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Modeling and analysis of motivation-driven network behavior communication

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This paper presents a new *SIRI* propagation model based on *SIR* model. Behavioral motivation value and neighbor infection number influence value were added to the model to study the effects of these two quantities on behavior transmission. In order to make the model closer to the real situation, self-motivation value and neighbor influence value are added to the behavior motivation value. Compared with previous studies, our proposed experimental model is closer to the real world, improves the possibility of behavior prediction, and is validated by simulation. Finally, this paper takes Lanzhou Polytechnic University campus cartoon consumption data as an example to test and validate the model.

Keywords: Behavior; spread; motivation; infections.

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1. Introduction

Behavioral communication is widely used in our life now. Each of us has our own social circle. These circles are intertwined to form a large and complex social network. In such a social network, people can share various information, public opinions, attitudes and behaviors about things at any time. And people can interact with each other.[1]

In fact, if we put all kinds of communication phenomena in the framework of complex networks, we can get more practical theoretical models and get different new conclusions.[2] Compared with the spread of infectious diseases, the spread of ideas and behaviors has the characteristics of memory, reinforcement and interaction.

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In 2010, Damon Canola, an eco-sociologist at the Massachusetts Institute of Technology, published a study on behavioral communication on Sciences, drawing attention to behavioral communication.[3] There are also some new developments in behavioral communication.[4–8] The dissemination of behavior has an important social and economic impact on daily life, so behavior dissemination and behavior prediction are the research direction in recent years.[9–13]

In previous studies, behavioral communication has only been studied at the level of behavioral science.[14] With the development of complex networks in recent years, new ideas and methods are provided for all kinds of communication advance and research in society.[15] For example, some papers have studied the behavior of user forwarding information, and proposed a user forwarding behavior based on weighted non-negative matrix decomposition. At the same time, different behaviors of garbage users and normal users are studied, and a garbage user identification model based on improved artificial immune algorithm is proposed. The influence of user personality traits on user behavior is also studied.[16–18]

For example, some papers analyze individual decision-making behavior of consumers from a micro-individual perspective, set up an information diffusion model under the influence of individual decision-making, explored how the social influence of consumers affects consumers' purchase decision-making, and proposed the important role of social influence, especially dynamic social influence factors, in user decision-making analysis.[19–22] In addition, some papers further study behavioral communication focusing on group behavior, not just individual behavior.[23–26] What's more, considering the user behavior characteristics that affect forwarding behavior, the existing classification algorithm is used to determine the ultimate user redistribution behavior. At the same time, we design and implement a prediction model for the scale of microblog information dissemination.[27–30] When users enter a complete information, the system can predict the scale of microblog dissemination. There are also papers on how to use the information dissemination path to predict users' information sharing behavior, including users' preference information on content, sources, etc.[31–34]

Based on the papers presented above, the authors put forward different influencing factors in communication, and through the study of these factors, analyze their impact on behavior communication. In the process of behavior dissemination, the disseminator and receiver of the behavior will be affected by many factors, such as their own influence, the influence of neighbors, and the time of information dissemination in the network. Furthermore, the speed of dissemination is influenced by the behavioral decision makers and the surrounding behavioral groups. Based on these comprehensive factors, we build a new model on the basis of the traditional model.

The rest of the paper is organized as follows. We first describe in detail the details of behavioral communication in Sec. 2. However, in Sec. 3, we carried out experimental simulations and validated our ideas with real data. Finally, in Sec. 4, we draw a conclusion.

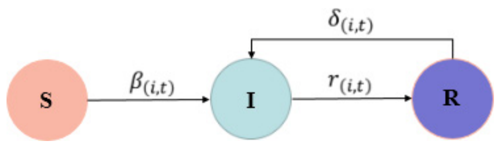


Fig. 1. Network node relationship.

2. System Model and Analysis

In the BA scale-free network, nodes are divided into susceptible population (S), infected population (I) and immune population (R) based on the idea of partitioning by epidemiological dynamic model. The susceptible group here refers to people who do not know but can accept this behavior. Infected people are people who know about this behavior and can spread it to their neighbors. Immunized people are people who know the behavior but do not spread it to their neighbors. In order to conform to the reality, we improve the SIR model to form a new $SIRI$ model. In the $SIRI$ model, the state transition of nodes in behavior propagation is shown in Fig. 1.

For students in the canteen dining process, there will be different windows to provide different food. The students line up outside the window to pick the food they wanted. Some students never choose food in one of the windows, so these students are susceptible nodes. When they are influenced by their neighbors and choose the food in this window, they will have the probability of infection transmission. When they continuously choose the same food in this window for a period of time, due to the boredom of eating these foods continuously, the node will become immune node. Those who are immunized know about it, but they don't spread it out. After a period of time, these nodes become less boring. When they are affected again, they will become nodes again with probability, and the behavior will happen again. Here, $S(t)$, $I(t)$, $R(t)$ is used to represent the density of S, I, R nodes in the network at t-time.

Definition 1. Let $\beta(i,t)$ be the behavior propagation rate of node i at time t, then

$$\beta(i,t) = \beta_0 + \omega_i(t)P_i(t). \tag{1}$$

Here, β_0 is the initial propagation probability,$\omega_i(t)$ is the behavior motivation value, $P_i(t)$ is the neighbor infection number influence value. The behavioral transmission rate was related to the behavior motivation value and the number of neighbor infections.

In our real life, when students choose food in the dining room window, if the number of people who choose food in this window is small, we generally think that the food in this window is not popular, then fewer students come to the dining room and choose food in this window. If this window has a large number of people who choose food, we generally think that the food in this window is popular, but if there are too many people who choose food in this window, the longer the waiting time will be, the fewer students will choose food in this window. When the number of students choosing the food in this window is appropriate, the students will think that the food in this window is popular and they don't need to wait too long. More students will choose this window.

Based on this phenomenon, we propose the influence value of the number of neighbor infections. When there are too many neighbors, the probability of this behavior will be small for node i. However, if there are too few neighbors, node i will have a small probability of this behavior. Only when the number of neighbors of node i is appropriate, it will have a greater impact on node i and node it will be more prone to this behavior.

Definition 2. Let $P_i(t)$ be the influence value of neighbor infection number at time t of node i, then

$$P_i(t) = -\frac{a}{k_i^2} I_i(t)[I_i(t) - (k_i - R_i(t))], \tag{2}$$

where k_i is the degree of node i, a is a constant, $I_i(t)$ is the infection density of node i at time t and $R_i(t)$ is the recovery density of node i at time t.

Based on the assumption that the node i can repeat a particular behavior and the current behavior will be affected by its historical behavior and neighbor behavior, we propose a behavioral motivation value $\omega_i(t)$.

Definition 3. Let $\omega_i(t)$ be the behavior motivation value of node i at time t, the behavior motivation value refers to the probability of such behavior, then

$$\omega_i(t) = \begin{cases} \omega_0, \\ [\omega_i(t-1) + F(i,t)/\omega_{\mathrm{Max}}]/\omega_{\mathrm{Max}}, \end{cases} \tag{3}$$

where ω_{Max} is the maximum behavioral motivation value among all nodes of the whole network. $F(i,t)$ is the neighbor influence value, and the surrounding nodes with this behavior will also have an impact on the node i. ω_0 is the behavior motivation value at the initial time. When the behavior begins to spread, the value of behavior motivation will be affected by itself and neighbors.

Definition 4. Let $F(i,t)$ be the influence value of neighbor j on node i at time t, then

$$F(i,t) = \sum_{j \in K_i} a_{i,j}\omega_j(t), \tag{4}$$

where K_i is the set of neighbors whose node i has this behavior. j is a neighbor around the node i. $a_{i,j}$ is the edge weight between node i and node j.

Here, A is the adjacency matrix of the network, then

$$A = \begin{Bmatrix} a_{1,1} & \cdots & a_{1,N} \\ \vdots & \ddots & \vdots \\ a_{N,1} & \cdots & a_{N,N} \end{Bmatrix}. \tag{5}$$

Definition 5. Let γ be the recovery probability of node i at time t, then

$$\gamma = \gamma_0 \omega_i(t)^{|t-b|}, \tag{6}$$

where γ_0 is the initial recovery probability, $\omega_i(t)$ is the behavioral motivation value, and b is the constant. In this model, a node becomes tired of repeating this behavior over a continuous period of time. When this behavior does not occur at the node, the probability of recovery is minimal. The probability of recovery is the highest when the nodes continue to behave for b days.

Definition 6. Let δ be the reinfection probability of node i at time t, then

$$\delta = \delta_0. \tag{7}$$

Here, when a node becomes R state, after a period of time, the node is affected by its neighbors and again becomes I state with probability δ_0 .

The differential equation description of the new SIRI propagation model is as follows:

$$\begin{cases} \dfrac{ds(t)}{dt} = -[\beta_0 + \omega_i(t)p_i(t)]si, \\ \dfrac{di(t)}{dt} = [\beta_0 + \omega_i(t)p_i(t)]si - [\gamma_0\omega_i(t)^{|t-b|}]i + \delta\gamma, \\ \dfrac{d\gamma(t)}{dt} = [\gamma_0\omega_i(t)^{|t-b|}]i - \delta\gamma. \end{cases} \tag{8}$$

Here, $s(t)$, $i(t)$ and $r(t)$ are the susceptible, infected, and immune individuals in the network, respectively. $s(t) + i(t) + r(t) \equiv 1(0 < s, i, r <= 1)$ the first item on the right side of the second equation represents that the unknown becomes the propagator with probability $\beta_{(i,t)}$. The second item on the right side of the second equation indicates that the propagator becomes immune with probability $\gamma_{(i,t)}$. The third term of the second equation indicates that the immune becomes the propagator again with probability $\delta_{(i,t)}$. The initial state of the node is S. The initial infection status of the node is as follows:

$$N = S_0 + I_0 + R_0, \tag{9}$$

$$\begin{cases} S_0 = N - I_0, \\ I_0 = [\beta_0 + \omega_i(t)P_i(t)]N, \\ R_0 = 0. \end{cases} \tag{10}$$

Equation (9) represents the initial state of the nodes in the network. In formula (10), S_0, I_0, R_0 are the initial values for the unknown, the propagator, and the immunizer, respectively, and t is the time.

3. Experimental Simulation

3.1. *Related data settings*

In our simulation, the total number of nodes in the BA scale-free network is N and the degree of nodes is K. In the simulation, $N = 20000$, $K = 5$. The value of the constant is $\beta_0 = 0.4$, $\gamma_0 = 0.05$, $\delta_0 = 0.2$, $\omega_0 = 0$, $a = 4$, $b = 7$.

3.2. *Simulation and analysis*

In this section, we do the corresponding experimental verification based on the theoretical and model analysis in the previous section. In the experiment, 120 rounds of simulation were performed on a scale-free network with a total number of nodes of 20,000 and a degree of 5. In the experiment, we divide all nodes into three segments according to the network average degree. By dividing the three segments, the transmission probability, behavioral motivation value, and the change of the influence of the number of neighbor infections of the network were studied. At the same time, we also studied the effect of behavioral motivation value and neighbor infection number impact value on transmission rate.

Firstly, in a scale-free network, the nodes are divided into three segments according to the network average degree to study the propagation probability. From Fig. 2, we can see that the trend of three-segment transmission rate increases first and then decreases. This is because this paper is based on the SIRI model, when infection begins to spread, the increased transmission rate will increase, and after a period of time node infection will recover, so the transmission rate will decrease. At the same time, we can see that one period of the decline in transmission rate was moderate because some immune nodes reinfection in the model would increase the transmission rate and then resume the decline of transmission rate. From the

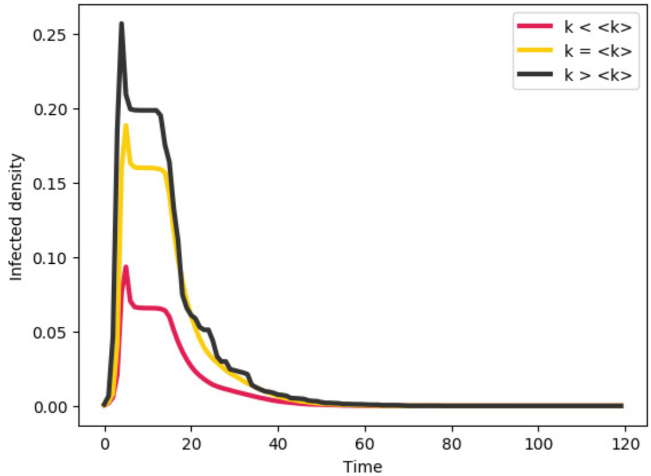


Fig. 2. Propagation rate of nodes.

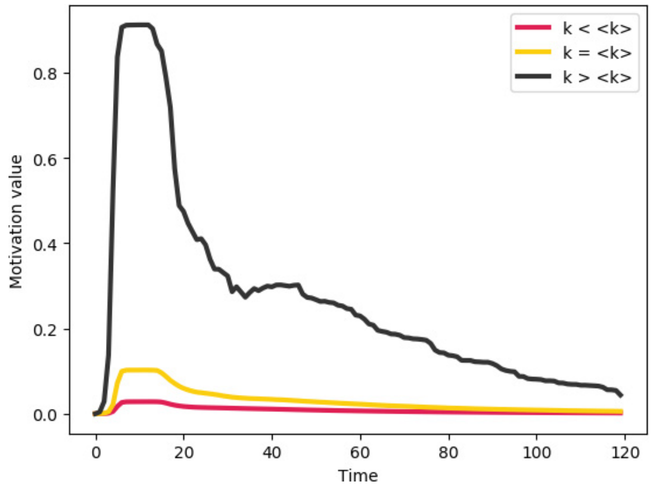


Fig. 3. Behavioral motivation values of nodes.

three different propagation rates in the graph, it can be seen that the degree of a node affects the propagation probability. A node with a larger degree has more neighbors and more propagation capability, so the propagation rate of a node with a larger degree is larger than that of a node with a smaller degree.

Figure 3 shows a behavioral motivation value diagram based on the SIRI model in a scale-free network. Nodes are divided into three segments according to the network average. The graph shows that behavioral motivation values begin to grow, reach their peak, and then begin to decline. According to our formula, this is because at the very beginning, the value of behavioral motivation increases with the value of its own incentive and the influence of neighbors. When this behavior is repeated, the individual becomes bored with it, so the value of motivation decreases. According to the model individual, after a period of boredom, the behavior is affected again, so a slow period occurs when the value of behavioral motivation decreases. It can also be concluded from the difference of the three segments that the node with large degree of behavior motivation value is higher because the value of behavior motivation will be affected by the neighbors, and when there are more nodes around this node, the influence on this node will be greater.

Figure 4 shows the number of neighbor infections in a scale-free network based on the SIRI model. The nodes are divided into three segments according to the average degree of the network. It can be seen from the graph that the influence value of the number of neighbor infections increases first and then decreases. According to our model analysis, the impact value is affected by the number of infections, the number of infections is small at first, and the number of neighbors has a smaller impact value. As the number of infections increases over time, the impact of the number of neighbors will be smaller. It can also be seen from the figure that when the number of infections is in a certain amount, the number of neighbors has a

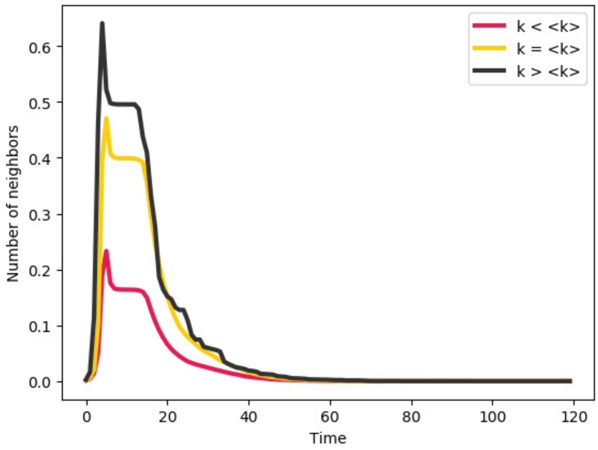


Fig. 4. Influence value of the number of neighbors.

greater impact. This indicates that if the number of infections is too small or too large, the influence value of the number of neighbors is relatively small, and only when the number of infections is a certain amount, the influence value will be large. At the same time, there is a slow decline in the figure, which is because the number of infections increases over time, but some nodes become restored, causing the number of infections to change, so the number of neighbors will fluctuate. Here, we combine our sample campus cartoon consumption data. Analyze the number of people who selected food in each window over a period of time. When this window has fewer people to choose food, that is, fewer infections, fewer people will choose to eat in this window. When the window chooses more food, that is, more infections, fewer people eat in it. When the number of people who choose food in the window is within a certain range, that is, the appropriate number of infections, the number of people who choose to eat in this window will be larger. At the same time, it can be concluded from the difference of three sections that more nodes around the node with large degree of behavior will have a greater impact on the node.

These analyses analyzed the variation of transmission rate, behavioral motivation values, and the number of neighbors affected by infection. Next, we will examine the relationship between behavioral motivation values and the number of neighbor infections on transmission rates. To fully illustrate their relationship, we obtained three-dimensional stereographs of behavioral motivation values and the number of neighbor infections, transmission rates and time, as well as projections of three-dimensional maps, respectively. Here, we also divide the nodes into three segments according to the network average. Figures 5 and 8 are based on degree less than network average, and Fig. 8 is a projection of Fig. 5. Figures 6 and 9 are based on degree equal to network average, and Fig. 9 is a projection of Fig. 6. Figures 7 and 10 are based on degree greater than network average, and Fig. 10 is a projection of Fig. 7.

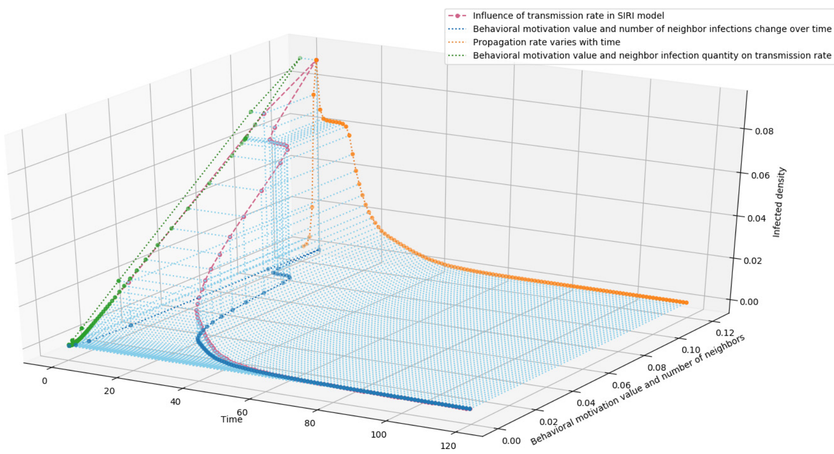


Fig. 5. Three-dimensional graph with degree less than network average.

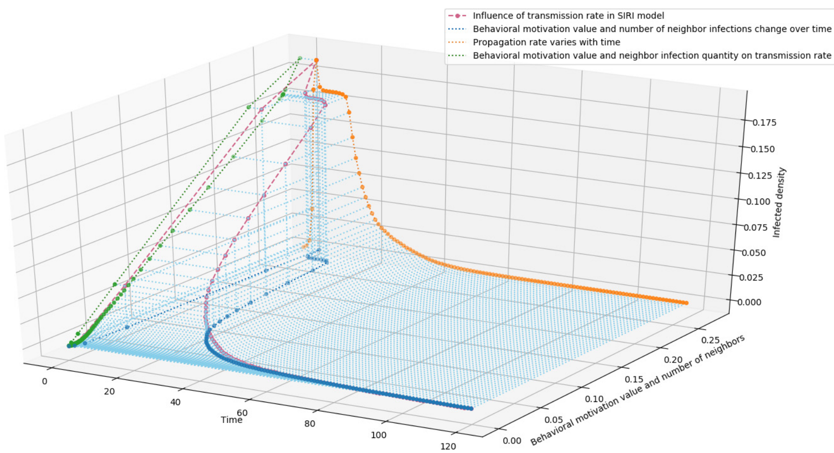


Fig. 6. Three-dimensional graph with degree equal to network average.

Figures 5–7 are all three-dimensional spatial maps. The figure reflects the influence of neighborhood infection number impact value and behavioral motivation value on transmission rate. It can be seen that the number of neighbor infections' impact value and behavioral motivation value increase the transmission rate initially, and then decrease slowly after reaching the maximum value. The difference between the three graphs is that due to the degree difference, the influence value of Fig. 7 is larger than Fig. 6, Fig. 5 is the smallest, the largest node, the more neighbors, and the value of behavioral motivation will increase, which will increase the propagation rate.

The (a) in Figs. 8–10 are projections of the X–Y axis, which reflect the change of the value of behavioral motivation and the number of neighbors' influence over

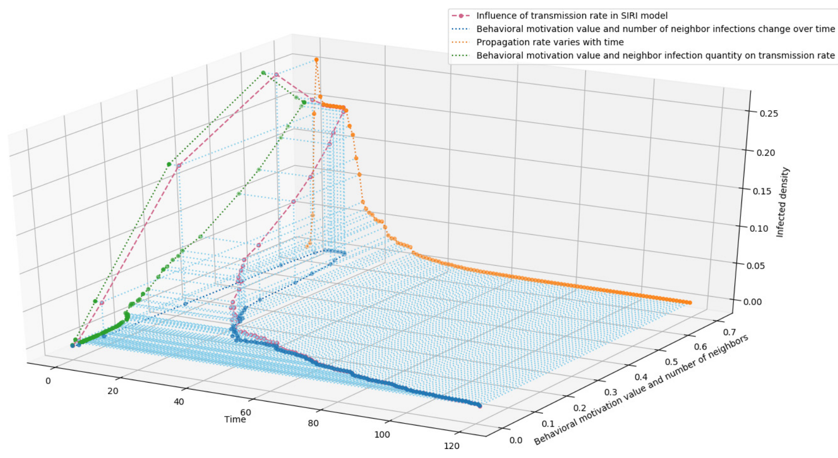
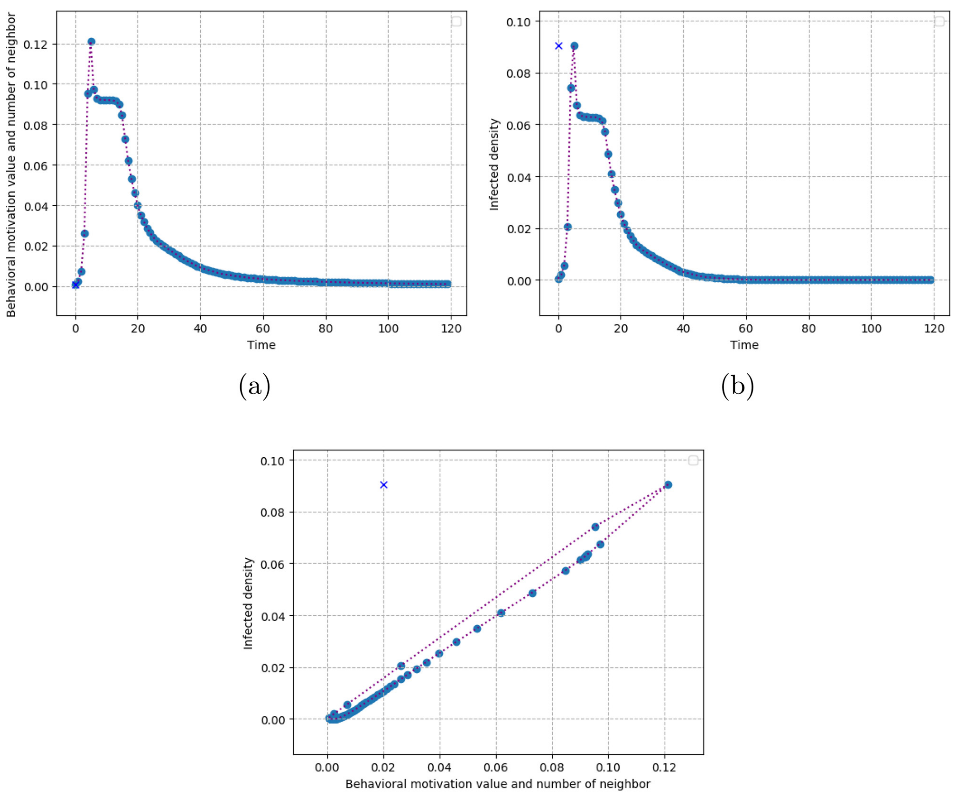


Fig. 7. Three-dimensional graph with degree greater than network average.



(a) (b)

(c)

Fig. 8. (a) is a projection of the X–Y plane, (b) is a projection of the X–Z plane (c) is a projection of the Y–Z plane.

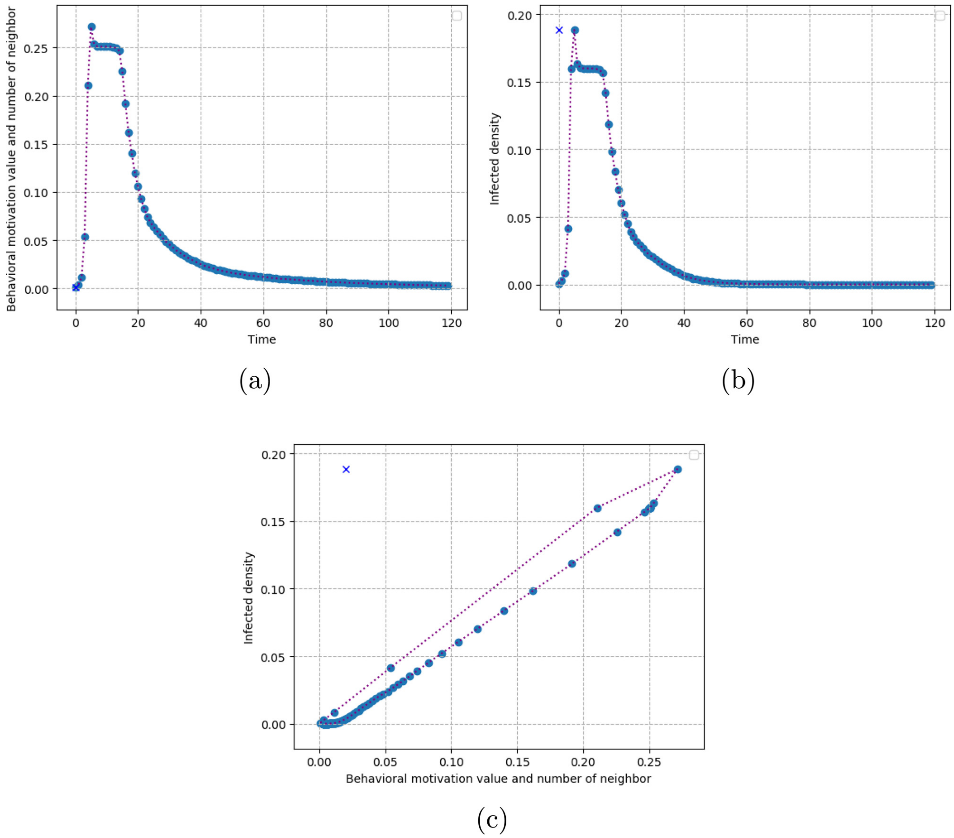


Fig. 9. (a) is a projection of the X–Y plane, (b) is a projection of the X–Z plane, (c) is a projection of the Y–Z plane.

time. It can be seen from the figure that they increase first and then decrease with the development of time. Over time, the number of infections begins to increase, which increases the value of behavioral motivation. Infected nodes become immune nodes after a period of time, which reduces the value of behavioral motivation. Moreover, with the change of time, the number of neighbor infections increases, and the impact value of the number of neighbor infections will increase, but when the number of infections is large, the impact value will become smaller. Therefore, the curve first increases and then decreases. The difference between these three graphs is that, due to the different degrees, the nodes with large degrees, the neighbors will increase the value of behavioral motivation, and the influence value of the number of neighbors infected will also increase. Therefore, the influence value of Fig. 10(a) is larger than that of Fig. 9(a) and that of Fig. 8(a) is the smallest.

The (b) in Figs. 8–10 are projections of X–Z axis, which are graphs of propagation rate over time. The propagation increases first and then decreases with time. The difference between the three graphs is that due to different degrees, the

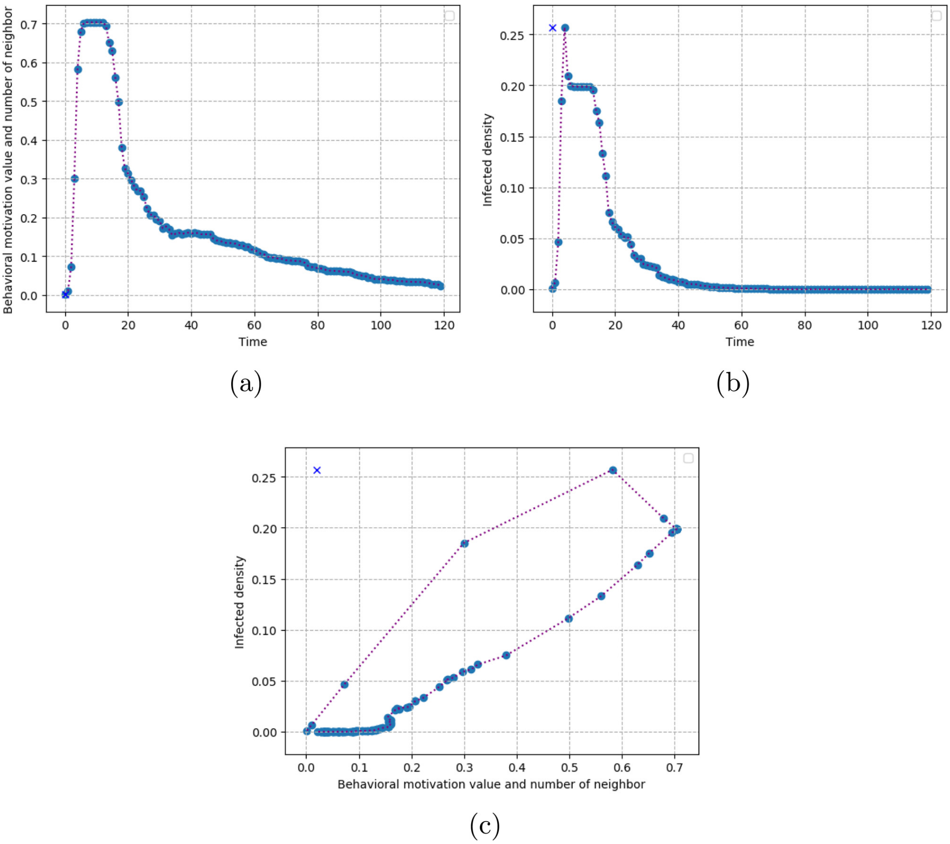


Fig. 10. (a) is a projection of the X–Y plane, (b) is a projection of the X–Z plane, (c) is a projection of the Y–Z plane.

influence value of Fig. 10(b) is greater than that of Fig. 9(b) and Fig. 8(b) is the smallest, which indicates that nodes with higher degree have more neighbors, which will lead to higher node propagation rate.

The (c) in Figs. 8–10 are projections of Y–Z axis, which describe the influence of behavior motivation value and neighborhood number influence value on propagation rate. At the beginning of transmission, the number of infected nodes increased, and the value of behavior motivation increased. With the increase of time, due to the recovery of node infection, the behavior motivation value decreases, and then the transmission rate decreases. At the same time, the influence value of the number of neighbor infection began to increase with the increase of the number of infections. With the increase of time, the number of infections was too much, but the influence value of neighbor infection quantity decreased. Therefore, the transmission rate was first increased and then decreased by the behavior motivation value and the number of neighbor infections. The difference between the three graphs is that the influence value of Fig. 10(c) is greater than that of Fig. 9(c) due to different degrees, and Fig. 8(c) is the smallest. The larger the degree of nodes have more neighbors, the behavior motivation value will increase, and the influence value of neighbor infection number will also increase, so the transmission rate will increase.

In summary, we can see that with the increase of degree, the influence of neighbor infection number impact value and behavior motivation value on transmission rate will increase, indicating that nodes with more neighbors are more likely to spread. In summary of the above experiments, the values of neighborhood influence and behavioral motivation influence the change of transmission rate.

3.3. *Comparison with real cases*

We use Lanzhou Polytechnic University Campus Card Consumption Data, which is "Student of LUT" data in this paper. The main object of study is to change the consumption figures of all windows at the same time. Figure 11 shows the statistical analysis of the data. By analyzing the graph, we find that the influence value of neighbor number in BA scale-free network based on the improved SIRI model is close to the trend of "Student of LUT" data. This result demonstrates that our proposed behavior propagation model is consistent with the actual process. The reason for this is that the real network has the characteristics of scale-free network and the state of the real network has the characteristics of the improved SIRI model. That is, students eat in the dining hall and choose food in the window in accordance with our hypothesis. When the number of people selecting food in the window is too small or too large, the number of people choosing food in the window is small; when the number of people choosing food in the window is appropriate, the number of people choosing food increases. This also corresponds to the fact that when the number of people who select food in the window is too small, students will think

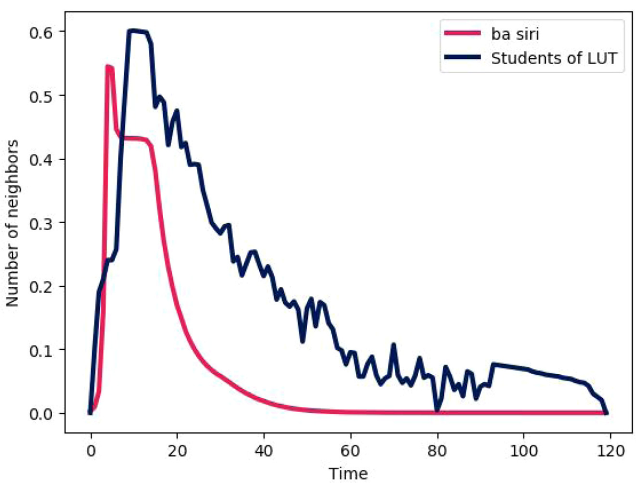


Fig. 11. Comparison of students of LUT neighbor number impact value trend with simulation.

that the food in the window is not popular when they select it, and the number of people who select the window will be small. When there are too many people eating in the window, students will think it is too costly to select the window, so there will be fewer people choosing the window. When the number of people choosing food in this window is small and large, the students will think that popularity and time cost are both most appropriate, so they will choose more people.

4. Conclusion

In this paper, behavioral motivation values and the number of neighbor infections are added to the transmission process, and the two factors are proved to have an impact on the transmission rate. Behavioral motivation values are affected by self-motivation and neighbor influence values, initially increasing, with time changing, immune nodes increasing, resulting in a decrease in behavioral motivation values. Therefore, the effect of behavioral motivation value on transmission rate increases first and then decreases. The number of infected nodes will affect the impact value of the number of neighbor infections, and the impact value of the number of infected nodes will be smaller if there are too few and too many infected nodes. Only if there is a certain number of infections, the number of neighbors will have a greater impact. When there are very few people and too many people around to do this, then there will be fewer people to do it. Therefore, the transmission rate is affected by these two factors, and increases first and then decreases with time. Moreover, the greater the degree of infection nodes and the faster the behavior spread during the transmission process, which also verifies the star effect.

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References

1. J. Gao, Z. Jia and J. Hao, *J. Guilin Univ. Technol.* **38**(001) (2018) 160.
2. F. Nian and H. Diao, *IEEE Trans. Netw. Sci. Eng.* **7**(3) (2020) 1394.
3. C. M. McBride *et al.*, *Amer. J. Prevent. Med.* **38**(5) (2010) 556.
4. D. Acemoglu and A. Ozdaglar, *Dyn. Games Appl.* **1**(1) (2011) 3.
5. F. Amblard and G. Deffuant, *Phys. A: Statist. Mech. Appl.* **343** (2004) 725.
6. V. Capraro and M. Perc, *Front. Phys.* **6** (2018) 107.
7. J. Lorenz, *Physics* **18**(12) (2008) 2007.
8. J. Lorenz, *Phys. A: Stat. Mech. Appl.* **355**(1) (2007) 217.
9. Q. Feng *et al.*, *Comput. Syst. Appl.* **027**(001) (2018) 28.
10. B. Lin *et al.*, *Athens World Sci. Eng. Acad. Soc.* (2008) 52.
11. X. Luo *et al.*, *Future Gen. Comput. Syst.* **93** (2019) 1023.

12. T. Mori *et al.*, Behavior prediction based on daily-life record database in distributed sensing space, *IEEE/RSJ Int. Conf. Intelligent Robots & Systems* (IEEE, 2005), pp. 1703–1709.
13. Y. Quan *et al.*, *J. Comput. Sci.* **28** (2018) 217.
14. R.-F. Song, *J. Tongren Voc. Tech. Coll.* **3** (2008) 72.
15. M. Boguñá, C. Castellano and R. Pastor-Satorras, *Phys. Rev. Lett.* **111** (2013) 068701.
16. Z. Jing and Z. Ling, *J. Beijing Univ. Aeronaut. Astronaut.* **27**(1) (2014) 76.
17. M.-S. Lee, *J. Korea Soc. Comput. Inf.* **21**(10) (2016) 43.
18. S. Li, Q. Pan and X. Zhuang, *J. Natl. Lib. China* **35**(3) (2015) 410.
19. M. A. Guang-Qi and Z. Lin-Yun, *J. Heb Univ. Sci. Technol.* **03** (2008) 9.
20. K. Sahar, A Purchase Decision-Making Process Model of Online Consumers and Its Influential Factora Cross Sector Analysis (University of Manchester, 2013).
21. Q. Kong and X. Liang, *Int. Rev. Comput. Soft.* **6**(7) (2011) 1320.
22. S. Senecal, P. J. Kalczynski and J. Nantel, *J. Business Res.* **58**(11) (2005) 1599.
23. J. R. Alberts, *Dev. Psychobiol.* **49**(1) (2010) 22.
24. D. I. Shapiro-Ilan, E. E. Lewis and P. Schliekelman, *Int. J. Parasitol.* **44**(1) (2014) 49.
25. N. De Lay and S. Gottesman, *J. Bacteriol.* **191**(2) (2009) 461.
26. N. Tausch *et al.*, *J. Pers. Soc. Psychol.* **101**(1) (2011) 129.
27. W. Guan *et al.*, *Phys. A: Stat. Mech. Appl.* **395** (2014) 340.
28. S. Kim, J. Y. Bak and A. H. Oh, Discovering emotion influence patterns in online social network conversations, *Int. Conf. Ubiquitous Intelligence & Computing & Int. Conf. Autonomic & Trusted Computing* (IEEE, 2012).
29. X. Lin, K. A. Lachlan and P. R. Spence, *Comput. Human Behav.* **65** (2016) 576.
30. H. Liu and Y. Li, *Math. Probl. Eng.* **2015**(9) (2015) 876218.1.
31. L. Hou *et al.*, *Sci. Rep.* **4** (2014) 6560.
32. K.-Y. Jung and J.-H. Lee, *IEICE Trans. Inf. Syst.* **87**(12) (2004) 2781.
33. F. L. Barroso *et al.*, *Landscape Urban Plan.* **104**(3–4) (2012) 329.
34. S. Li *et al.*, On market-based coordination of thermostatically controlled loads with user preference, in *Proc. IEEE Conf. Decision Control*, Vol. 2015, 2015, pp. 2474–2480.