文章编号：1673-5196(2021)03-0070-08

一类受到未知外部干扰的多智能体系统学习协同控制

杨娜娜*[1]，孟新友[1]，玄海燕[2]

（1.兰州理工大学 理学院，甘肃 兰州　730050；2.兰州理工大学 经济管理学院，甘肃 兰州　730050）

摘要：针对一类受到未知外部干扰的多智能体系统，在迭代学习控制框架下，结合自适应控制，首先设计了具有微分型参数自适应律的时变增益，同时，为了补偿未知外部干扰，设计了辅助控制器；通过构造复合能量函数，基于类Barbalat引理，证明了区间$[0,T]$上的完全一致性.其次，借助坐标变换，将编队问题转化为一致性问题.最后，通过一个仿真算例验证了所提算法的有效性.

关键词：多智能体系统；迭代学习控制；未知外部干扰；一致性

中图分类号：TP301.6；O415.5　　文献标志码：A

Learning cooperative control of a class of multi-agent systems with unknown external disturbances

YANG Na-na[1]，MENG Xin-you[1]，XUAN Hai-yan[2]

(1. School of Science，Lanzhou Univ. of Tech.，Lanzhou　730050，China；2. School of Economics and Management，Lanzhou Univ. of Tech.，Lanzhou　730050，China)

Abstract：For the multi-agent systems with unknown external interferences，in the framework of iterative learning control，combined with adaptive control，firstly，a time-varying gain with differential parameter adaptive law is designed，at the same time，in order to compensate the unknown external interferences，the auxiliary control protocol is proposed. By constructing the composite energy function and based on Barbalat-like lemma，the perfect consensus on the interval $[0,T]$ is proved. Secondly，the formation problem is transformed into the consensus problem by the coordinate transformation.Finally，the effectiveness of the proposed algorithm is verified using a simulation example.

Key words：multi-agent systems；iterative learning control；unknown external disturbances；consensus

多智能体系统(MASs)协同控制因其广泛的应用背景，如无人机编队、航天器飞行、传感器网络等，受到多学科专家们的极大关注[1-2].一致性问题作为MASs协同问题研究中最基本且重要的课题，是指在分布式控制协议下，智能体的状态或输出通过与其邻居共享信息而达到一个共同值[3-4]，包括带头节点一致性[5-7]和无头节点一致性[8-10]，现如今已成为MASs领域的一个研究热点.

此外，实际中，存在大量的系统是可重复运行的[11].在这样的实际背景下，迭代学习控制(ILC)方法作为一种智能控制算法被提出[12-13].ILC可以有效解决有限时间区间上可重复运行系统的高精度轨迹跟踪问题，具有控制算法简单、不需要知道具体的数学模型等优点.目前，随着对ILC的深入研究，已经与一些先进控制相结合，产生了诸多新型的控制算法，如自适应控制[14]、模糊控制[15]、最优控制[16]等.

值得一提的是，许建新等[12-16]都是将ILC方法用于单个系统的研究，而近些年来，很多学者[17-22]已将ILC用于MASs协同问题的研究，其中文献[17～20]利用自适应ILC的方法分别研究了一阶、二阶和高阶非线性MASs的协同控制问题；Meng等[21]将ILC与输出反馈方法相结合，处理了高阶非线性MASs的有限时间一致性问题；Yang等[22]提出了最优控制器增益的设计方法，使得一致性误差的λ-范数以最快的速度收敛，但这些文献[17-22]都没

收稿日期：2020-07-27
基金项目：国家自然科学基金(11661050,11261031)
通讯作者：杨娜娜(1986-)，女，甘肃永靖人，博士，讲师.
Email：xjyangnana@126.com

有考虑到未知外部干扰的影响.实际工程中,由于复杂环境等因素,系统的外部干扰是比较常见的,会对系统的稳定产生一定影响,有时可能导致系统不稳定.因此,在系统的控制器设计时,有必要考虑外部干扰,且具有一定的实际意义.

因此,本文在一类 MASs 中考虑未知外部干扰,并通过坐标变换,将一致性问题推广到编队问题,解决了区间$[0,T]$上的完全一致性和编队问题,即 MASs 的协同控制问题.

1　问题描述

在重复环境下,考虑一类 MASs:

$$\begin{aligned}&\text{从节点:}\ \dot{x}_i^k(t)=\boldsymbol{A}x_i^k(t)+u_i^k(t)+\tilde{\omega}_i^k(t)\\&\text{头节点:}\ \dot{x}_0^k(t)=\boldsymbol{A}x_0(t)\end{aligned}\tag{1}$$

其中:k 是迭代次数;$t\in[0,T]$;$x_i^k(t)$、$u_i^k\in R^n$ 分别表示第 i 个从节点的状态和输入向量;$\tilde{\omega}_i^k(t)\in R^n$ 是未知的时变有界外部干扰,$i=1,2,\cdots,N$;$x_0(t)\in R^n$ 是头节点的状态向量.

假设1　$\tilde{\omega}_i^k(t)$的分量满足$\|\tilde{\omega}_{ij}^k(t)\|\leqslant\tilde{\omega}_{ij}^*$,其中 $\tilde{\omega}_{ij}^*$ 是未知时不变常数$(j=1,2,\cdots,n)$,且记 $\tilde{\omega}_i^*=[\tilde{\omega}_{i1}^*,\tilde{\omega}_{i2}^*,\cdots,\tilde{\omega}_{in}^*]^{\mathrm{T}}$.

假设2　所有的智能体满足对接条件,即:

$$x_i^k(0)=x_i^{k-1}(T),\ x_0(0)=x_0(T)$$

定义第 i 个从节点与头节点之间的一致性误差向量为

$$\delta_i^k=x_i^k-x_0\tag{2}$$

本文的主要目标是寻找合适的控制协议序列$\{u_i^k(t),0\leqslant t\leqslant T;i=1,2,\cdots,N;k=0,1,2,\cdots\}$,使得每个从节点随着 k 趋于无穷,在区间$[0,T]$上完全跟踪上头节点,即实现完全一致性,$\lim\limits_{k\to\infty}\|\delta_i^k\|=0$,$i=1,2,\cdots,N$,$\forall t\in[0,T]$.

再定义第 i 个从节点的分布式误差向量为

$$e_i^k=\sum_{j=1}^{N}a_{ij}(x_j^k-x_i^k)+b_i(x_0-x_i^k)\tag{3}$$

且由误差(2,3),得:

$$\delta_x^k=x^k-1_N\otimes x_0\tag{4}$$

$$\begin{aligned}e^k=&-((L+B)\otimes I_n)(x^k-1_N\otimes x_0)=\\&-(H\otimes I_n)\delta^k\end{aligned}$$

其中:

$$\delta^k=[(\delta_1^k)^{\mathrm{T}},(\delta_2^k)^{\mathrm{T}},\cdots,(\delta_N^k)^{\mathrm{T}}]^{\mathrm{T}}\in R^{Nn}$$
$$x^k=[(x_1^k)^{\mathrm{T}},(x_2^k)^{\mathrm{T}},\cdots,(x_N^k)^{\mathrm{T}}]^{\mathrm{T}}\in R^{Nn}$$
$$e^k=[(e_1^k)^{\mathrm{T}},(e_2^k)^{\mathrm{T}},\cdots,(e_N^k)^{\mathrm{T}}]^{\mathrm{T}}\in R^{Nn}$$

注1　本文中,从节点之间的通信拓扑图 G 为无向连通图,而头节点与 G 构成的通信拓扑图G'为有向图,即头节点只能将其信息传递给从节点,而不能获取任意从节点的信息,且记邻接矩阵为 $\boldsymbol{A}=[a_{ij}]_{N\times N}$.若第 i 个从节点可以获得第 j 个从节点的信息,则 $a_{ij}=a_{ji}=1$,否则,$a_{ij}=a_{ji}=0$,且 $a_{ii}=0$.无向连通通信拓扑图 G 的 Laplace 矩阵为 $\boldsymbol{L}=\boldsymbol{D}-\boldsymbol{A}$,其中 $\boldsymbol{D}=\mathrm{diag}\{d_1,d_2,\cdots,d_N\}$,$d_i=\sum_{j=1}^{N}a_{ij}$.此外,$\boldsymbol{B}=\mathrm{diag}\{b_1,b_2,\cdots,b_N\}$,其中当第 i 个从节点可以获得头节点信息时,$b_i=1$,否则 $b_i=0$.本文假设至少存在一个从节点可以直接获得头节点信息,则 $\boldsymbol{H}=\boldsymbol{L}+\boldsymbol{B}$ 是对称正定矩阵[18-21].

另外,由误差(2),得误差动态为

$$\dot{\delta}_i^k=\boldsymbol{A}\delta_i^k+u_i^k(t)+\tilde{\omega}_i^k\tag{5}$$

2　控制协议设计

基于以上的误差动态(5),设计第 i 个从节点的分布式控制协议为

$$u_i^k(t)=\beta_i^k(t)+\hat{r}_i^k(t)e_i^k\tag{6}$$

$$\beta_i^k(t)=\hat{\tilde{\omega}}_i^k(t)\tanh\left(\frac{(e_i^k)^{\mathrm{T}}\boldsymbol{Q}\hat{\tilde{\omega}}_i^k(t)}{\Delta_{k+1}}\right)\tag{7}$$

其中:Q 是任意正定矩阵.

此时,误差动态(5)可集中写为

$$\begin{aligned}\dot{\delta}^k=&(\boldsymbol{I}_N\otimes\boldsymbol{A})\delta^k-\\&(\hat{R}(t)\boldsymbol{H}\otimes\boldsymbol{I}_n)\delta^k+\beta^k+\tilde{\omega}^k\end{aligned}\tag{8}$$

其中:

$$\hat{R}(t)=\mathrm{diag}\{\hat{r}_1^k(t),\hat{r}_2^k(t),\cdots,\hat{r}_N^k(t)\}$$
$$\beta^k=[(\beta_1^k)^{\mathrm{T}},(\beta_2^k)^{\mathrm{T}},\cdots,(\beta_N^k)^{\mathrm{T}}]^{\mathrm{T}}\in R^{Nn}$$
$$\tilde{\omega}^k=[(\tilde{\omega}_1^k)^{\mathrm{T}},(\tilde{\omega}_2^k)^{\mathrm{T}},\cdots,(\tilde{\omega}_N^k)^{\mathrm{T}}]^{\mathrm{T}}\in R^{Nn}$$

设计 $\hat{r}_i^k(t)$和 $\hat{\tilde{\omega}}_i^k(t)$的微分型参数自适应律为

$$\begin{cases}\dot{\hat{r}}_i^k(t)=\varphi_i(e_i^k)^{\mathrm{T}}\boldsymbol{Q}e_i^k\\\hat{r}_i^k(0)=\hat{r}_i^{k-1}(T)\end{cases}\tag{9}$$

$$\begin{cases}\dot{\hat{\tilde{\omega}}}_i^k(t)=\Psi_i\,|\boldsymbol{Q}e_i^k|\\\hat{\tilde{\omega}}_i^k(0)=\hat{\tilde{\omega}}_i^{k-1}(T)\end{cases}\tag{10}$$

其中:

$$\varphi_i>0$$
$$\Psi_i=\mathrm{diag}\{\psi_{i1},\psi_{i2},\cdots,\psi_{in}\},\psi_{ij}>0$$
$$\hat{\tilde{\omega}}_i^0(0)=[\hat{\tilde{\omega}}_{i1}^0(0),\hat{\tilde{\omega}}_{i2}^0(0),\cdots,\hat{\tilde{\omega}}_{in}^0(0)]^{\mathrm{T}},\hat{\tilde{\omega}}_{ij}^0(0)\geqslant 0$$
$$(i=1,2,\cdots,N;j=1,2,\cdots,n)$$

3　学习一致性分析

定理1　对于具有通信拓扑图 G'的 MASs

(1)，当假设 1、2 成立时，设计的控制协议(6,7)以及参数自适应律(9,10)能够使得所有从节点在区间 $[0,T]$ 上随着迭代次数的无限增加与头节点达到完全一致，即

$$\lim_{k\to\infty}\|\delta_i^k\|=0,\quad \forall t\in[0,T],\quad i=1,2,\cdots,N$$

同时，闭环系统内的所有信号有界.

证明 构造如下的复合能量函数(CEF)：

$$E^k(t)=(\delta^k)^{\mathrm T}(\boldsymbol H\otimes\boldsymbol Q)\delta^k+\sum_{i=1}^N\frac{1}{\varphi_i}(\tilde r_i^k)^2+\sum_{i=1}^N(\tilde{\tilde\omega}_i^k(t))^{\mathrm T}\boldsymbol\Psi_i^{-1}\tilde{\tilde\omega}_i^k(t)\tag{11}$$

其中：$\tilde r_i^k=r-\hat r_i^k$；$\tilde{\tilde\omega}_i^k(t)=\tilde\omega_i^*-\hat{\tilde\omega}_i^k(t)$.

(1) 考查 $E^k(t)$ 第 k 次和 $k-1$ 次之间的差分：

$$\begin{aligned}\Delta E^k(t)=&E^k(t)-E^{k-1}(t)=\\&(\delta^k)^{\mathrm T}(\boldsymbol H\otimes\boldsymbol Q)\delta^k-(\delta^{k-1})^{\mathrm T}(\boldsymbol H\otimes\boldsymbol Q)\delta^{k-1}+\\&\sum_{i=1}^N\frac{1}{\varphi_i}[(\tilde r_i^k)^2-(\tilde r_i^{k-1})^2]+\\&\sum_{i=1}^N[(\tilde{\tilde\omega}_i^k(t))^{\mathrm T}\boldsymbol\Psi_i^{-1}\tilde{\tilde\omega}_i^k(t)-\\&(\tilde{\tilde\omega}_i^{k-1}(t))^{\mathrm T}\boldsymbol\Psi_i^{-1}\tilde{\tilde\omega}_i^{k-1}(t)]\end{aligned}\tag{12}$$

其中，由式(8,12)中的第 1 项可变形为

$$\begin{aligned}&(\delta^k)^{\mathrm T}(\boldsymbol H\otimes\boldsymbol Q)\delta^k=\\&2\int_0^t(\delta^k)^{\mathrm T}(\boldsymbol H\otimes\boldsymbol Q)\dot\delta^k\mathrm d\rho+\\&(\delta^k(0))^{\mathrm T}(\boldsymbol H\otimes\boldsymbol Q)\delta^k(0)=\\&\int_0^t(\delta^k)^{\mathrm T}[\boldsymbol H\otimes(\boldsymbol{QA}+\boldsymbol A^{\mathrm T}\boldsymbol Q)]\delta^k\mathrm d\rho-\\&2\int_0^t(\delta^k)^{\mathrm T}(\boldsymbol H\hat{\boldsymbol R}(\rho)\boldsymbol H\otimes\boldsymbol Q)\delta^k\mathrm d\rho+\\&(\delta^k(0))^{\mathrm T}(\boldsymbol H\otimes\boldsymbol Q)\delta^k(0)+\\&2\int_0^t(\delta^k)^{\mathrm T}(\boldsymbol H\otimes\boldsymbol Q)(\beta^k+\tilde\omega^k)]\mathrm d\rho\end{aligned}\tag{13}$$

再由式(9)，第 3 项变形为

$$\begin{aligned}&\sum_{i=1}^N\frac{1}{\varphi_i}[(\tilde r_i^k(t))^2-(\tilde r_i^{k-1}(t))^2]=\\&\sum_{i=1}^N\frac{2}{\varphi_i}\int_0^t\tilde r_i^k(\rho)\dot{\tilde r}_i^k(\rho)\mathrm d\rho+\\&\sum_{i=1}^N\frac{1}{\varphi_i}[(\tilde r_i^k(0))^2-(\tilde r_i^{k-1}(t))^2]=\\&-\sum_{i=1}^N\frac{2}{\varphi_i}\int_0^t\tilde r_i^k(\rho)(e_i^k)^{\mathrm T}\boldsymbol Qe_i^k\mathrm d\rho+\\&\sum_{i=1}^N\frac{1}{\varphi_i}[(\tilde r_i^k(0))^2-(\tilde r_i^{k-1}(t))^2]=\\&-2r\int_0^t(\delta^k)^{\mathrm T}(\boldsymbol H^2\otimes\boldsymbol Q)\delta^k\mathrm d\rho+\\&2\int_0^t(\delta^k)^{\mathrm T}(\boldsymbol H\hat{\boldsymbol R}(\rho)\boldsymbol H\otimes\boldsymbol Q)\delta^k\mathrm d\rho+\\&\sum_{i=1}^N\frac{1}{\varphi_i}[(\tilde r_i^k(0))^2-(\tilde r_i^{k-1}(t))^2]\end{aligned}\tag{14}$$

由式(10)，第 5 项变为

$$\begin{aligned}&\sum_{i=1}^N[(\tilde{\tilde\omega}_i^k(t))^{\mathrm T}\boldsymbol\Psi_i^{-1}\tilde{\tilde\omega}_i^k(t)-\\&(\tilde{\tilde\omega}_i^{k-1}(t))^{\mathrm T}\boldsymbol\Psi_i^{-1}\tilde{\tilde\omega}_i^{k-1}(t)]=\\&2\sum_{i=1}^N\int_0^t(\tilde{\tilde\omega}_i^k(\rho))^{\mathrm T}\boldsymbol\Psi_i^{-1}\dot{\tilde{\tilde\omega}}_i^k(\rho)\mathrm d\rho-\\&\sum_{i=1}^N(\tilde{\tilde\omega}_i^{k-1}(t))^{\mathrm T}\boldsymbol\Psi_i^{-1}\tilde{\tilde\omega}_i^{k-1}(t)+\\&\sum_{i=1}^N(\tilde{\tilde\omega}_i^k(0))^{\mathrm T}\boldsymbol\Psi_i^{-1}\tilde{\tilde\omega}_i^k(0)=\\&-2\sum_{i=1}^N\int_0^t(\tilde{\tilde\omega}_i^k(\rho))^{\mathrm T}|\boldsymbol Qe_i^k|\mathrm d\rho-\\&\sum_{i=1}^N(\tilde{\tilde\omega}_i^{k-1}(t))^{\mathrm T}\boldsymbol\Psi_i^{-1}\tilde{\tilde\omega}_i^{k-1}(t)+\\&\sum_{i=1}^N(\tilde{\tilde\omega}_i^k(0))^{\mathrm T}\boldsymbol\Psi_i^{-1}\tilde{\tilde\omega}_i^k(0)\end{aligned}\tag{15}$$

再将式(13～15)代入式(12)，得：

$$\begin{aligned}\Delta E^k(t)\leqslant&\int_0^t(\delta^k)^{\mathrm T}\{\boldsymbol H\otimes[(\boldsymbol{QA}+\boldsymbol A^{\mathrm T}\boldsymbol Q)-\\&2r\lambda_{\min}(\boldsymbol H)\lambda_{\min}(\boldsymbol Q)\boldsymbol I]\}\delta^k\mathrm d\rho+\\&2\int_0^t(\delta^k)^{\mathrm T}(\boldsymbol H\otimes\boldsymbol Q)(\beta^k+\tilde\omega^k)]\mathrm d\rho-\\&2\sum_{i=1}^N\int_0^t(\tilde{\tilde\omega}_i^k(\rho))^{\mathrm T}|\boldsymbol Qe_i^k|\mathrm d\rho+\\&(\delta^k(0))^{\mathrm T}(\boldsymbol H\otimes\boldsymbol Q)\delta^k(0)-\\&(\delta^{k-1}(t))^{\mathrm T}(\boldsymbol H\otimes\boldsymbol Q)\delta^{k-1}(t)+\\&\sum_{i=1}^N\frac{1}{\varphi_i}[(\tilde r_i^k(0))^2-(\tilde r_i^{k-1}(t))^2]+\\&\sum_{i=1}^N(\tilde{\tilde\omega}_i^k(0))^{\mathrm T}\boldsymbol\Psi_i^{-1}\tilde{\tilde\omega}_i^k(0)-\\&\sum_{i=1}^N(\tilde{\tilde\omega}_i^{k-1}(t))^{\mathrm T}\boldsymbol\Psi_i^{-1}\tilde{\tilde\omega}_i^{k-1}(t)\end{aligned}\tag{16}$$

其中：$\lambda_{\min}(\boldsymbol H)$ 和 $\lambda_{\min}(\boldsymbol Q)$ 是正定矩阵 $\boldsymbol H$ 和 $\boldsymbol Q$ 的最小特征值.

又因为

$$\begin{aligned}&2\int_0^t(\delta^k)^{\mathrm T}(\boldsymbol H\otimes\boldsymbol Q)(\beta^k+\tilde\omega^k)]\mathrm d\rho-\\&2\sum_{i=1}^N\int_0^t(\tilde{\tilde\omega}_i^k(\rho))^{\mathrm T}|\boldsymbol Qe_i^k|\mathrm d\rho=-2\sum_{i=1}^N\int_0^t(e_i^k)^{\mathrm T}\boldsymbol Q\beta_i^k\mathrm d\rho-\\&2\sum_{i=1}^N\int_0^t(e_i^k)^{\mathrm T}\boldsymbol Q\tilde\omega_i^k\mathrm d\rho-2\sum_{i=1}^N\int_0^t(\tilde{\tilde\omega}_i^k)^{\mathrm T}|\boldsymbol Qe_i^k|\mathrm d\rho\leqslant\end{aligned}$$

$$-2\sum_{i=1}^{N}\int_0^t (e_i^k)^{\mathrm T}\boldsymbol{Q}\hat{\tilde{\omega}}_i^k(t)\tanh\left(\frac{(e_i^k)^{\mathrm T}\boldsymbol{Q}\hat{\tilde{\omega}}_i^k(t)}{\Delta_{k+1}}\right)\mathrm{d}\rho +$$
$$2\sum_{i=1}^{N}\int_0^t |(e_i^k)^{\mathrm T}\boldsymbol{Q}|\tilde{\omega}_i^*\mathrm{d}\rho -$$
$$2\sum_{i=1}^{N}\int_0^t |(e_i^k)^{\mathrm T}\boldsymbol{Q}|(\tilde{\omega}_i^k)\mathrm{d}\rho =$$
$$2\sum_{i=1}^{N}\int_0^t |(e_i^k)^{\mathrm T}\boldsymbol{Q}|\hat{\tilde{\omega}}_i^k -$$
$$2\sum_{i=1}^{N}\int_0^t (e_i^k)^{\mathrm T}\boldsymbol{Q}\hat{\tilde{\omega}}_i^k(t)\tanh\left(\frac{(e_i^k)^{\mathrm T}\boldsymbol{Q}\hat{\tilde{\omega}}_i^k(t)}{\Delta_{k+1}}\right)\mathrm{d}\rho \leqslant$$
$$2NT\varepsilon\Delta_{k+1} \tag{17}$$

其中最后一个不等式由文献[21]中的 Lemma 2.2 得到.因此：

$$\Delta E^k(t)=\int_0^t (\delta^k)^{\mathrm T}\{\boldsymbol{H}\otimes[(\boldsymbol{QA}+\boldsymbol{A}^{\mathrm T}\boldsymbol{Q})-$$
$$2r\lambda_{\min}(\boldsymbol{H})\lambda_{\min}(\boldsymbol{Q})\boldsymbol{I}]\}\delta^k\mathrm{d}\rho +$$
$$(\delta^k(0))^{\mathrm T}(\boldsymbol{H}\otimes\boldsymbol{Q})\delta^k(0)-$$
$$(\delta^{k-1}(t))^{\mathrm T}(\boldsymbol{H}\otimes\boldsymbol{Q})\delta^{k-1}(t)+$$
$$\sum_{i=1}^{N}\frac{1}{\varphi_i}[(\tilde{r}_i^k(0))^2-(\tilde{r}_i^{k-1}(t))^2]+$$
$$2NT\varepsilon\Delta_{k+1}+\sum_{i=1}^{N}(\tilde{\omega}_i^k(0))^{\mathrm T}\boldsymbol{\Psi}_i^{-1}\tilde{\omega}_i^k(0)-$$
$$\sum_{i=1}^{N}(\tilde{\omega}_i^{k-1}(t))^{\mathrm T}\boldsymbol{\Psi}_i^{-1}\tilde{\omega}_i^{k-1}(t) \tag{18}$$

此时，可以选择充分大的 $r>0$，使得

$$(\boldsymbol{QA}+\boldsymbol{A}^{\mathrm T}\boldsymbol{Q})-2r\lambda_{\min}(\boldsymbol{H})\lambda_{\min}(\boldsymbol{Q})\boldsymbol{I}\leqslant-\sigma\boldsymbol{I} \tag{19}$$

对于 $\sigma>0$ 总成立.从而有

$$\Delta E^k(t)\leqslant-\sigma\lambda_{\min}(\boldsymbol{H})\sum_{i=1}^{N}\int_0^t (\delta_i^k)^{\mathrm T}\delta_i^k\mathrm{d}\rho +$$
$$(\delta^k(0))^{\mathrm T}(\boldsymbol{H}\otimes\boldsymbol{Q})\delta^k(0)-$$
$$(\delta^{k-1}(t))^{\mathrm T}(\boldsymbol{H}\otimes\boldsymbol{Q})\delta^{k-1}(t)+$$
$$\sum_{i=1}^{N}\frac{1}{\varphi_i}[(\tilde{r}_i^k(0))^2-(\tilde{r}_i^{k-1}(t))^2]+$$
$$2NT\varepsilon\Delta_{k+1}+\sum_{i=1}^{N}(\tilde{\omega}_i^k(0))^{\mathrm T}\boldsymbol{\Psi}_i^{-1}\tilde{\omega}_i^k(0)-$$
$$\sum_{i=1}^{N}(\tilde{\omega}_i^{k-1}(t))^{\mathrm T}\boldsymbol{\Psi}_i^{-1}\tilde{\omega}_i^{k-1}(t) \tag{20}$$

令 $t=T$，则由假设 2 和自适应律(9,10)可知：

$$\Delta E^k(T)\leqslant$$
$$-\sigma\lambda_{\min}(\boldsymbol{H})\sum_{i=1}^{N}\int_0^t (\delta_i^k)^{\mathrm T}\delta_i^k\mathrm{d}\rho+2NT\varepsilon\Delta_{k+1} \tag{21}$$

即

$$E^k(T)\leqslant E^{k-1}(T)+2NT\varepsilon\Delta_{k+1} \tag{22}$$

(2) 证明闭环系统所有信号的有界性.

由式(12)，可得：

$$E^k(t)=\Delta E^k(t)+E^{k-1}(t)\leqslant$$
$$-\sigma\lambda_{\min}(\boldsymbol{H})\sum_{i=1}^{N}\int_0^t (\delta_i^k)^{\mathrm T}\delta_i^k\mathrm{d}\rho +$$
$$(\delta^k(0))^{\mathrm T}(\boldsymbol{H}\otimes\boldsymbol{Q})\delta^k(0)-$$
$$(\delta^{k-1}(t))^{\mathrm T}(\boldsymbol{H}\otimes\boldsymbol{Q})\delta^{k-1}(t)+$$
$$\sum_{i=1}^{N}\frac{1}{\varphi_i}[(\tilde{r}_i^k(0))^2-(\tilde{r}_i^{k-1}(t))^2]+$$
$$2NT\varepsilon\Delta_{k+1}+\sum_{i=1}^{N}(\tilde{\omega}_i^k(0))^{\mathrm T}\boldsymbol{\Psi}_i^{-1}\tilde{\omega}_i^k(0)-$$
$$\sum_{i=1}^{N}(\tilde{\omega}_i^{k-1}(t))^{\mathrm T}\boldsymbol{\Psi}_i^{-1}\tilde{\omega}_i^{k-1}(t)+$$
$$(\delta^k(t))^{\mathrm T}(\boldsymbol{H}\otimes\boldsymbol{Q})\delta^k(t)+\sum_{i=1}^{N}\frac{1}{\varphi_i}(\tilde{r}_i^k(t))^2+$$
$$\sum_{i=1}^{N}(\tilde{\omega}_i^k(t))^{\mathrm T}\boldsymbol{\Psi}_i^{-1}\tilde{\omega}_i^k(t)=$$
$$-\sigma\lambda_{\min}(\boldsymbol{H})\sum_{i=1}^{N}\int_0^t (\delta_i^k)^{\mathrm T}\delta_i^k\mathrm{d}\rho +$$
$$(\delta^k(0))^{\mathrm T}(\boldsymbol{H}\otimes\boldsymbol{Q})\delta^k(0)+\sum_{i=1}^{N}\frac{1}{\varphi_i}(\tilde{r}_i^k(0))^2+$$
$$\sum_{i=1}^{N}(\tilde{\omega}_i^k(0))^{\mathrm T}\boldsymbol{\Psi}_i^{-1}\tilde{\omega}_i^k(0)+2NT\varepsilon\Delta_{k+1}=$$
$$-\sigma\lambda_{\min}(\boldsymbol{H})\sum_{i=1}^{N}\int_0^t (\delta_i^k)^{\mathrm T}\delta_i^k\mathrm{d}\rho +$$
$$E^{k-1}(T)+2NT\varepsilon\Delta_{k+1} \tag{23}$$

则

$$E^k(t)\leqslant E^{k-1}(T)+2NT\varepsilon\Delta_{k+1} \tag{24}$$

且

$$E^k(t)\leqslant E^{k-1}(T)+2NT\varepsilon\Delta_{k+1}\leqslant$$
$$E^{k-2}(T)+2NT\varepsilon\Delta_k+2NT\varepsilon\Delta_{k+1}\leqslant$$
$$\cdots\leqslant E^0(T)+2NT\varepsilon\sum_{l=2}^{k+1}\Delta_l \tag{25}$$

另外，由文献[21]中的 Lemma 2.1：

$$\lim_{k\to\infty}2NT\varepsilon\sum_{l=2}^{k+1}\Delta_l\leqslant 4NT\varepsilon\mu$$

故 $2NT\varepsilon\sum_{l=2}^{k+1}\Delta_l$ 有界，对于 $\forall k$ 成立.不失一般性，记 $2NT\varepsilon\sum_{l=2}^{k+1}\Delta_l\leqslant K, K>0$，则

$$E^k(t)\leqslant E^0(T)+K \tag{26}$$

从式(26)可以看到，如果 $E^0(T)$ 有界，就能保证 $E^k(t)$ 是一致有界的.所以，下面证明 $E^0(t)$ 的有界性.因为

$$E^0(t)=(\delta^0)^{\mathrm T}(\boldsymbol{H}\otimes\boldsymbol{Q})\delta^0+\sum_{i=1}^{N}\frac{1}{\varphi_i}(\tilde{r}_i^0)^2+$$

$$\sum_{i=1}^{N}(\tilde{\omega}_i^0(t))^{\mathrm{T}}\boldsymbol{\Psi}_i^{-1}\tilde{\omega}_i^0(t) \tag{27}$$

对式(27)两端求导,得

$$\begin{aligned}\dot{E}^0(t)=&2(\delta^0)^{\mathrm{T}}(\boldsymbol{H}\otimes\boldsymbol{Q})\dot{\delta}^0+\\&\sum_{i=1}^{N}\frac{2}{\varphi_i}(\tilde{r}_i^0)\dot{\tilde{r}}_i^0+\\&2\sum_{i=1}^{N}(\tilde{\omega}_i^0(t))^{\mathrm{T}}\boldsymbol{\Psi}_i^{-1}\dot{\tilde{\omega}}_i^0(t)\leqslant\\&-\sigma\lambda_{\min}(\boldsymbol{H})(\delta_i^0)^{\mathrm{T}}[(\boldsymbol{QA}+\boldsymbol{A}^{\mathrm{T}}\boldsymbol{Q})-\\&2r\lambda_{\min}(\boldsymbol{H})\lambda_{\min}(\boldsymbol{Q})\boldsymbol{I}]\delta_i^0+\\&2(\delta^0)^{\mathrm{T}}(\boldsymbol{H}\otimes\boldsymbol{Q})(\beta^0+\tilde{\omega}^0)-\\&2\sum_{i=1}^{N}(\tilde{\omega}_i^0)^{\mathrm{T}}|\boldsymbol{Q}e_i^0|\end{aligned} \tag{28}$$

其中:

$$\begin{aligned}&2(\delta^0)^{\mathrm{T}}(\boldsymbol{H}\otimes\boldsymbol{Q})(\beta^0+\tilde{\omega}^0)-2\sum_{i=1}^{N}(\tilde{\omega}_i^0)^{\mathrm{T}}|\boldsymbol{Q}e_i^0|=\\&-2\sum_{i=1}^{N}(e_i^0)^{\mathrm{T}}\boldsymbol{Q}\beta_i^0-2\sum_{i=1}^{N}(e_i^0)^{\mathrm{T}}\boldsymbol{Q}\tilde{\omega}_i^0-\\&2\sum_{i=1}^{N}(\tilde{\omega}_i^0)^{\mathrm{T}}|\boldsymbol{Q}e_i^0|\leqslant\\&-2\sum_{i=1}^{N}(e_i^0)^{\mathrm{T}}\boldsymbol{Q}\hat{\tilde{\omega}}_i^0(t)\tanh\left(\frac{(e_i^0)^{\mathrm{T}}\boldsymbol{Q}\hat{\tilde{\omega}}_i^0(t)}{\Delta_1}\right)+\\&2\sum_{i=1}^{N}|(e_i^0)^{\mathrm{T}}\boldsymbol{Q}|\tilde{\omega}_i^*-2\sum_{i=1}^{N}|(e_i^0)^{\mathrm{T}}\boldsymbol{Q}|(\tilde{\omega}_i^0)=\\&2\sum_{i=1}^{N}|(e_i^0)^{\mathrm{T}}\boldsymbol{Q}|\hat{\tilde{\omega}}_i^0-\\&2\sum_{i=1}^{N}(e_i^0)^{\mathrm{T}}\boldsymbol{Q}\hat{\tilde{\omega}}_i^0(t)\tanh\left(\frac{(e_i^0)^{\mathrm{T}}\boldsymbol{Q}\hat{\tilde{\omega}}_i^0(t)}{\Delta_1}\right)\leqslant\\&2NT\varepsilon\Delta_1=2N\varepsilon a\end{aligned} \tag{29}$$

则

$$\begin{aligned}\dot{E}^0(t)\leqslant&-\sigma\lambda_{\min}(\boldsymbol{H})\sum_{i=1}^{N}(\delta_i^0)^{\mathrm{T}}\delta_i^0+\\&2N\varepsilon a\leqslant 2N\varepsilon a\triangleq F_0\end{aligned} \tag{30}$$

所以:

$$\begin{aligned}E^0(t)\leqslant&|E^0(0)|+\int_0^t|\dot{E}^0(\rho)|\mathrm{d}\rho\leqslant\\&(\delta^0(0))^{\mathrm{T}}(\boldsymbol{H}\otimes\boldsymbol{Q})\delta^0(0)+TF_0+\\&\sum_{i=1}^{N}\frac{(\tilde{r}_i^0(0))^2}{\varphi_i}+\sum_{i=1}^{N}(\tilde{\omega}_i^0(0))^{\mathrm{T}}\boldsymbol{\Psi}_i^{-1}\tilde{\omega}_i^0(0)<\infty\end{aligned} \tag{31}$$

由式(31),知 $E^0(t)$ 有界,$\forall t\in[0,T]$,则 $E^0(T)$ 有界,得到了 $E^k(t)$ 在$[0,T]$上的一致有界性,从而得 δ_i^k、$\hat{r}_i^k(t)$和 $\hat{\tilde{\omega}}_i^k(t)$是一致有界的.再由式(6,7)知,$u_i^k(t)$一致有界.

(3) 证明$[0,T]$上的完全一致性.

从式(21),可得:

$$E^k(T)=E^0(T)+\sum_{l=1}^{k}\Delta E^l(T) \tag{32}$$

则

$$\begin{aligned}E^k(T)\leqslant& E^0(T)+2NT\varepsilon\sum_{l=2}^{k+1}\Delta_l-\\&\sigma\lambda_{\min}(\boldsymbol{H})\sum_{l=1}^{k}\sum_{i=1}^{N}\int_0^t(\delta_i^l)^{\mathrm{T}}\delta_i^l\mathrm{d}\rho\end{aligned} \tag{33}$$

由 $E^k(T)>0$,$E^0(T)$ 有界以及级数 $2NT\varepsilon\sum_{l=2}^{k+1}\Delta_l$ 收敛,可得级数 $\sum_{l=1}^{k}\sum_{i=1}^{N}\int_0^t(\delta_i^l)^{\mathrm{T}}\delta_i^l\mathrm{d}\rho$ 收敛.故

$$\lim_{k\to\infty}\int_0^t(\delta_i^k)^{\mathrm{T}}\delta_i^k\mathrm{d}\rho=0\qquad(i=1,2,\cdots,N)$$

再由式(5),$\dot{\delta}_i^k$ 一致有界.结合类 Barbalat 引理[23],$\lim_{k\to\infty}\|\delta_i^k\|=0$,$\forall t\in[0,T]$,$i=1,2,\cdots,N$.

综上,定理1得证.

4 编队问题

对于$\forall t\in[0,T]$,若每个从节点与头节点之间能够形成期望的距离,这意味着 MASs(1)实现了编队控制.

定义:

$$\bar{x}_{i,1}^k=x_{i,1}^k-\Delta_i \tag{34}$$

其中:Δ_i 是第 i 个从节点与头节点之间期望的距离.

第 i 个从节点与头节点之间的编队误差为

$$\delta_{i,1}^k(t)=\bar{x}_{i,1}^k(t)-x_{0,1}(t) \tag{35}$$

和 $\delta_{i,s}^k(t)$,$s=2,3,\cdots,n$.因此,可以将编队问题重新描述为一致性问题,即$\lim_{k\to\infty}\|\delta_i^k\|=0$,$\forall t\in[0,T]$.

此外,第 i 个从节点的分布式编队误差定义为

$$\begin{cases}e_{i,1}^k=\sum_{j=1}^{N}a_{ij}(\bar{x}_{j,1}^k-\bar{x}_{i,1}^k)+b_i(x_{0,1}-\bar{x}_{i,1}^k)\\e_{i,s}^k=\sum_{j=1}^{N}a_{ij}(x_{j,s}^k-x_{i,s}^k)+b_i(x_{0,s}-x_{i,s}^k)\end{cases} \tag{36}$$

假设3 假设所有智能体满足对接条件,即

$$\bar{x}_{i,1}^k(0)=\bar{x}_{i,1}^{k-1}(T),\quad x_{i,s}^k(0)=x_{i,s}^{k-1}(T)$$
$$(i=1,2,\cdots,N;\ s=2,3,\cdots,n)$$

且 $x_0(0)=x_0(T)$.

定理2 对于具有通信拓扑图 G'的 MASs(1),当假设1、3成立时,基于误差(36)设计的控制协议(6,7)以及参数自适应律(9,10),能够使得所有从节点在区间$[0,T]$上随着迭代次数的无限增加而与头

节点形成期望的编队，即 $\lim_{k\to\infty}\|\delta_i^k\|=0,\ \forall t\in[0,T]$，$i=1,2,\cdots,N$；同时，闭环系统内的所有信号有界.

5　仿真

为了验证本文算法的有效性和实用性，给出一个网络化 LC 振荡器系统[24]，此系统可以看作是一个 MASs，由 5 个从振荡器和 1 个头振荡器组成，其网络通信拓扑图如图 1 所示.

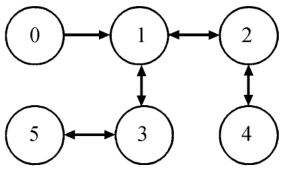

图 1　通信拓扑图(0 表示头节点)

Fig.1　Communication topology graph(0 denotes the leader)

由图 1 得到，只有第 1 个从节点可以得到头节点的信息，且 L 和 B 分别为

$$L=\begin{bmatrix}2&-1&-1&0&0\\-1&2&0&-1&0\\-1&0&2&0&-1\\0&-1&0&1&0\\0&0&-1&0&1\end{bmatrix}$$

$$B=\mathrm{diag}\{1,0,0,0,0\}$$

第 i 个振荡器的动态描述为

$$\begin{cases}\dfrac{\mathrm{d}v_i(t)}{\mathrm{d}t}=\dfrac{1}{C}c_i(t)\\ \dfrac{\mathrm{d}c_i(t)}{\mathrm{d}t}=-\dfrac{1}{L'}v_i(t)\end{cases}\tag{37}$$

其中：$i=1,2,\cdots,6$；$c_i(t)$、$v_i(t)$、L' 和 C 分别是电流、电压、电感和电容.

在重复环境下，将控制输入 $u_i^k(t)$ 施加到系统(37)上.假设每个从振荡器受到外部干扰 $\tilde{\omega}_i^k(t)$，如此，系统(37)可写为

$$\dot{x}_i^k=Ax_i^k+u_i^k+\tilde{\omega}_i^k(t)\tag{38}$$

其中：

$$A=\begin{bmatrix}0&\dfrac{1}{C}\\-\dfrac{1}{L'}&0\end{bmatrix},\quad x_i^k=\begin{bmatrix}x_{i1}^k\\x_{i2}^k\end{bmatrix}=\begin{bmatrix}v_i^k\\c_i^k\end{bmatrix}$$

$$\tilde{\omega}_i^k(t)=\begin{bmatrix}h_1\sin(l_1t)\\h_2\sin(l_2t)\end{bmatrix}$$

h_1、h_2、l_1 和 l_2 是[0,1]上的随机数，$t\in[0,2]$.

另外，$x_0(t)=[\sin(\pi t),\cos(\pi t)]^{\mathrm{T}}$，满足假设 2.

对于 MASs(38)，运用本文所设计的协议(6,7)和自适应律(10,11).

情形 1：完全一致性问题.

仿真中，选取 $C=1/\pi H$，$L'=1/\pi F$，$Q=\mathrm{diag}\{2,1\}$，$\hat{\tilde{\omega}}_i^0(0)=[1,2]^{\mathrm{T}}$，$i=1,2,3,4,5$.

经过 40 次迭代的仿真结果如图 2 和图 3 所示.图 2 是智能体第 40 次迭代的状态轨迹图，可以看到所有从节点的状态向量与头节点的状态向量在[0,2]上完全重合.图 3 是一致性误差沿迭代轴的演化曲线图，显示所有从节点与头节点的一致性误差是

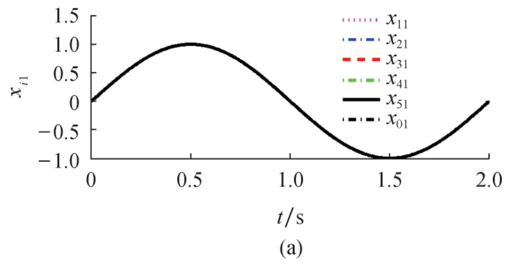

图 2　第 40 次迭代的状态轨迹

Fig.2　State trajectories the 40th iteration

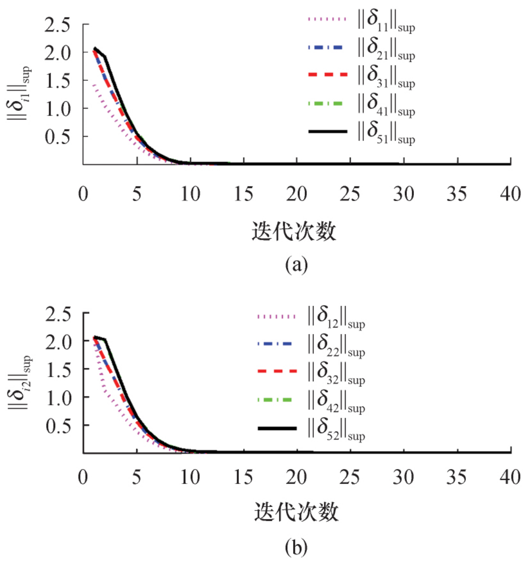

图 3　一致性误差沿迭代轴的演化曲线

Fig.3　Evolution curve of the consensus errors along the iterative axis

一致趋于零的.故图2和图3都说明,在区间[0,2]上,所有从节点随着迭代次数的无限增加,能够完全跟踪上头节点,实现了完全一致性.图4表明,闭环系统的其他变量都有界,进而验证了定理1的成立.

情形2:编队问题.

仿真中,选取 $C=1/\pi\boldsymbol{H}$, $L'=1/\pi F$, $\boldsymbol{Q}=\mathrm{diag}\{2,1\}$, $\Psi_i=\mathrm{diag}\{0.02,3\}$, $\varphi_i=3$, $x_i^0(0)=[0.7,-1.5]^{\mathrm{T}}$, $\hat{\omega}_i^0(0)=[1,2]^{\mathrm{T}}$,其他取值与情形1相同.

迭代75次仿真结果如图5所示,可以看出,智能体的状态向量形成了期望的编队.

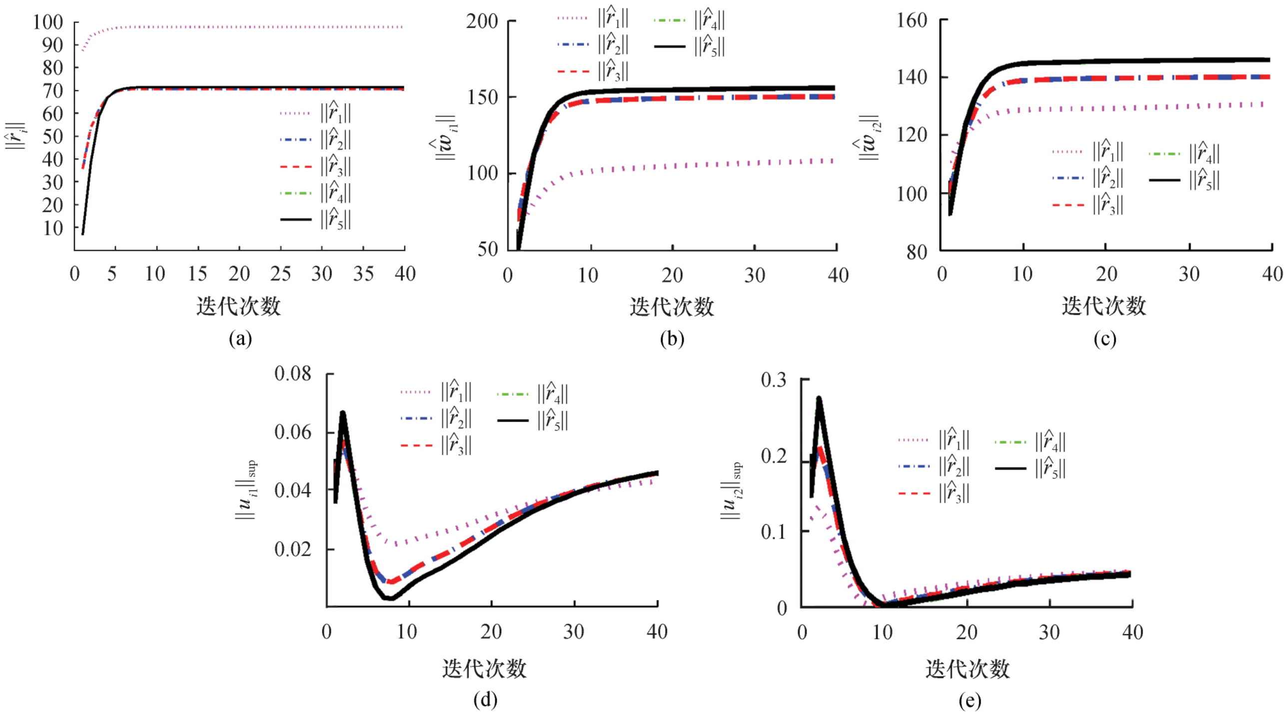

图4　其他变量沿迭代轴的演化曲线

Fig.4　Evolution curves of other variables along the iteration axis

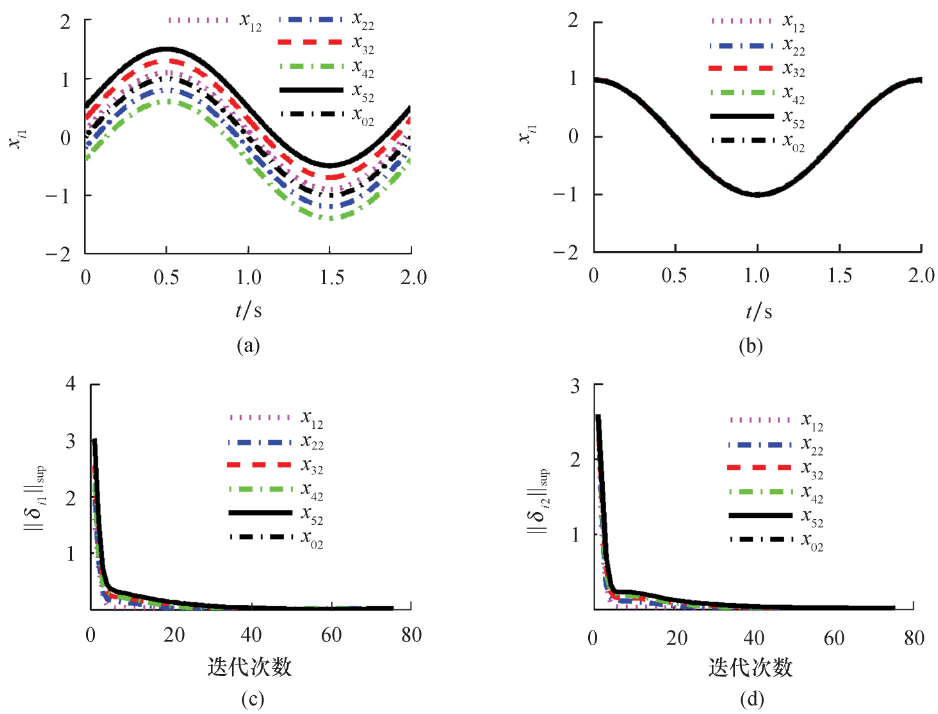

图5　情形2的仿真结果

Fig.5　Simulation results of case 2

6 结论

本文旨在解决一类 MASs 的学习协同控制问题,其中 MASs 受到外部时变有界干扰.通过设计具有微分型参数自适应律的时变增益,避免了控制增益对通信拓扑的依赖,同时,设计的辅助控制协议,补偿了从节点动态中的外部干扰.最终,实现了有限时间区间上的完全一致性,并通过定义新的状态变量与误差,将编队问题转化为一致性问题而解决.

参考文献:

[1] CAO Y C,YU W W,REN W,*et al*.An overview of recent progress in the study of distributed multi-agent coordination [J].IEEE Transactions on Industrial Informatics,2013,9(1):427-438.

[2] GE X H,YANG F,HAN Q L.Distributed networked control systems:a brief overview [J].Information Sciences,2017,380:117-131.

[3] OLFATI-SABER R,MURRAY R M.Consensus problems in networks of agents with switching topology and time-delays [J].IEEE Transactions on Automatic Control,2004,49(9):1520-1533.

[4] TANG Y T.Distributed optimization for a class of high-order nonlinear multi-agent systems with unknown dynamics [J].International Journal of Robust and Nonlinear Control,2018,28(17):5545-5556.

[5] REN C E,CHEN C L P,DU T,*et al*.Fuzzy adaptive leader-following consensus control for nonlinear multi-agent systems with unknown control directions [J].International Journal of Fuzzy Systems,2019,21(7):2066-2076.

[6] CAI Y L,ZHANG H G,HANG K,*et al*.Distributed leader-following consensus of heterogeneous second-order time-varying nonlinear multi-agent systems under directed switching topology [J].Neurocomputing,2019,325:31-47.

[7] MU Xiaowu,XIAO Xia,LIU Kai,*et al*.Leader-following consensus of multi-agent systems with jointly connected topology using distributed adaptive protocols [J].Journal of the Franklin Institute,2014,351:5399-5410.

[8] LI Zhongkui,REN Wei,LIU Xiangdong,*et al*.Consensus of multi-agent systems with general linear and Lipschitz nonlinear dynamics using distributed adaptive protocols [J].IEEE Transactions on Automatic Control,2013,58(7):1786-1791.

[9] REN W.On consensus algorithms for double-integrator dynamics [J].IEEE Transactions on Automatic Control,2008,53(6):1503-1509.

[10] YU W W,CHEN G R,CAO M,*et al*.Second-order consensus for multi-gent systems with directed topologies and nonlinear dynamics [J].IEEE Transactions on Systems,Man,and Cybernetics-Part B:Cybernetics,2010,40(3):881-891.

[11] 金 奎,孙明轩.控制方向未知非线性系统的自适应重复控制[J].控制理论与应用,2012,29(9):1176-1180.

[12] 许建新,侯忠生.学习控制的现状与展望[J].自动化学报,2005,31(6):943-955.

[13] BRISTOW D A,THARAYIL M,ALLEYNE A G.A survey of iterative learning control [J].IEEE Control Systems,2006,26(3):96-114.

[14] XU J X,TAN Y.A composite energy function-based learning control approach for nonlinear systems with time-varying parametric uncertainties [J].IEEE Transactions on Automatic Control,2002,47(11):1940-1945.

[15] CHIEN C J,HSU C T,YAO C Y.Fuzzy system-based adaptive iterative learning control for nonlinear plants with initial state errors [J].IEEE Transactions on Fuzzy Systems,2004,12(5):724-732.

[16] CHI R,LIU X,ZHANG R,*et al*.Constrained data-driven optimal iterative learning control [J].Journal of Process Control,2017,55:10-29.

[17] LI Jinsha,LI Junmin.Adaptive iterative learning control for consensus of multi-agent systems [J].IET Control Theory and Applications,2013,7(1):136-142.

[18] LI Jinsha,LI Junmin.Adaptive iterative learning control for coordination of second-order multi-agent systems [J].International Journal of Robust and Nonlinear Control,2014,24(18):3282-3299.

[19] YANG N N,LI J M.Fully distributed coordination learning control of second-order nonlinear multi-agent systems with input saturation [J].Asian Journal of Control,2020,22:1-14.DOI:10.1002/asjc.2330.

[20] YANG N N,LI J M.Fully distributed hybrid adaptive learning consensus protocols for a class of non-linearly parameterized multi-agent systems [J].Applied Mathematics and Computation,2020,375(125074):1-18.

[21] MENG D Y,JIA Y M.Finite-time consensus for multi-agent systems via terminal feedback iterative learning [J].IET Control Theory and Applications,2011,5(18):2098-2110.

[22] YANG S P,XU J X,HUANG D P,*et al*.Optimal iterative learning control design for multi-agent systems consensus tracking [J].Systems & Control Letters,2014,69:80-89.

[23] SUN M.A Barbalat-like lemma with its application to learning control [J].IEEE Transactions on Automatic Control,2009,54(9):2222-2225.

[24] SHI C X,YANG G H.Robust consensus control for a class of multi-agent systems via distributed PID algorithm and weighted edge dynamics [J].Applied Mathematics and Computation,2018,316:73-88.