{2}\right) > \frac{\beta' \alpha'}{6} \left(\frac{4}{9}\beta'^2 - 2\right)^{3/2}$$

存在同宿点, 系统可能发生混沌运动.

2) 当 $\omega=3$ 时,

$$g'(2\sqrt{2}\beta'^3 - 18\beta'^2 + 27\sqrt{2}\beta' + 27) > \frac{3\sqrt{2}}{8} \alpha' \beta' \left(\frac{4}{9}\beta'^2 - 2\right)^{5/2}$$

存在同宿点, 系统可能产生混沌现象.

图 1、2、3 是一系列数字仿真时程图, Poincaré 映射图及相轨图.

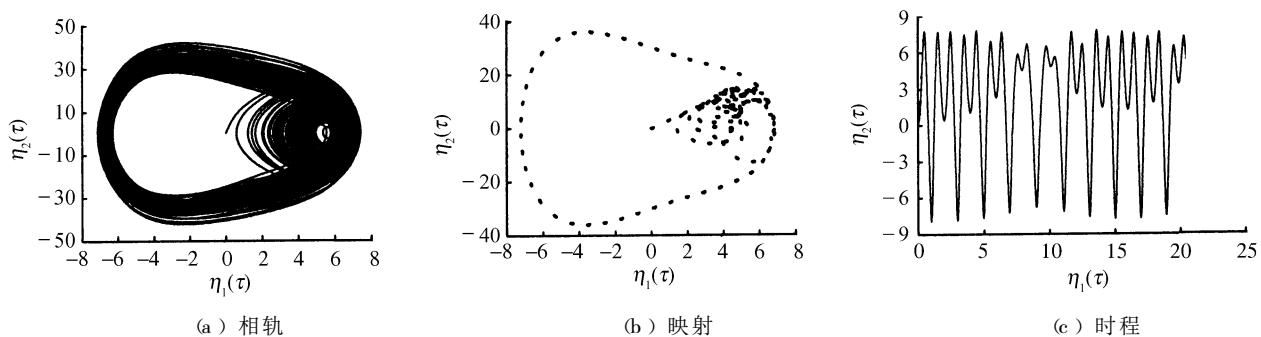


图 1 $\alpha'=0.03, g'=100$ 的数值模拟

Fig. 1 Numerical simulation for cases of $\alpha'=0.03, g'=100$

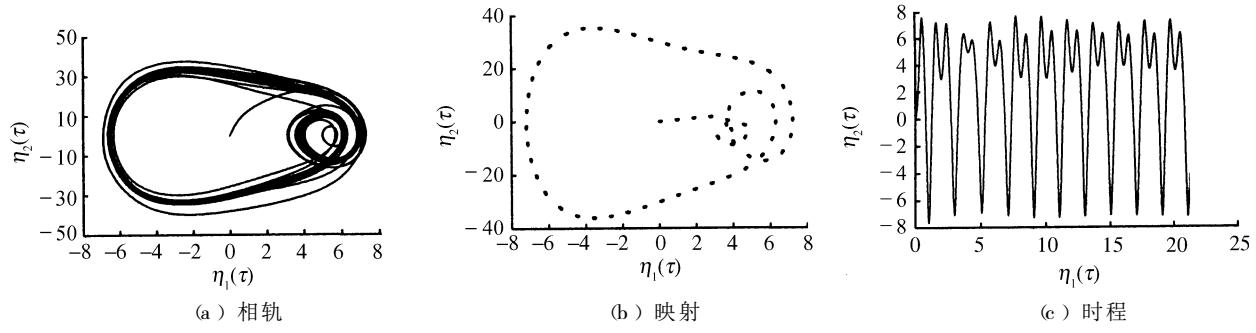
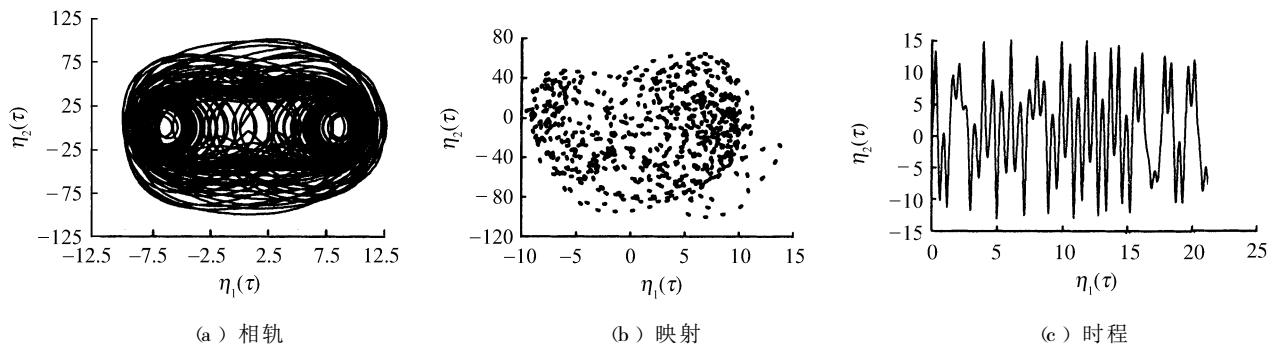


图 2 $\alpha'=0.3, g'=100$ 的数值模拟

Fig. 2 Numerical simulation for cases of $\alpha'=0.3, g'=100$

图 3 $\alpha'=0.09, g'=500$ 的数值模拟Fig. 3 Numerical simulation for cases of $\alpha'=0.09, g'=500$

4 结果分析

式(17)中 $f(t)$ 的系数含有静挠度,反映出静挠度(初始参数)对非线性动力系统的影响,系统在静载荷下可能发生的混沌运动临界载荷相应变小^[6]。这也是本文研究的在大挠度基础上的混沌意义和目的。从时程曲线图,Poincaré 映射图和相轨图分析知:当阻尼系数越小,外激励载荷越大时,此动力系统越容易产生混沌运动。

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